



Data assimilation in meteorology

Loïk Berre
Météo-France/CNRM

Plan of the talk

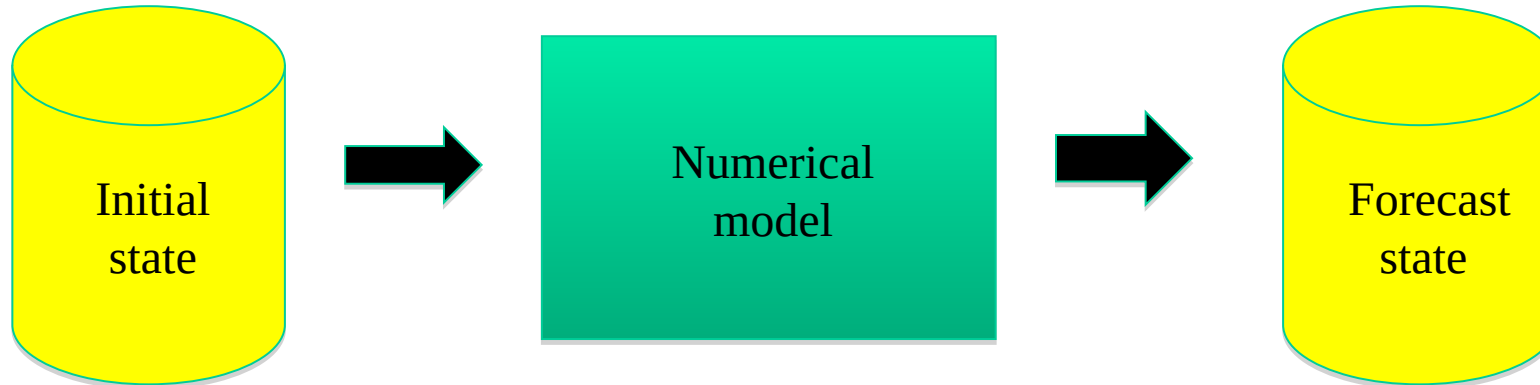
- Numerical Weather Prediction (NWP),
Data Assimilation (DA)
- Observations (in-situ and remote sensing)
- Error covariances : estimation and modelling



1. Numerical Weather Prediction and Data Assimilation

Numerical Weather Prediction

Numerical resolution of fluid mechanics equations (computer code), to **forecast the atmospheric evolution** from an **estimated initial state** (which is called the « analysis »).



Vigilance météorologique

La carte est actualisée au moins 2 fois par jour, à 6h et 16h.

- Une vigilance absolue s'impose des phénomènes dangereux d'intensité exceptionnelle sont prévus...
- Soyez très vigilant, des phénomènes dangereux sont prévus...
- Soyez attentif si vous pratiquez des activités sensibles au risque météorologique...
- Pas de vigilance particulière.

Vent violent	Neige-verglas
Pluie-inondation	Inondation
Orages	Vagues-submersion
Grand Froid	Avalanches

Les vigilances pluie-inondation et inondation sont élaborées avec le réseau de prévision des crues du Ministère du Développement durable



Diffusion : le lundi 30 janvier 2012 à 22h31
Validité : jusqu'au mardi 31 janvier 2012 à 16h00
Actualise la carte du lundi 30 janvier 2012 à 19h06

Consultez le [bulletin national](#)

Episode neigeux notable en cours des Pays de la Loire au Poitou et au Massif Central, et gagnant mardi en tout début de matinée Rhône-Alpes et PACA.

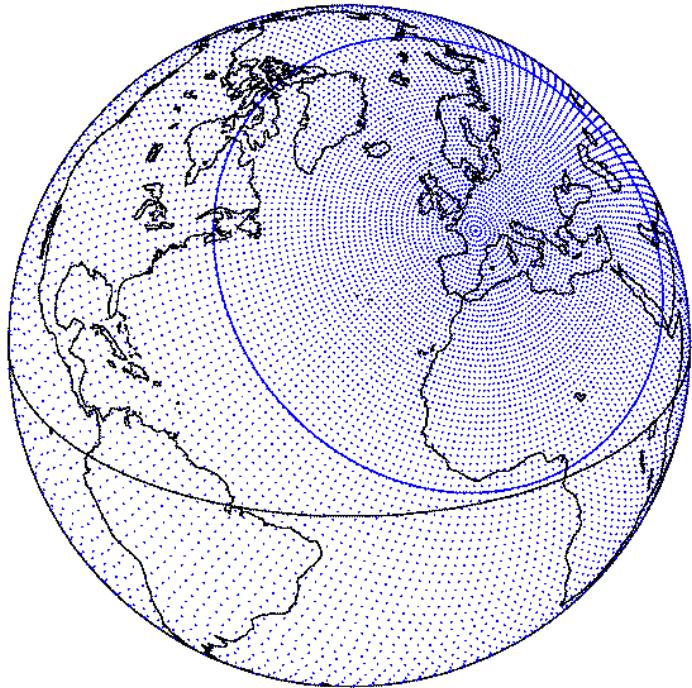
Cliquez sur la carte pour lire les [bulletins régionaux](#)

Conseils des pouvoirs publics :
Neige-Verglas/Orange - Soyez très prudents et vigilants si vous devez absolument vous déplacer. Renseignez-vous sur les conditions de circulation. - Respectez les restrictions de circulation et déviations. Prévoyez un équipement minimum en cas d'immobilisation prolongée. - Si vous devez installer un groupe électrogène, placez-le impérativement à l'extérieur des bâtiments. - N'utilisez jamais des chauffages d'appoint à combustion en continu.

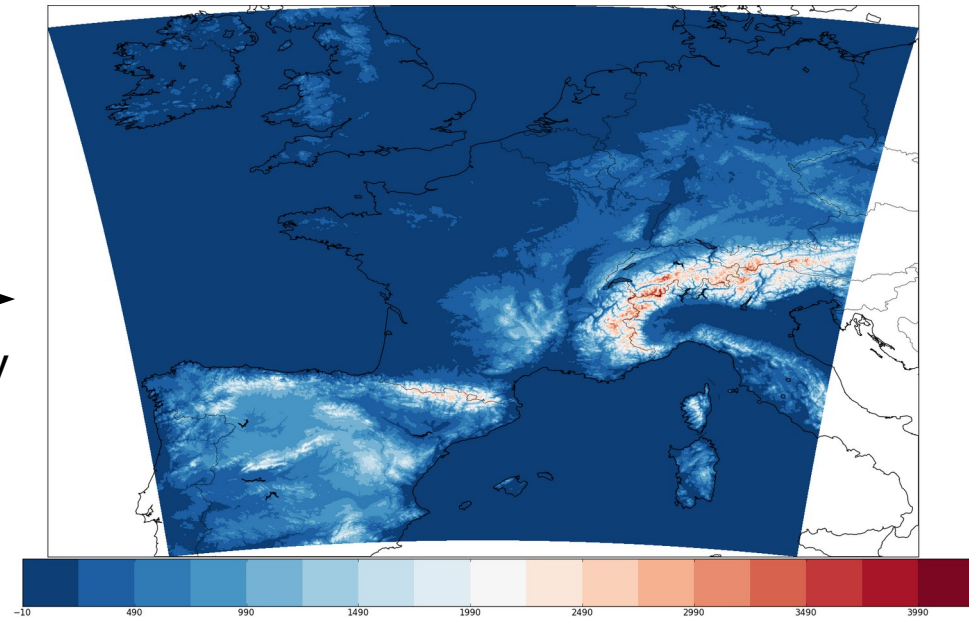
31 départements en Orange.

METEO FRANCE
Toujours un temps d'avance

NWP models at Météo-France (in collaboration with e.g. ECMWF)



Lateral Boundary
Conditions

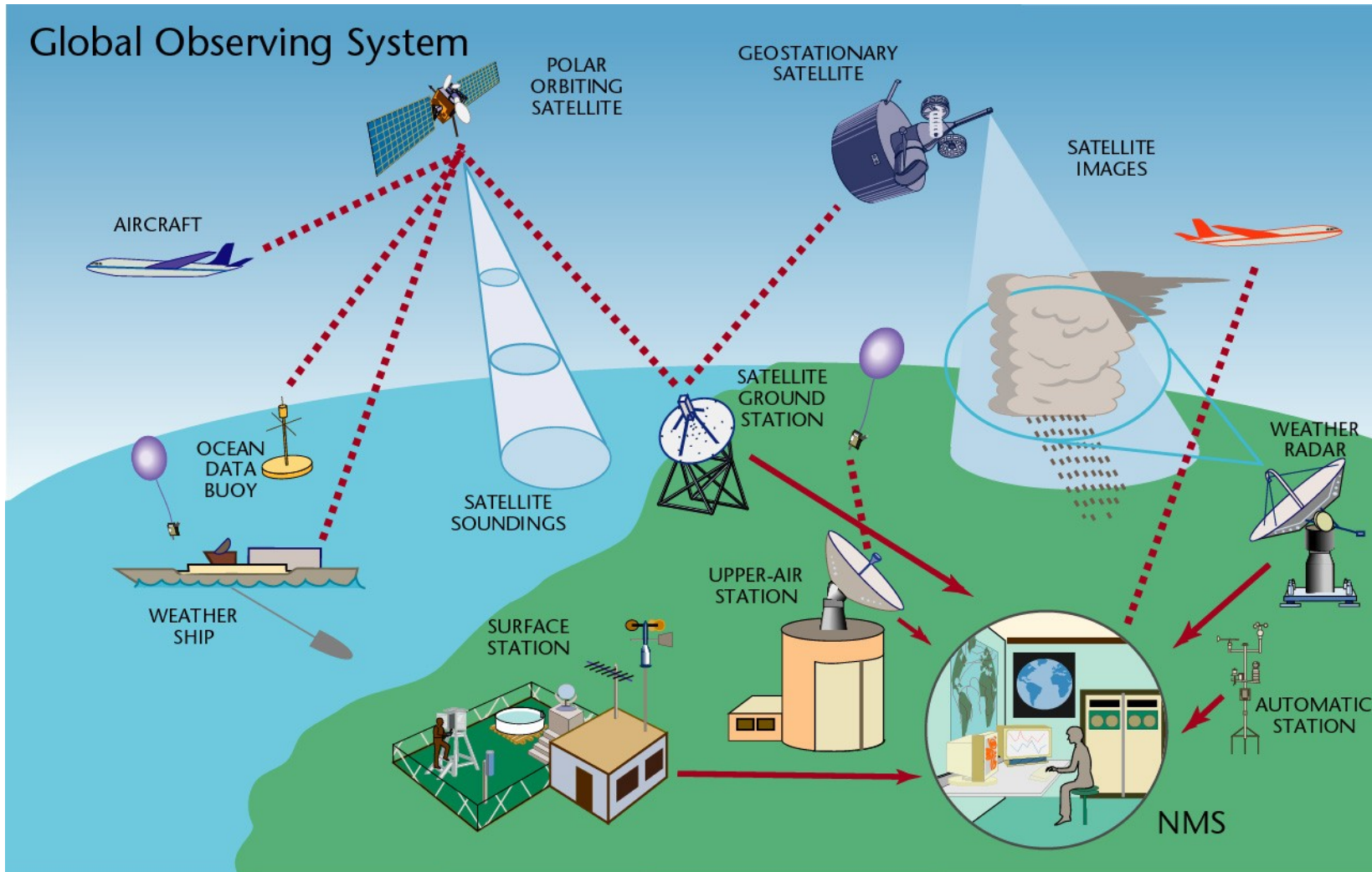


ARPEGE (5 km - 30 km)
 10^9 model variables

AROME (1.3 km)
 $1,4 \times 10^9$ model variables

Equations of dynamics and physical parametrizations (radiation, convection, ...) to predict the evolution of temperature, wind, humidity, etc.

Data which are assimilated in NWP models



ARPEGE

10^9 model variables

5×10^6 observations / 6h

90 % satellite

Computation time :

40min (over 6h window)

AROME

$1,4 \times 10^9$ model variables

2×10^5 observations / 6h

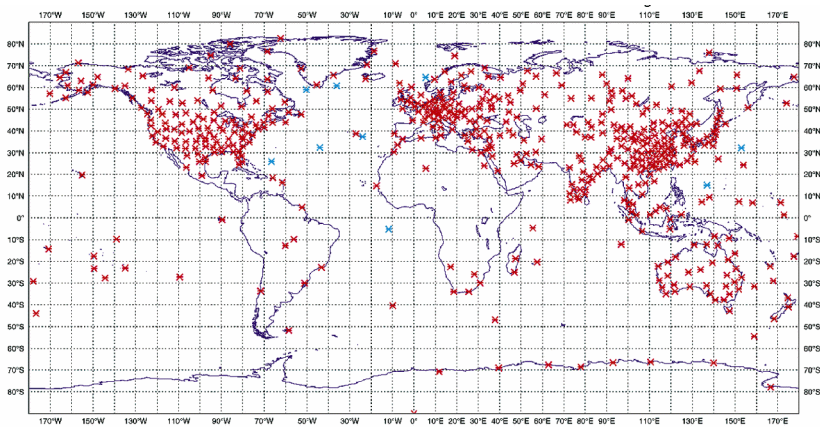
Up to 75 % radar, 10 % satellite

Computation time :

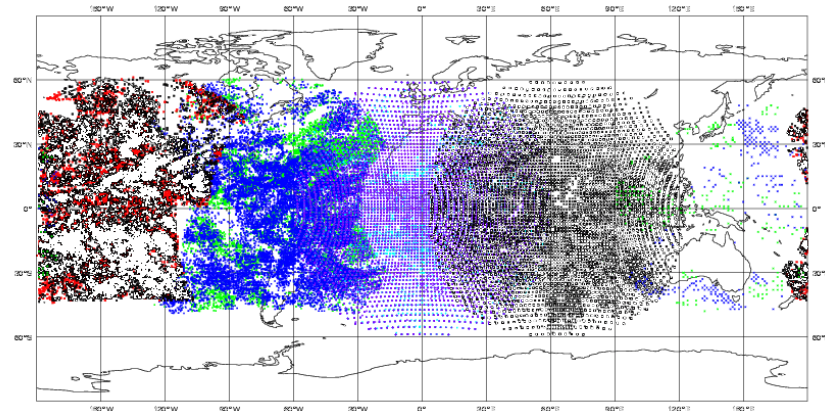
7 min (over 1h window)

Spatial coverage and density of observations

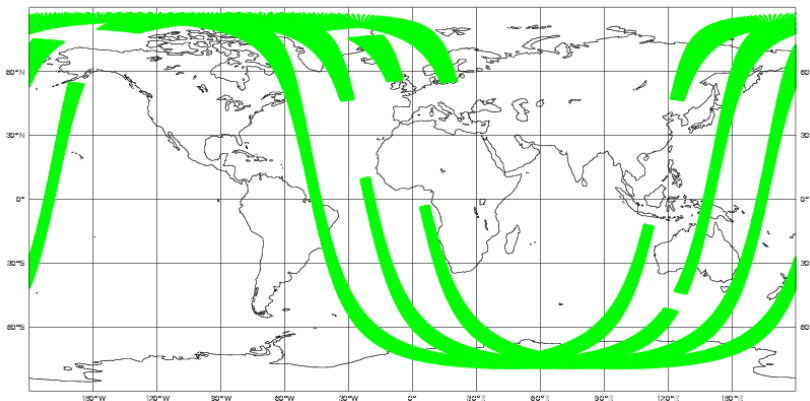
RADIOSONDE DATA



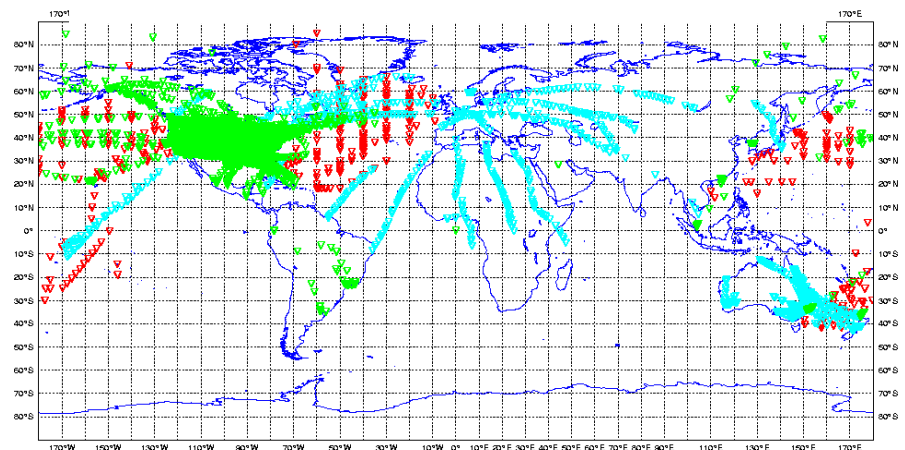
GEOSAT. WINDS



SCATTEROMETER

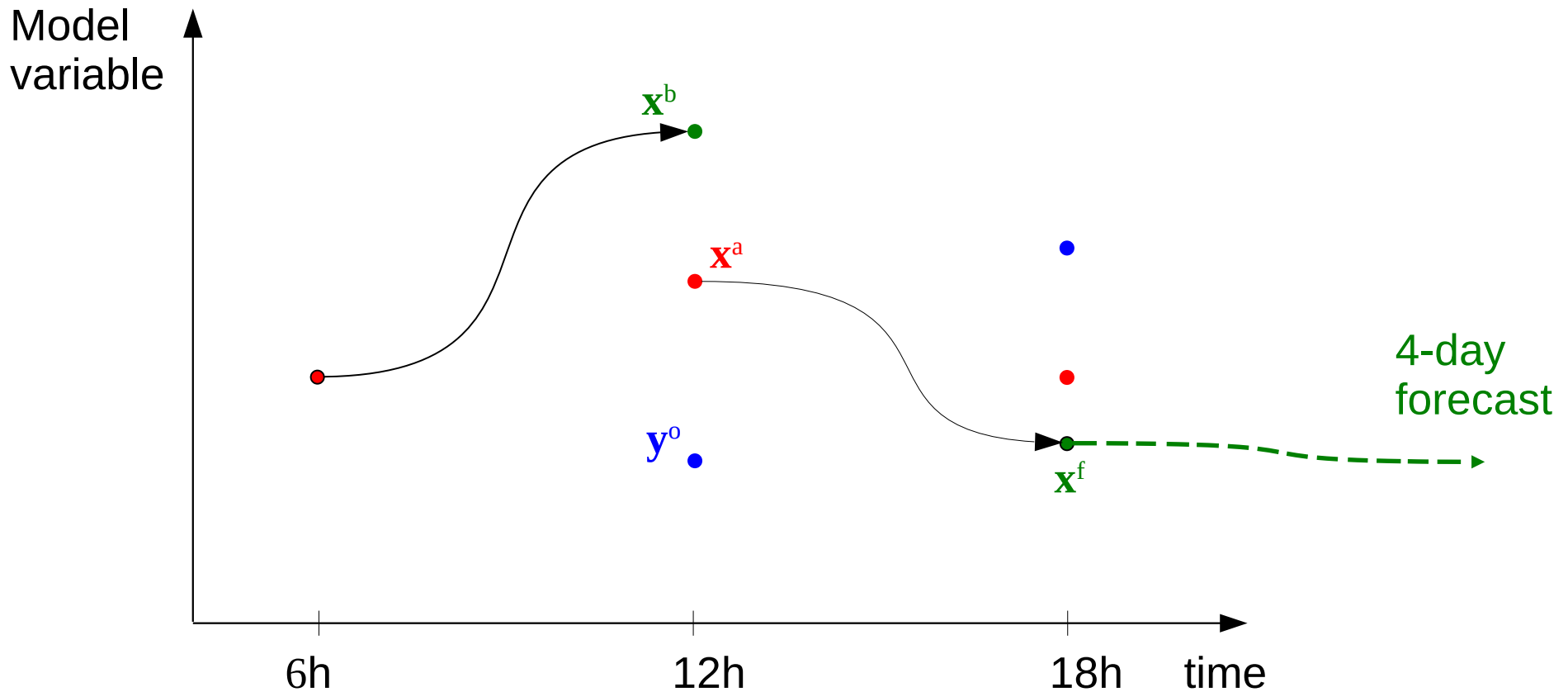


AIRCRAFT DATA



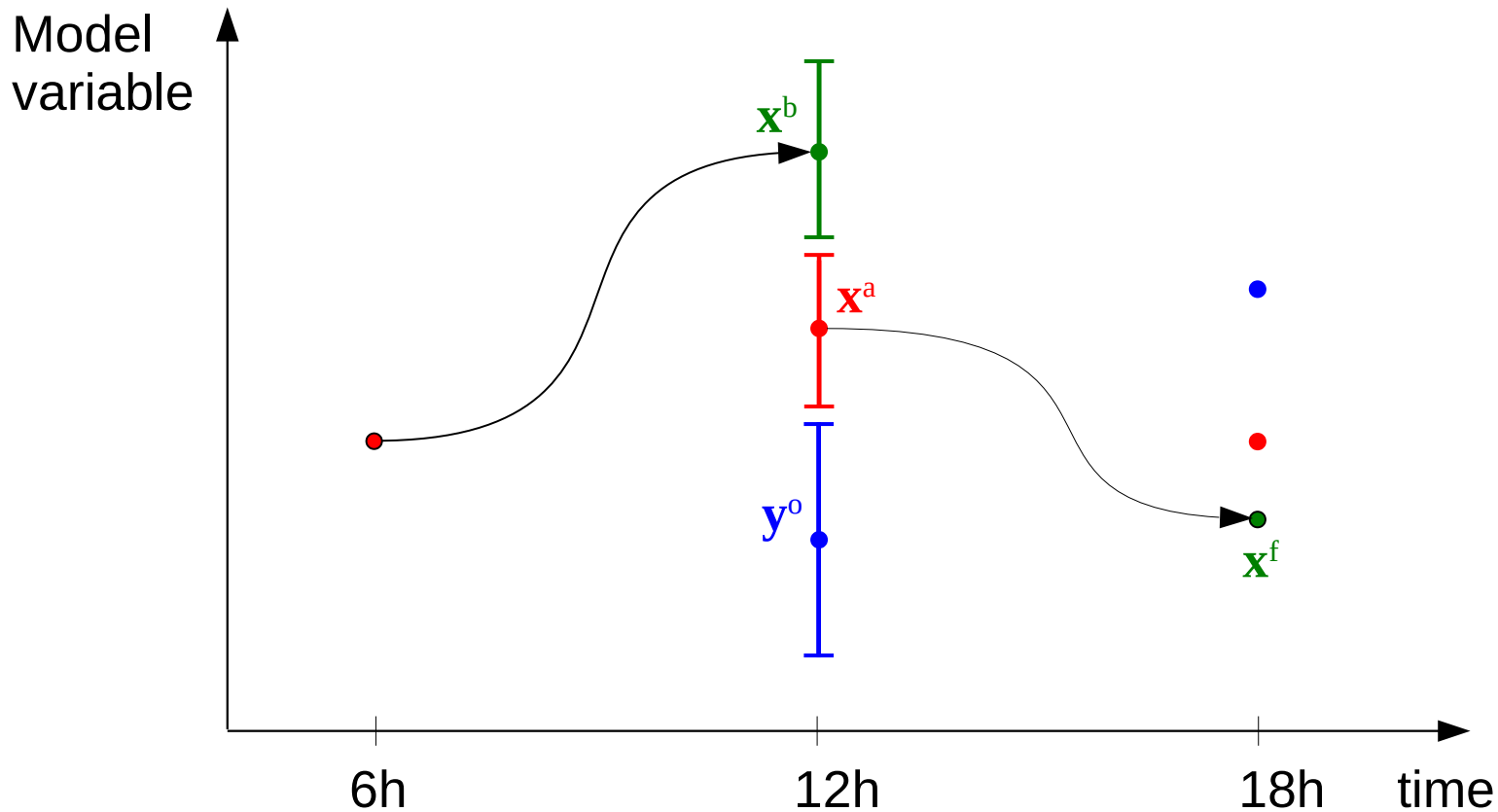
Observations are distributed in time

(ex : radiosondes every 6h) \Rightarrow **Data assimilation cycling** :
temporal succession of analysis and forecast steps



\Rightarrow The model state is propagated and then updated, e.g. every 6h :
the memory of DA system, contained in **the background**,
is updated ~ regularly in time with new observations, through this DA cycling.

Uncertainties in observations, background, analysis



Uncertainties are often measured by error variances
(ex : accurate observation \leftrightarrow small observation error variance).

Analysis equation

■ BLUE formalism : $\mathbf{x}^a = \mathbf{x}^b + \mathbf{K} (\mathbf{y}^o - H[\mathbf{x}^b])$

■ H = non linear observation operator

= projection from model space to observation space : $\mathbf{y} = H[\mathbf{x}]$

H ~ spatial interpolation : from model gridpoints to obs locations (ex: for radiosondes)

H ~ radiative transfer : from model temperature to satellite radiances ;

H ~ NWP model: for observations available at different times within 6h DA window.

■ \mathbf{K} ~ observation weights : $\mathbf{K} = \mathbf{B} \mathbf{H}^T (\mathbf{H} \mathbf{B} \mathbf{H}^T + \mathbf{R})^{-1}$

with \mathbf{H} = tangent linear version of H ,

\mathbf{B} = background error covariance matrix,

\mathbf{R} = observation error covariance matrix.

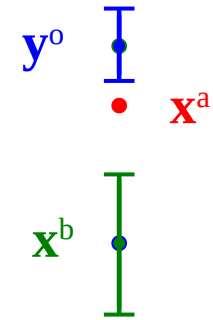
=> \mathbf{K} accounts for relative accuracy of observations, and for amplitudes &

spatial structures of background errors. => $\mathbf{y}^o - H[\mathbf{x}^b]$ are filtered & propagated.

Background error covariances : filtering and propagation of $y^o - H(x^b)$

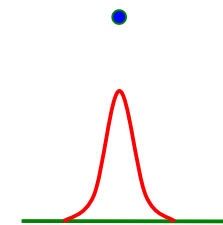
- Variances

- Weighting/filtering of observations.



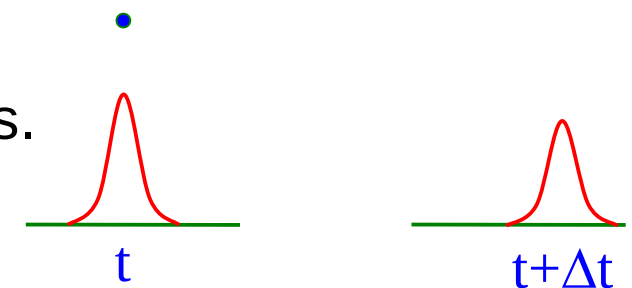
- 3D spatial correlations

- Spatial propagation of observations.
- Spatial coherence of analysis.

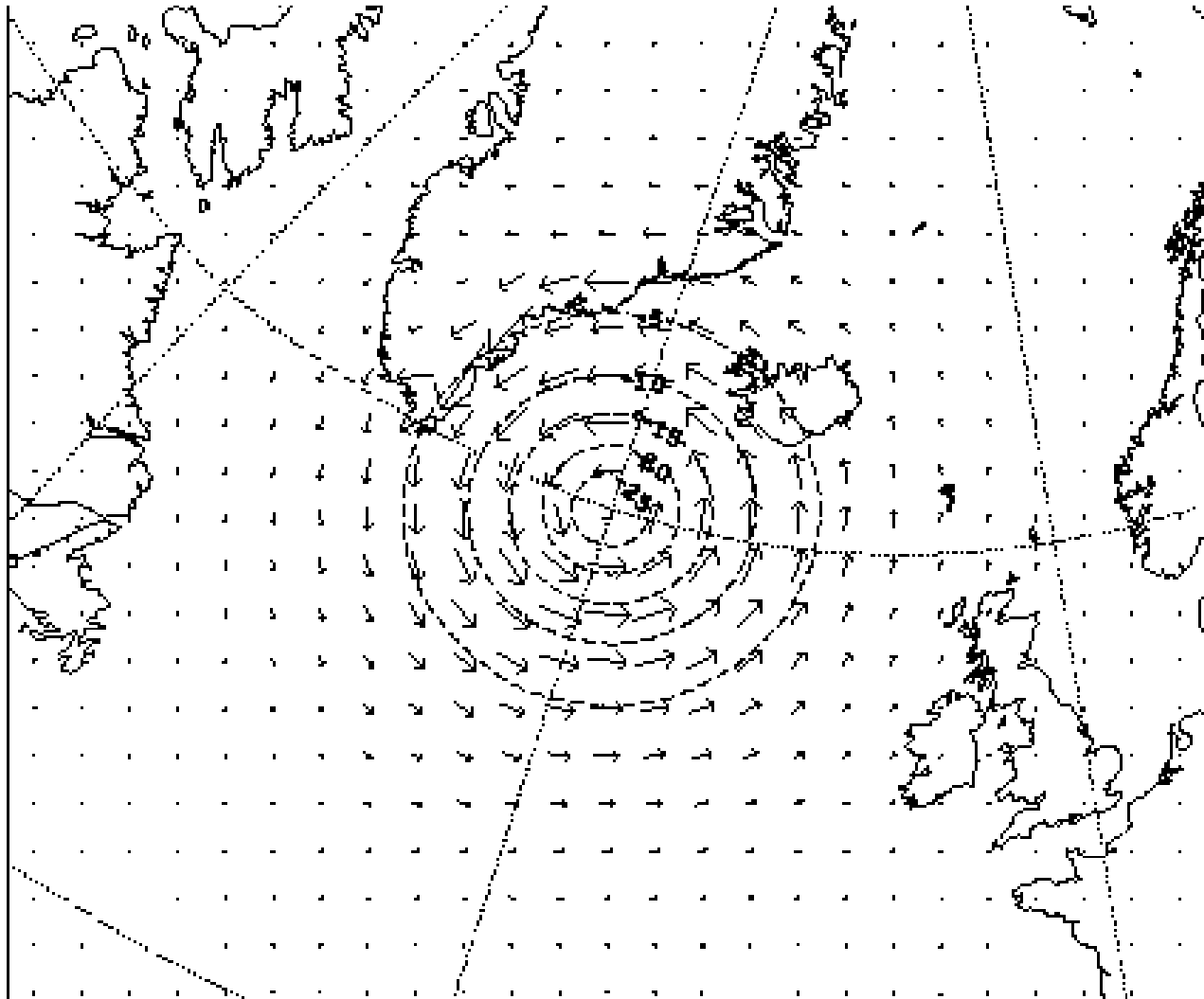


- 4D spatio-temporal correlations

- Spatial and temporal propagation of observations.
- Spatial and temporal coherence of trajectory.



Impact of one surface pressure observation on the pressure and wind analysis (2D)

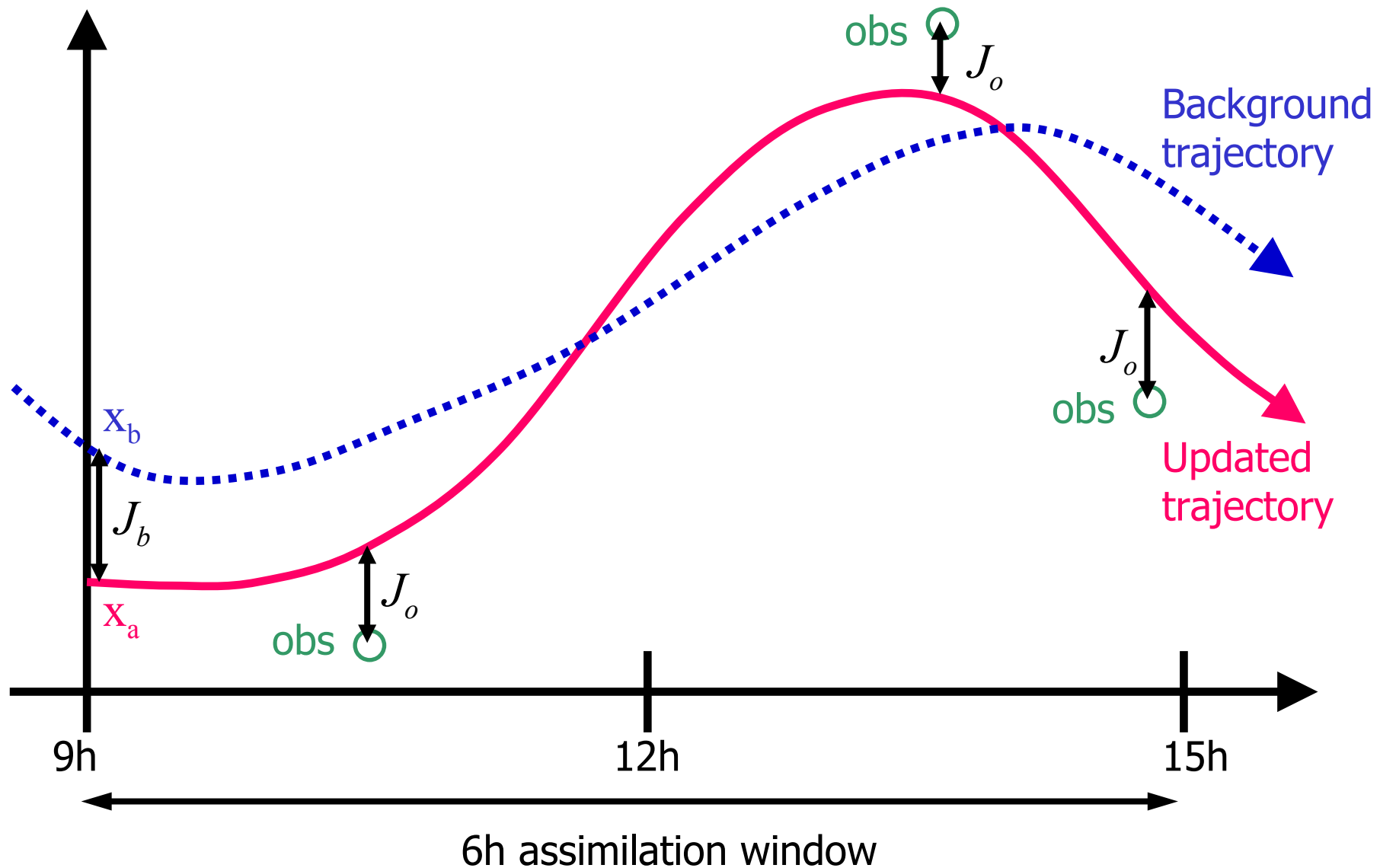


B contains & provides information about typical scales in the atmosphere and mass/wind couplings (such as geostrophy)

Variational analysis

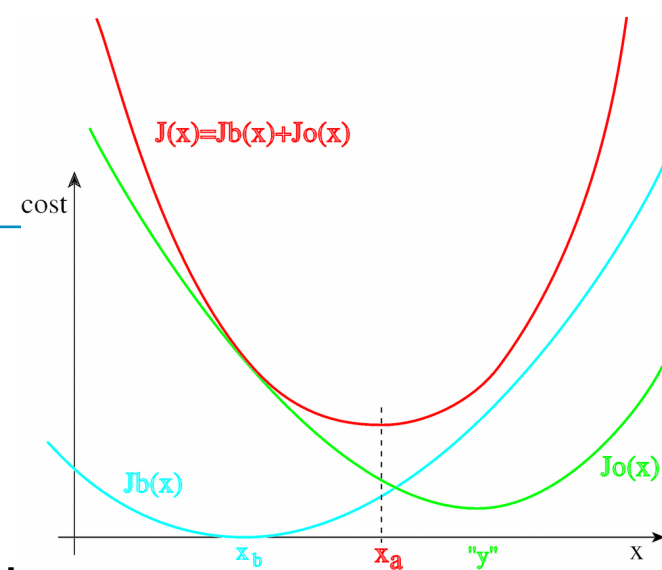
- Size of \mathbf{B} is huge : square of model size $\sim (10^9)^2 = 10^{18}$
=> error covariances need to be estimated, simplified and modelled.
- The matrix $(\mathbf{H} \mathbf{B} \mathbf{H}^T + \mathbf{R})$ in $\mathbf{K} = \mathbf{B} \mathbf{H}^T (\mathbf{H} \mathbf{B} \mathbf{H}^T + \mathbf{R})^{-1}$
is too large to be explicitly inverted.
=> minimize distance $J(\mathbf{x}^a)$ to \mathbf{x}^b and \mathbf{y}^o (variational assimilation),
without explicit matrix inversions (e.g. Talagrand and Courtier 1987).
- Some (weakly) non linear features are accounted for
in calculation of departures $\mathbf{y}^o - H(\mathbf{x}^b)$ (e.g. non linear radiative transfer),
and by updating the non linear trajectory in 4D-Var (non linear dynamics).

How can we handle observations distributed in time within a 6h window ? \Rightarrow Principle of 4D-Var assimilation



x_b at beginning ; use M to propagate x_b and to compare it with observations (M is part of H) ;
minimisation of $J=J_b+J_o$ allows an updated trajectory (x_a) to be computed,
consistent with observations at different times.

Implementation of 4D-Var



Variational formulation :

$$\text{cost function } J(\mathbf{x}^a) = \|\mathbf{x}^a - \mathbf{x}^b\|_{\mathbf{B}^{-1}}^2 + \|H(\mathbf{x}^a) - \mathbf{y}^o\|_{\mathbf{R}^{-1}}^2$$

minimised when gradient $J'(\mathbf{x}^a)=0$ (equivalent to BLUE).

- Computation of gradient J' : development and use of adjoint operators (i.e. transpose of tangent-linear operators).
- Generalized observation operator H : includes NWP model M , in order to assimilate observations distributed in time over a 6h window.
- Reduction of computation cost : analysis increment $\delta\mathbf{x} = \mathbf{x}^a - \mathbf{x}^b$ can be computed at low resolution (Courtier et al 1994).



2. In-situ observations and remote sensing data

Observation networks in meteorology : in situ measurements



- * Direct measurements of temperature, wind, humidity.
- * Relatively easy to compare with the model, and to assimilate.
- * High quality data, with relatively small biases.
- * Poor horizontal coverage over the globe (ex : South Hemisphere ; oceanic areas).

Geostationary satellites

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Fix position / earth, at 36 000 km height, above equator.

Same area of the globe (disk) is always observed.

□ Advantages

Very high temporal resolution (~ 15 min).

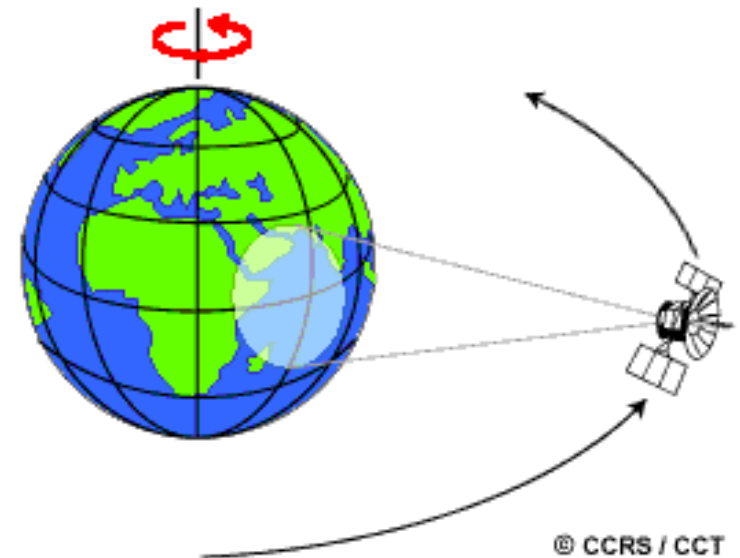
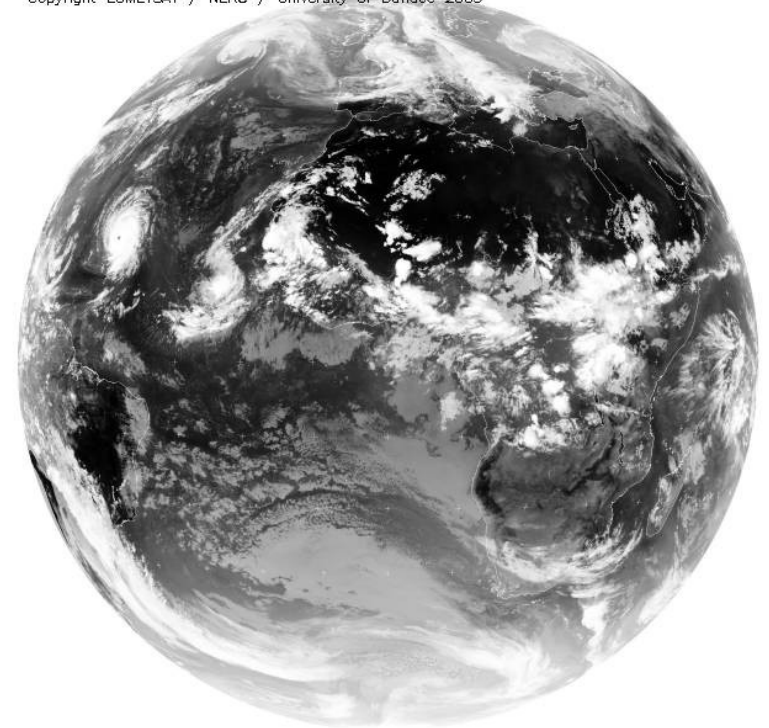
Useful for nowcasting
(= very short range forecasts, e.g. within the next 2 hours).

Dynamics of meteorological structures
(e.g. fronts, tropical cyclones).

□ Drawbacks

Insufficient spatial coverage of 1 satellite :
several satellites are needed to cover the whole globe.

Not adapted to polar regions, due to position.



Polar orbiting satellites

Low orbit satellites (800 km height) :

▣ Advantages

High spatial resolution (~ 10 km).

Global spatial coverage (twice a day)

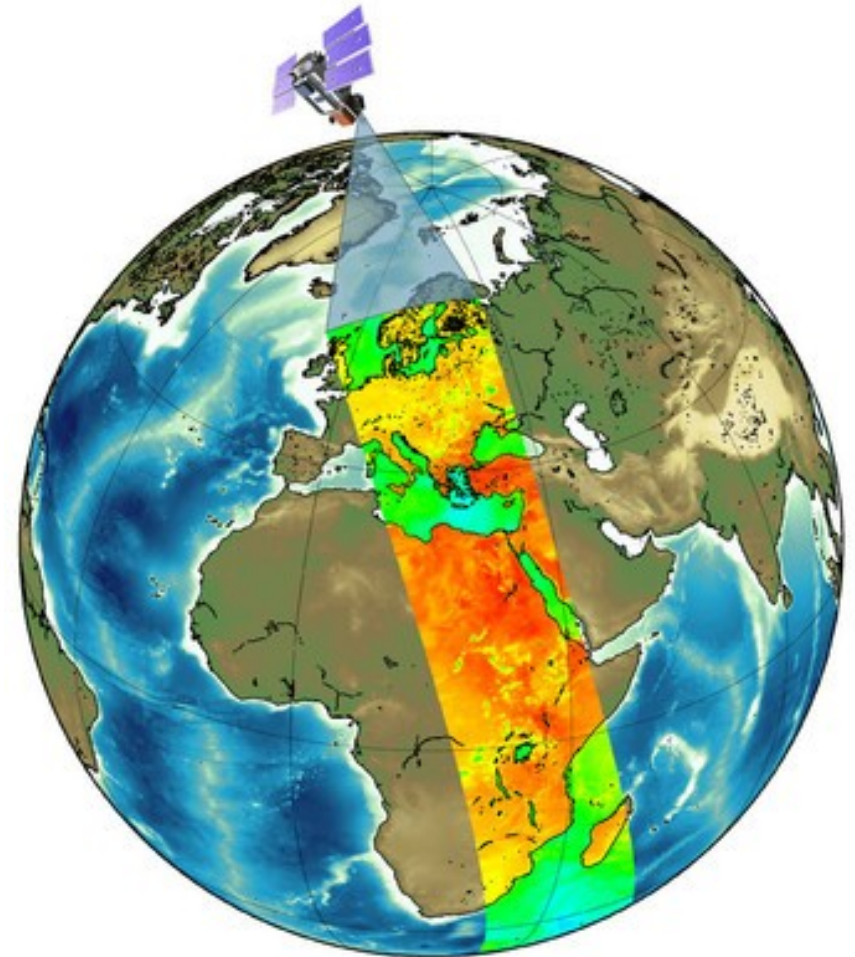
Sounding instruments
(over several vertical layers) :

\sim vertical profiles of T at different locations

▣ Drawbacks

Insufficient temporal resolution :
a given location is only observed every 12h

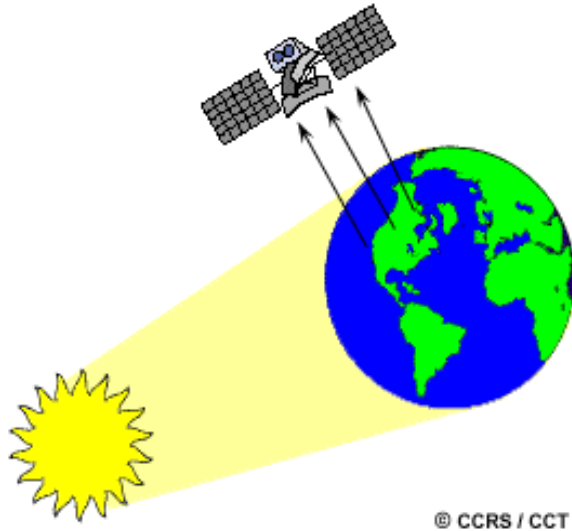
(several satellites are needed, to have
frequent observations over the same area)



Two types of satellite measurements

Passive measures

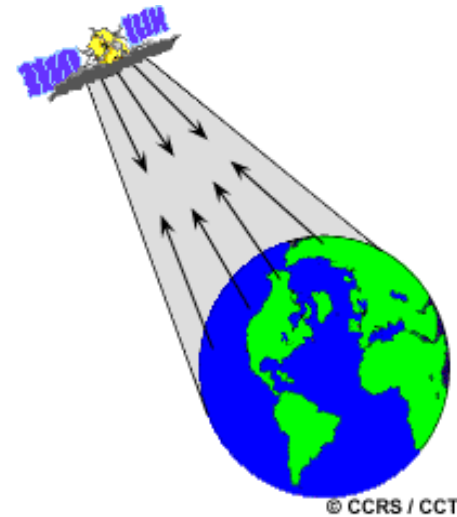
(no energy is emitted from instrument)



Measures natural radiation emitted by Earth or Atmosphere (with Sun origin)

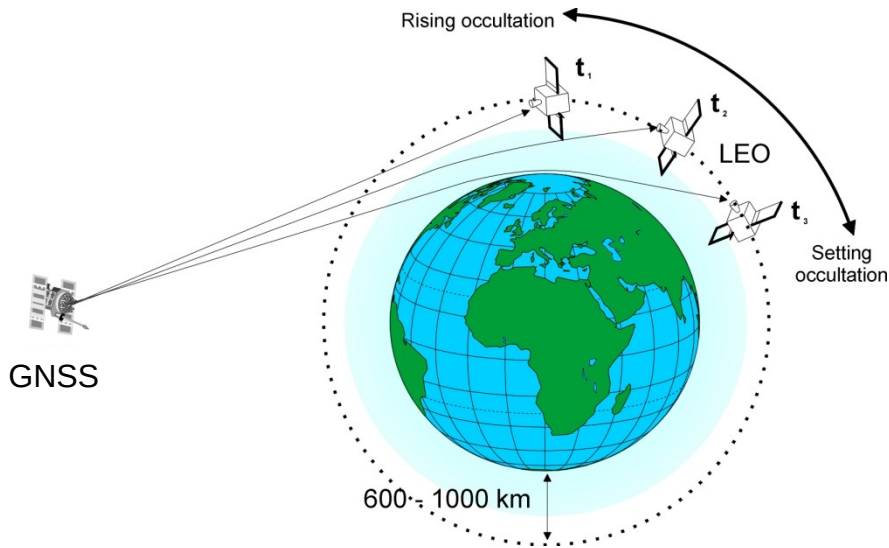
Active measures

(energy is emitted from instrument)

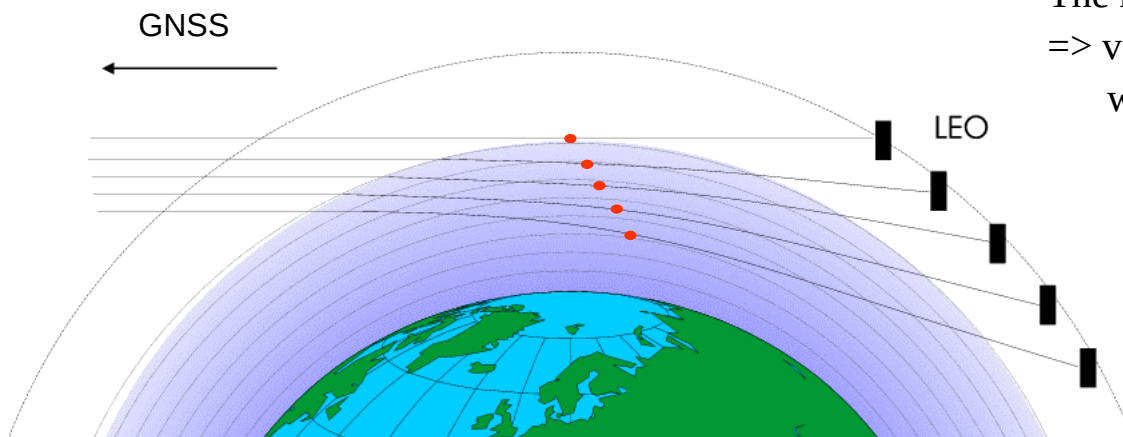


Measures radiation emitted by satellite and then reflected or diffused by Earth or Atmosphere

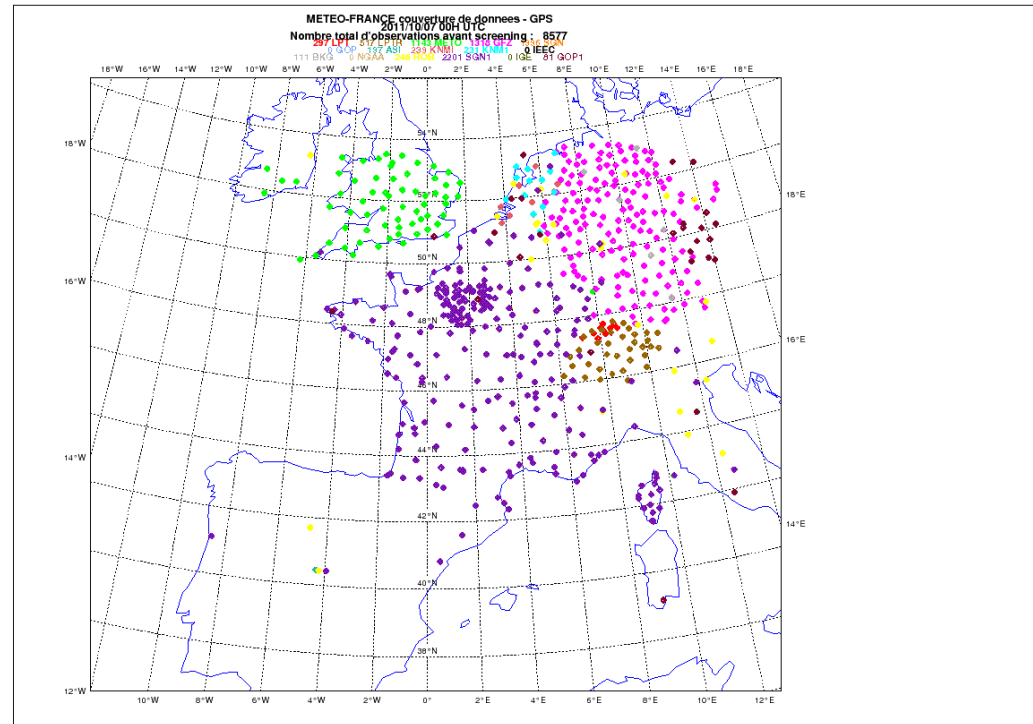
GNSS radio-occultation data (1st example of active remote sensing)



- GNSS is the Global Navigation Satellite System = GPS (USA) or Galileo (Europe).
- Low-Earth Orbit (LEO) satellites receive a signal emitted by a GNSS satellite.
- The GNSS signal passes through the atmosphere and it gets refracted along the way.
- The magnitude of the refraction depends on temperature, moisture and pressure near the tangent point (in red) of the path.
- The relative position of GNSS and LEO changes over time => vertical scanning of the atmosphere, with information on temperature and humidity.



Data from ground-based receiver stations of GNSS (2nd example of active remote sensing)

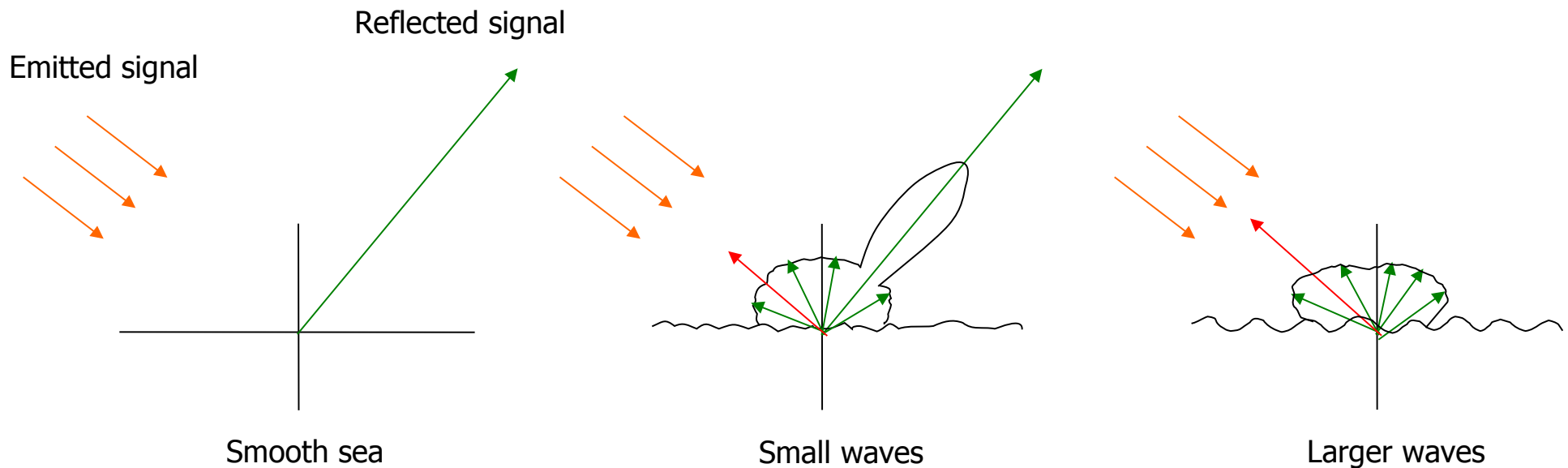


- Propagation of GNSS signal is slowed by atmosphere (dry air and water vapour) : the propagation delay provides information about humidity in particular.
- More than 900 GNSS stations over Europe provide an estimation of Zenith Total Delay (ZTD) in real time to weather centres.
 - “All weather” instrument (e.g. for either dry or rainy conditions) ;
 - High temporal resolution (=> follow dynamics of convective developments).

Scatterometers

They send out a microwave signal towards a sea target.

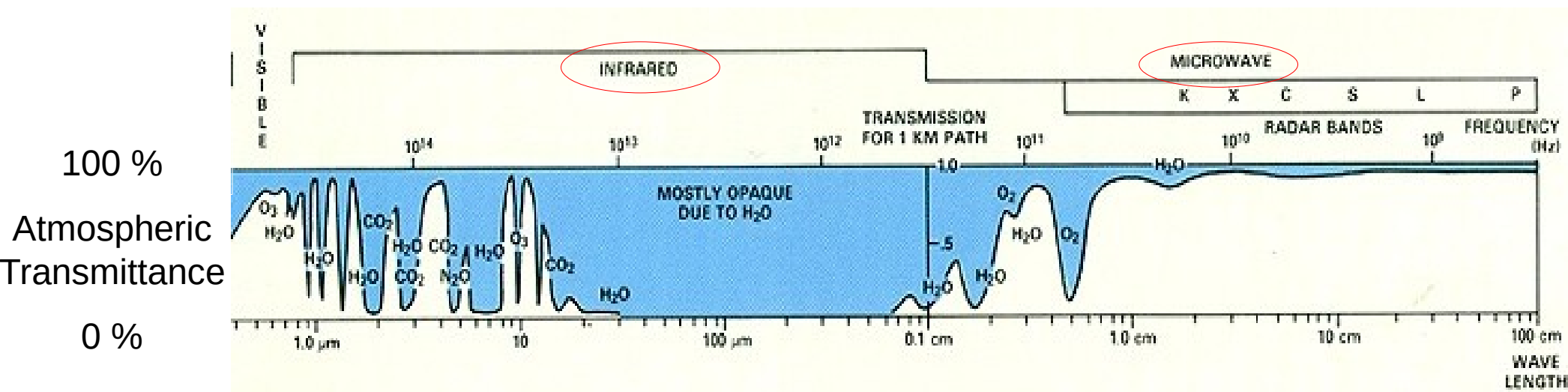
The fraction of energy returned to the satellite depends on wind speed and direction.



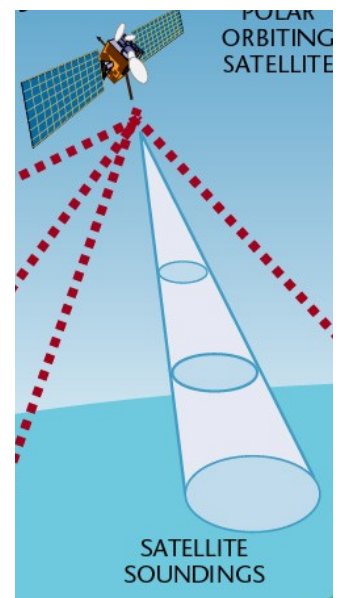
=> Measurements of near surface wind over the ocean,
through backscattering of microwave signal reflected by waves.

Passive remote sensing : what is measured by satellite sensors ?

- Sensors do not measure directly atmospheric temperature and humidity, but **electromagnetic radiation** : brightness temperature or radiance.
- Depending on **wave length**, indirect information on gas concentration (e.g. humidity) or on physical properties of atmosphere (temperature or pressure).
- Observations are often made in « **atmospheric windows** » (in white, below), e.g. in microwave and some infrared : frequencies with « high atmospheric transmittance » (= « low opacity » : radiation passes through the atmosphere to Earth surface, without being absorbed by gases) ; indirect info on T.



Passive remote sensing : radiative transfer equation



- What is observed is a **radiance** = quantity of energy per time unit, going through a surface, in a solid angle, and for a wave number interval of the radiation. [Unit : $\text{W}/\text{m}^2\text{Sr}\cdot\text{cm}^{-1}$]
- Planck function:
 $B_\nu(T)$ = radiance emitted by a *black body* at temperature T , for wave number ν .
- Intensity of the radiation, emitted by the *atmosphere* at wave number ν :

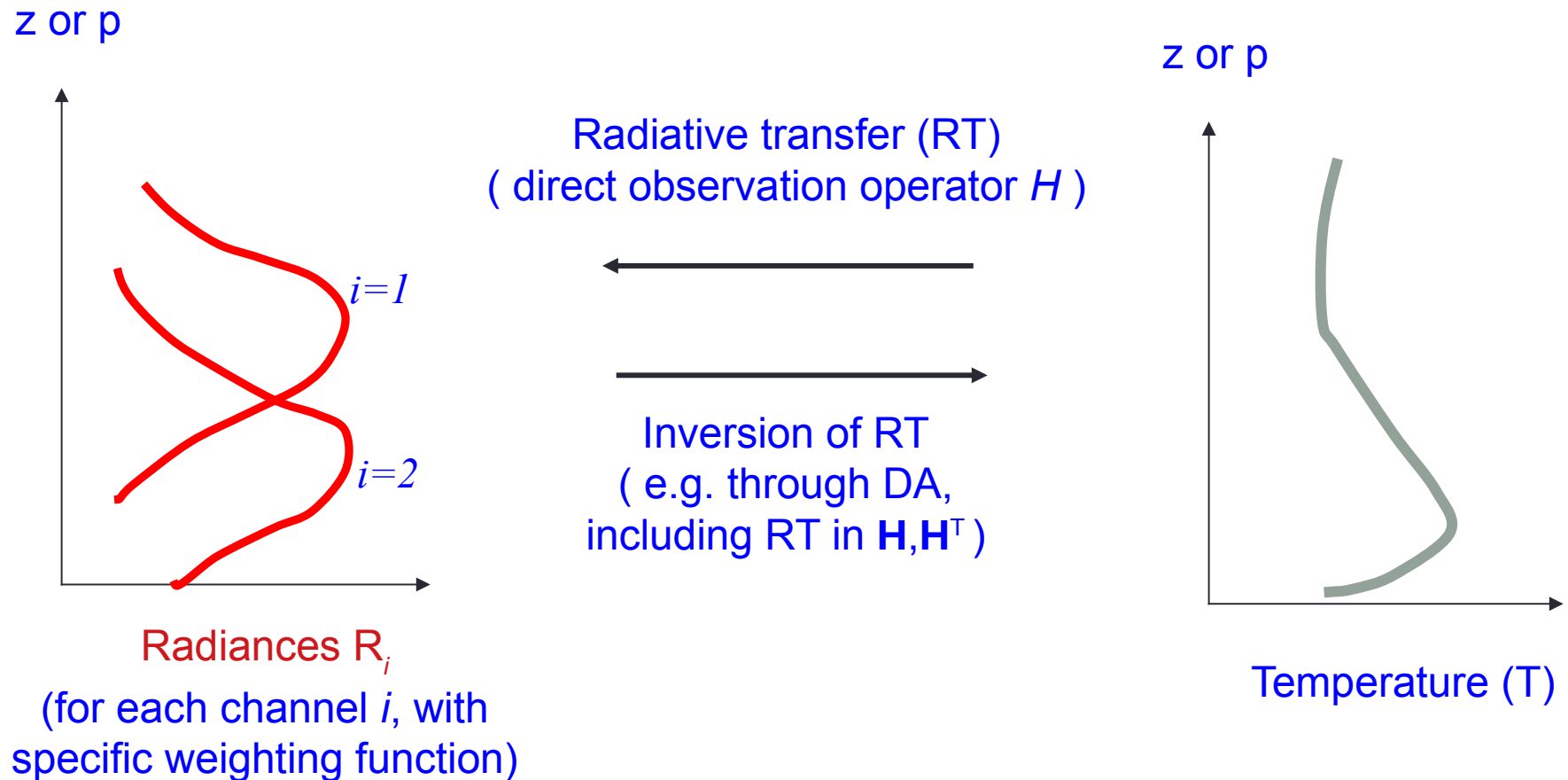
$$R_\nu = I_{0,\nu} \tau_\nu(z_0) + \int_{z_0} B_\nu[T(z)] [d\tau_\nu(z)/dz] dz$$

$I_{0,\nu}$ is the *surface emission* at altitude z_0 .

$\tau_\nu(z)$ is the *transmittance* from z to the top of the atmosphere :
it accounts for atmospheric absorption of radiation.

$K_\nu(z) = d\tau_\nu(z)/dz$ is called **weighting function** :
it weights the Planck function in the radiance equation, and it determines
the vertical layer of the atmosphere sounded at considered frequency ν .

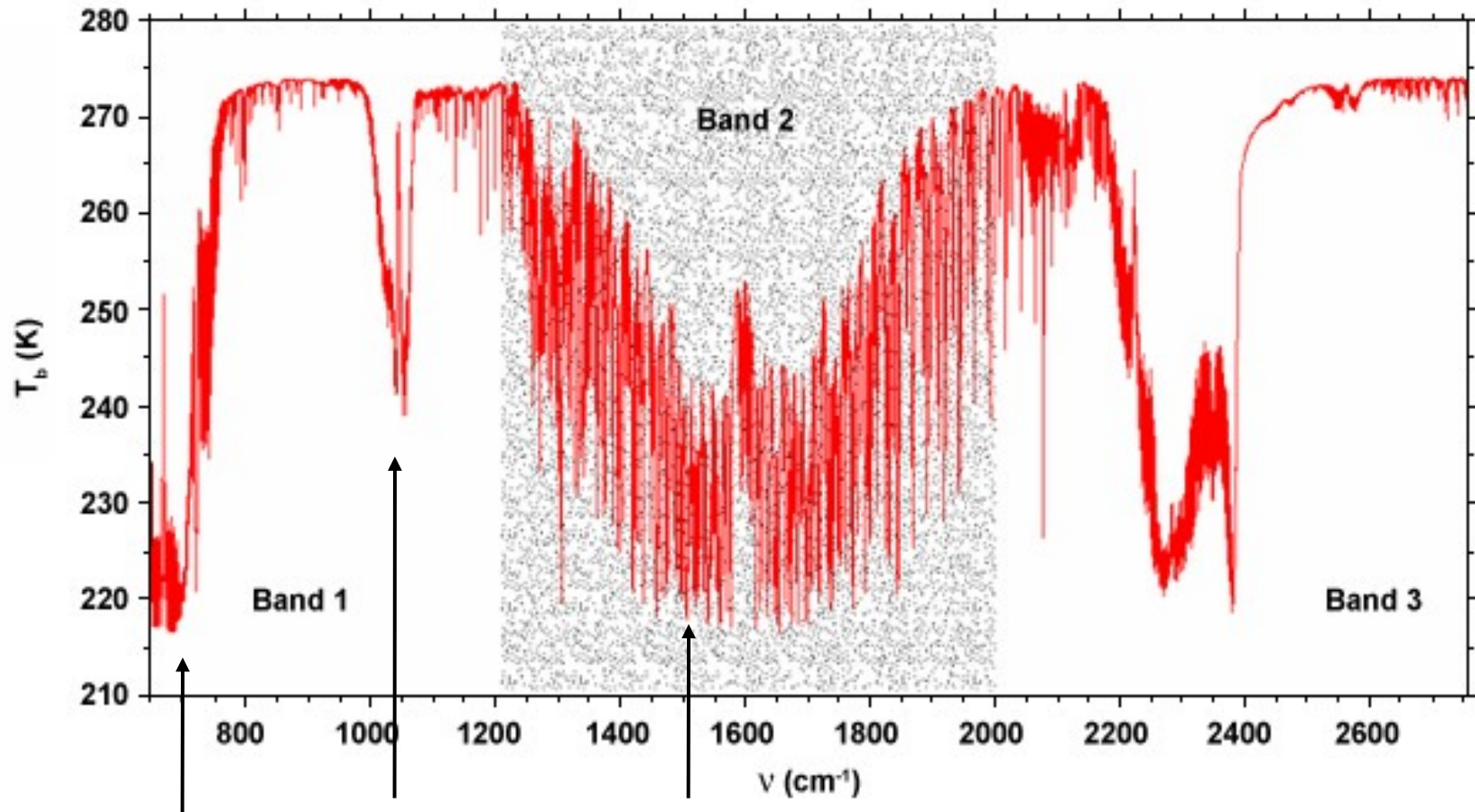
Radiative Transfer : compute « simulated radiances » from temperature profiles (+ associated inversion)



Using Radiative Transfer (in $H, \mathbf{H}, \mathbf{H}^T$) allows a large number of satellite radiances to be assimilated in NWP

IASI : infra-red interferometer developed by CNES and EUMETSAT

IASI offers a very high spectral resolution (~ 8000 channels)



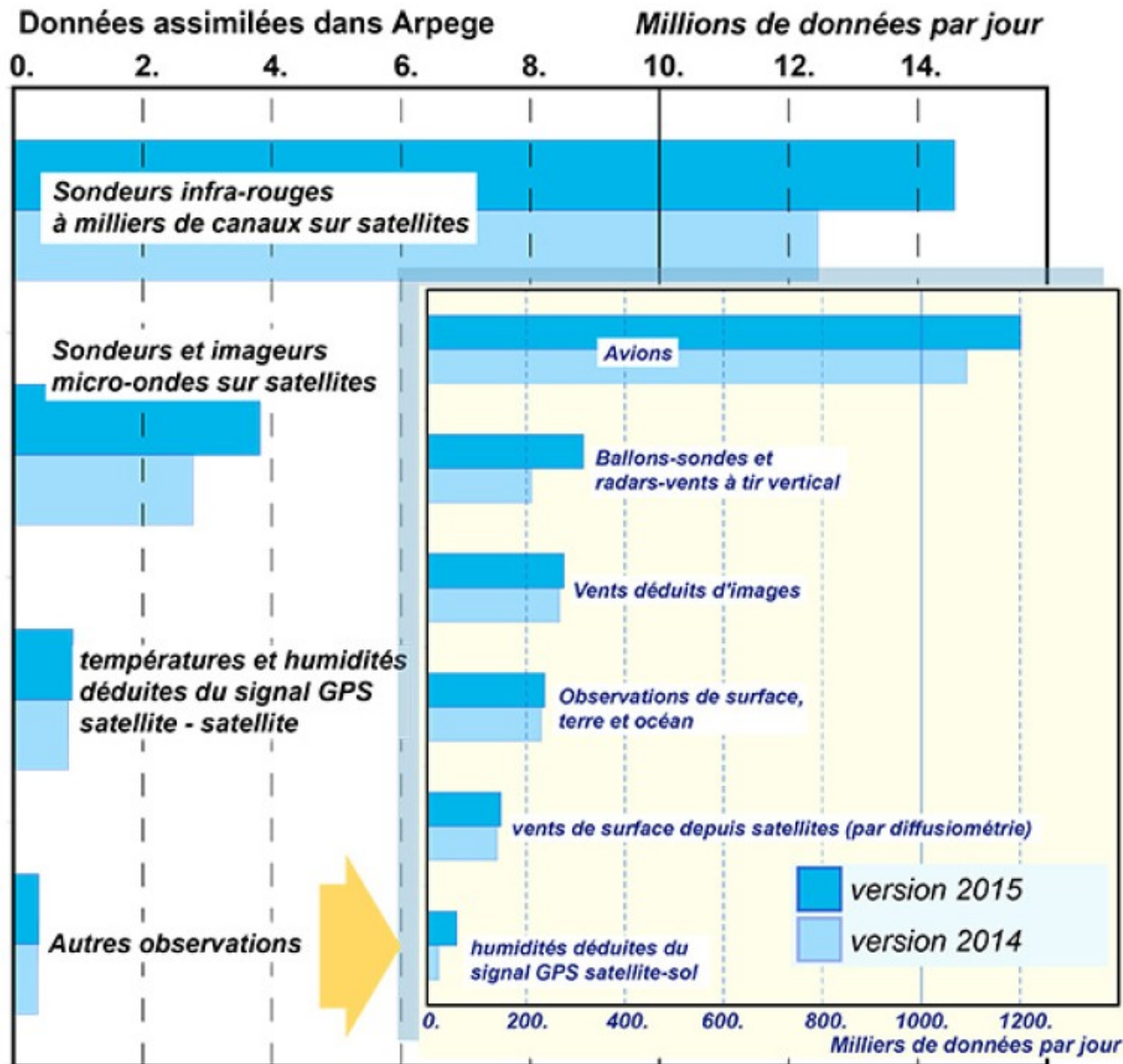
Temperature

ozone

Water vapor

©EUMETSAT, 2006

Number of observations used in ARPEGE (global DA at Météo-France)



Total ~ 20 million obs per day

How do observations meet global NWP requirements ?

■ Surface observations

good coverage over land, sparse coverage over sea;
observations not suited to describe upper levels.

■ Aircraft observations

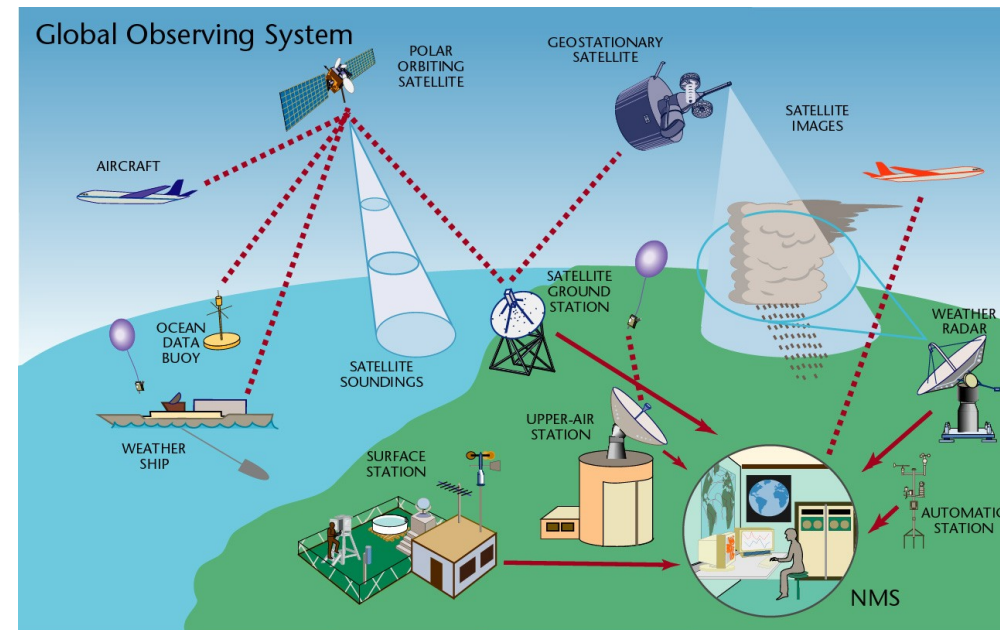
good accuracy,
but do not describe the 3D state of the atmosphere
(except near airports, during takeoff and landing).

■ Radiosonde data

good accuracy, good vertical resolution,
but poor horizontal coverage over the globe.

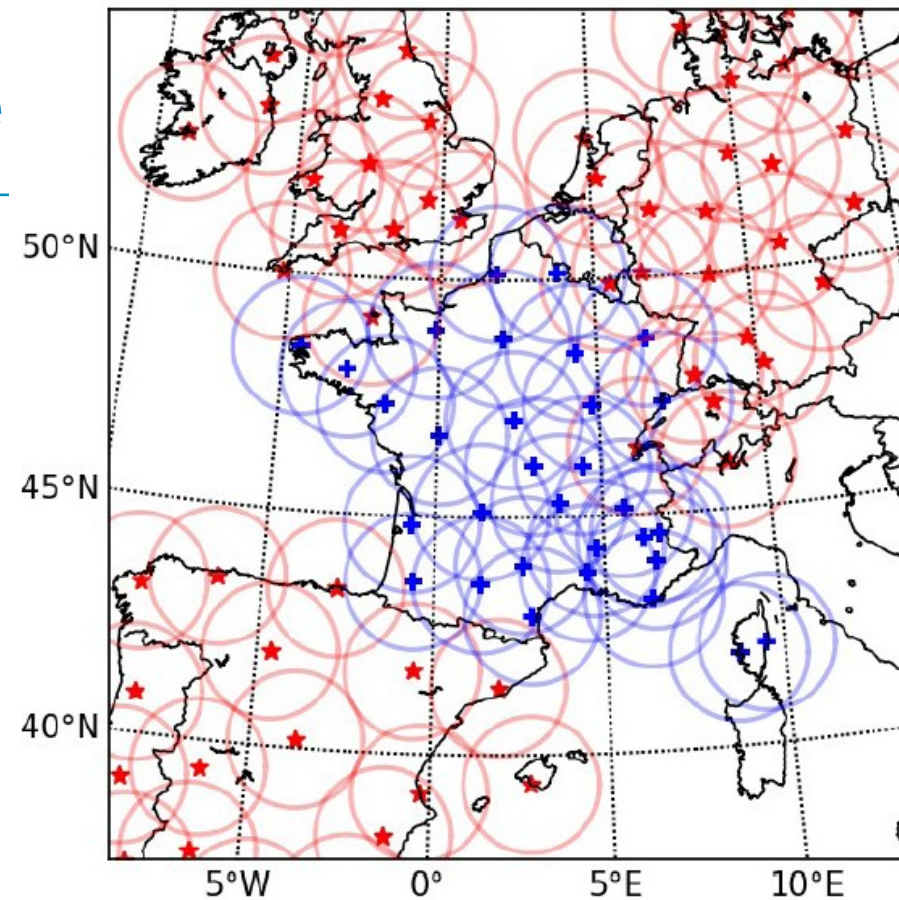
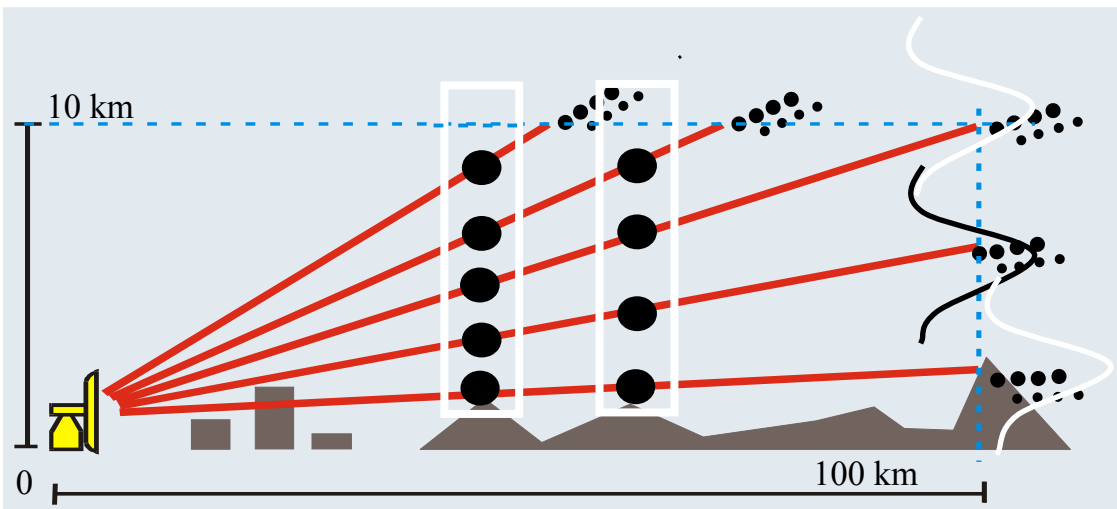
■ Satellite data

good horizontal coverage over the globe,
but poor vertical resolution (reduced to 1 level for satellite winds or imagers).



Radar network in Arôme-France

- 31 radars in France,
62 radars in neighbouring countries ;
every 15 minutes, at 1 km resolution.
- Observations :
reflectivities Z (related to precipitation) ;
radial winds V_r (doppler effect) :
the emitted microwave signal returns to the radar with a
modified frequency, when the target is moving (wind).
=> invert Doppler equation to obtain a wind observation.



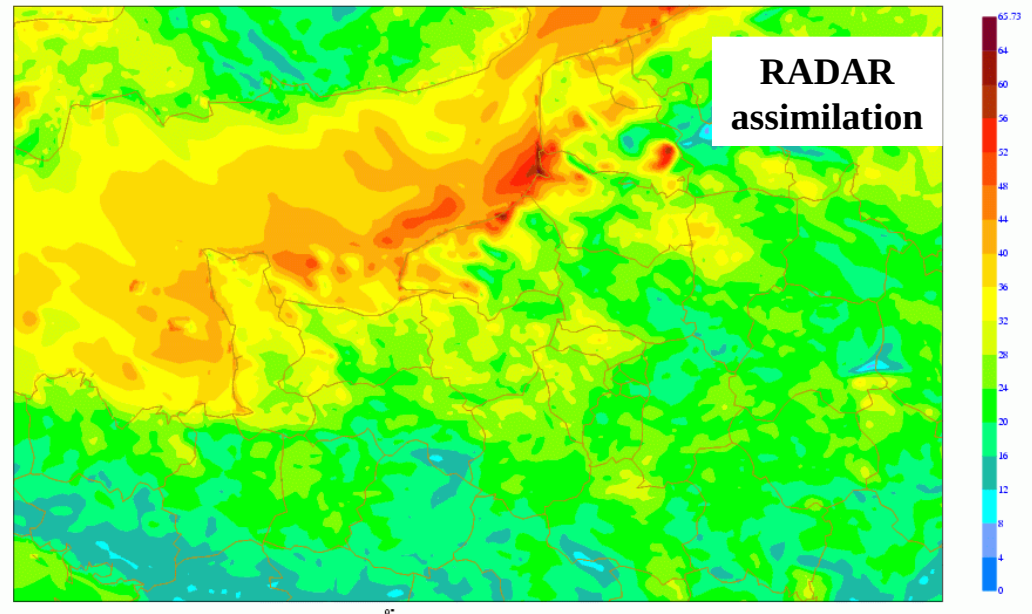
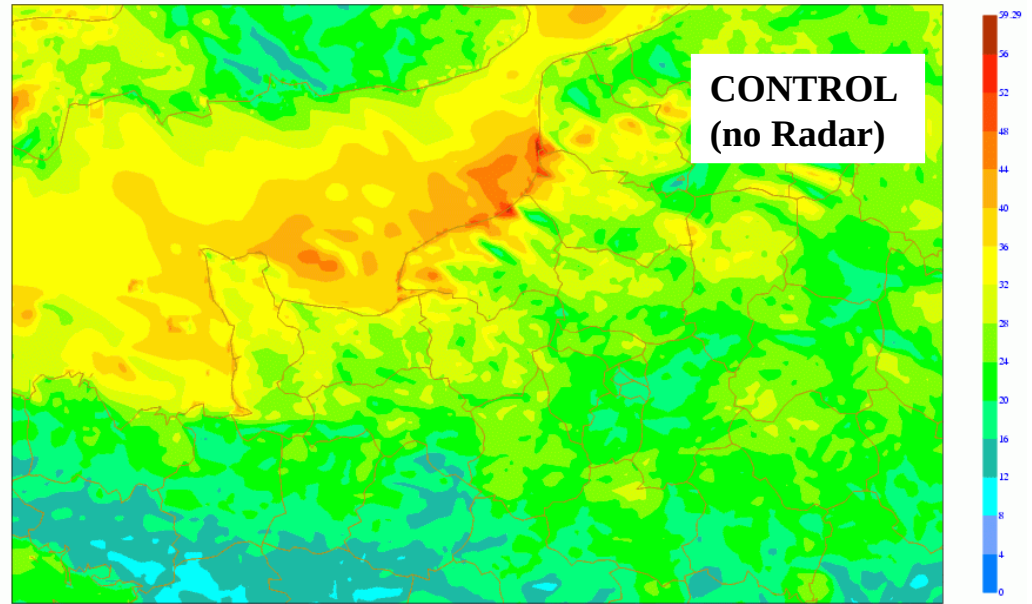
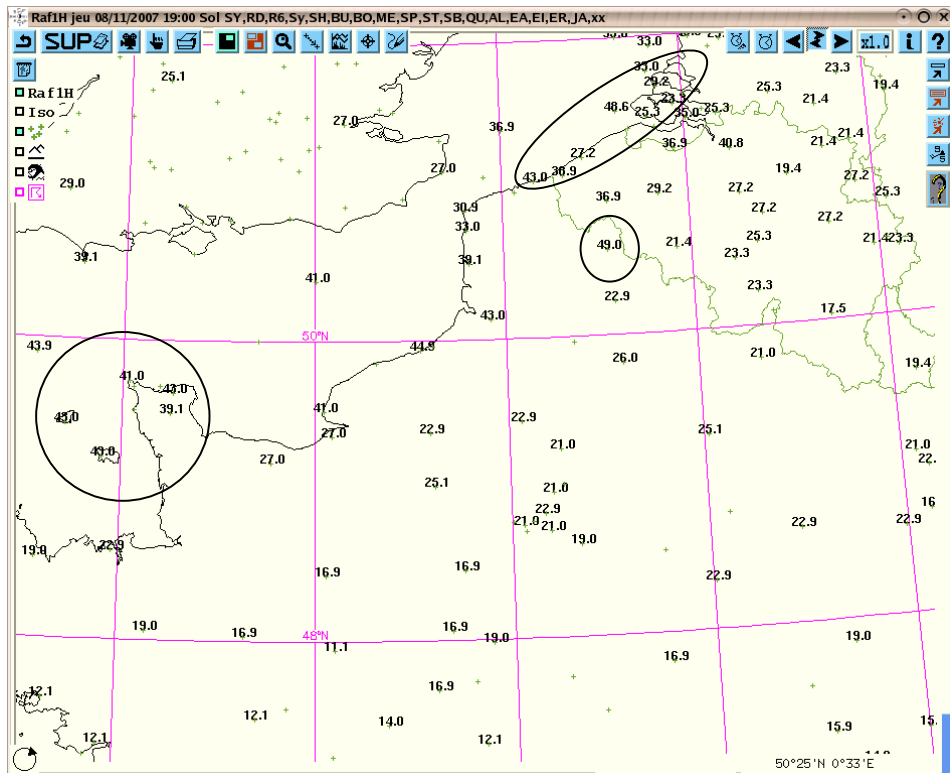
**Observations assimilated
as vertical profiles,
after estimating the pixel altitudes**

(Pixel altitude is computed using
a constant refractivity index along the path)
(= effective radius approximation)

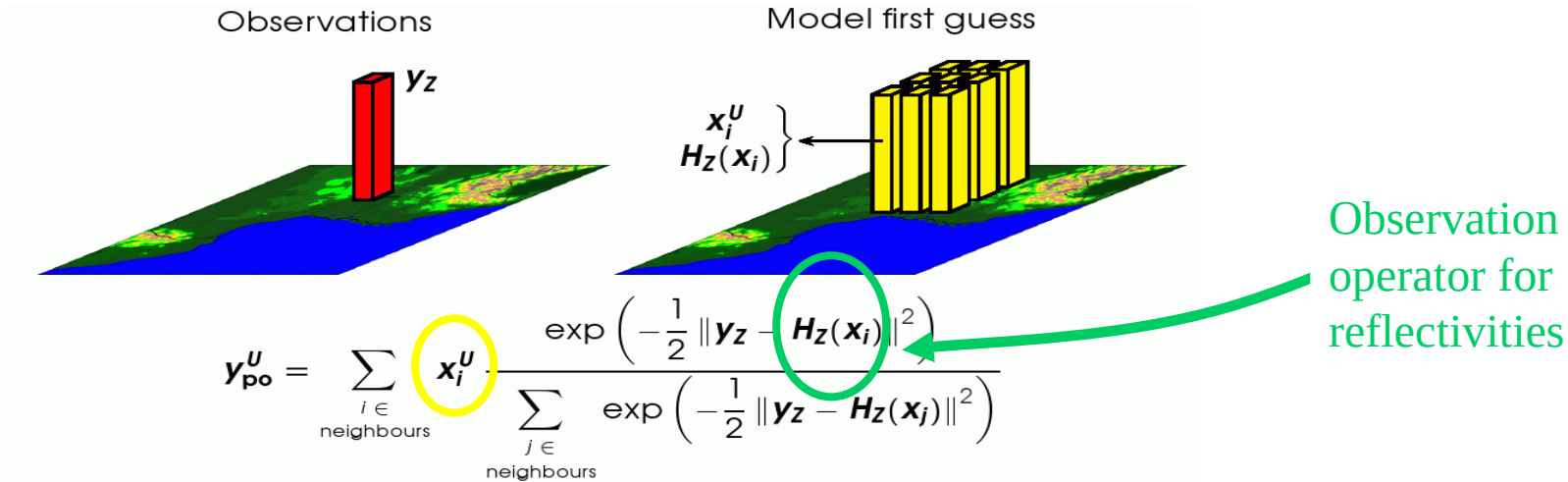
Assimilation of radar radial winds

Wind gust at 10 m (kt)
Forecast +1h (19 UTC)

OBS



Bayesian inversion of reflectivity profiles



y_{po}^u : column of pseudo-observed relative humidity,
 y_z : column of observed reflectivities,
 x_i^u : column of relative humidity,
 $H_z(x_j)$: column of simulated reflectivities.

Caumont, 2006: use model profiles in the neighborhood of each observation (in 3 steps), to provide *humidity retrievals*.

1. **Compute reflectivities simulated by the model**, from model relative humidity (RH) profiles (using *observation operator for relectivities*).
2. **Estimate likelihoods of model reflectiv.**, by comparing them with obs. reflectivities, and by computing associated *exponantials*.
3. Compute (pseudo-)observed RH profile (y_{po}), to be assimilated : **average of model RH profiles, weighted by their likelihoods**.

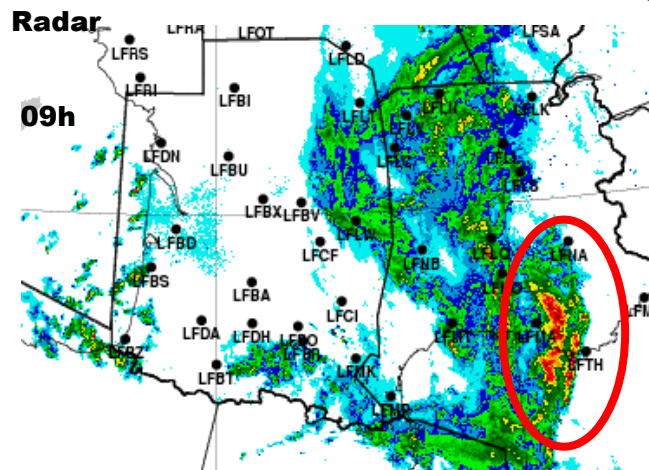
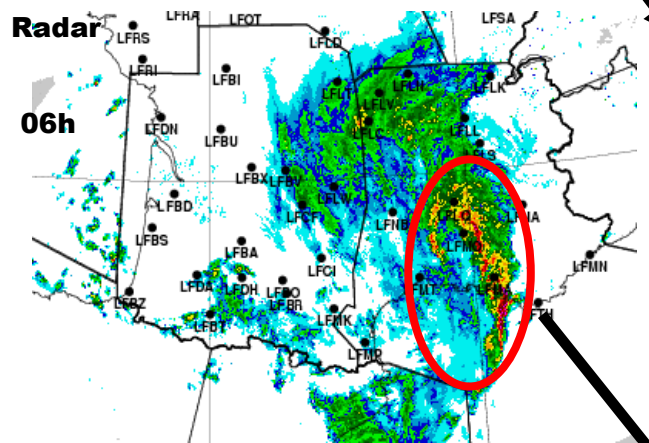
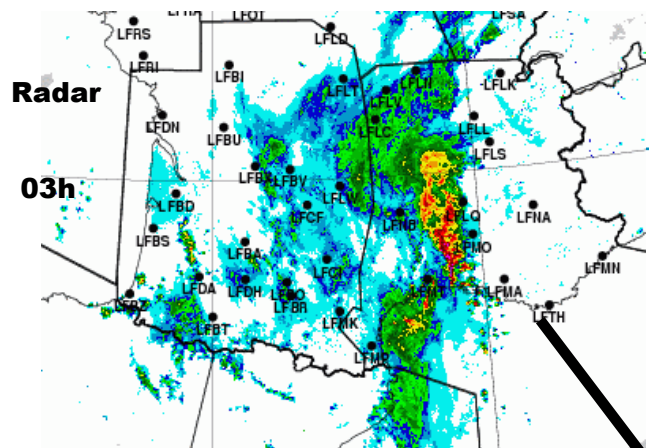
Impact of radar reflectivities

Example of Heavy Precipitation Event (South-East of France) :

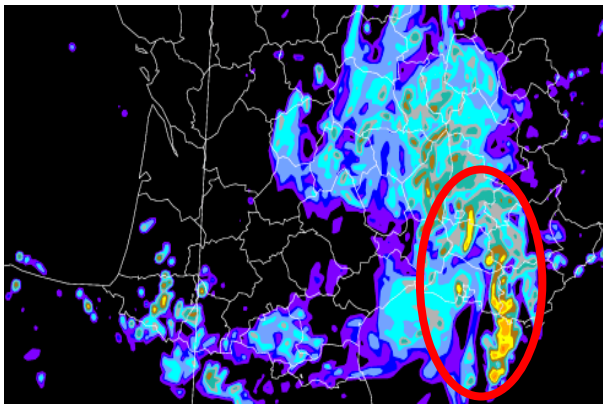
REFL
(assimilation of reflectivities)

VS

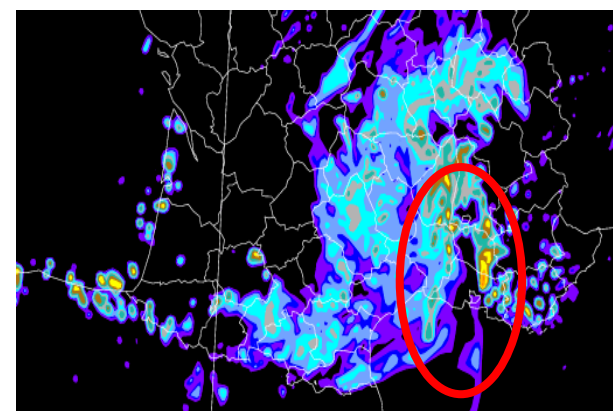
CONTROL
(no reflectivities in DA)



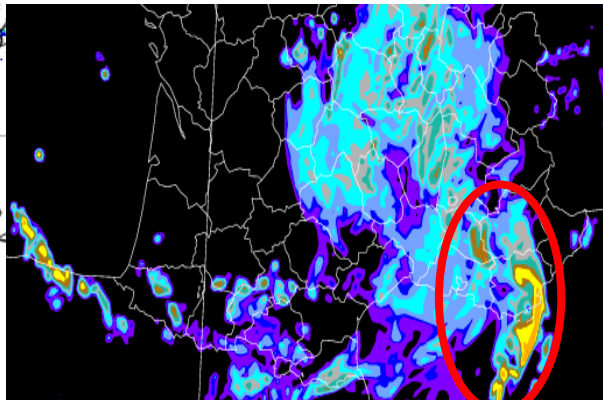
3h forecast issued at 3UTC



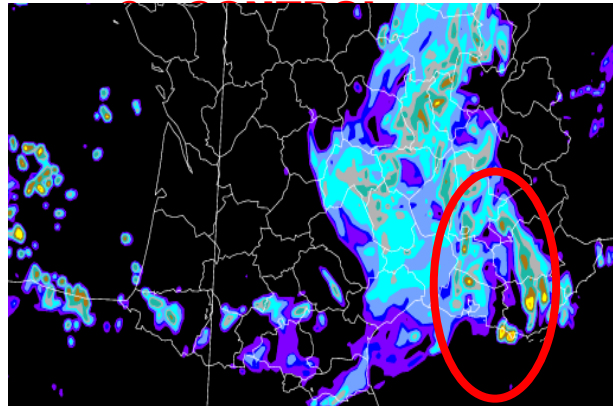
3h forecast issued at 3UTC



3h forecast issued at 6UTC



3h forecast issued at 6UTC



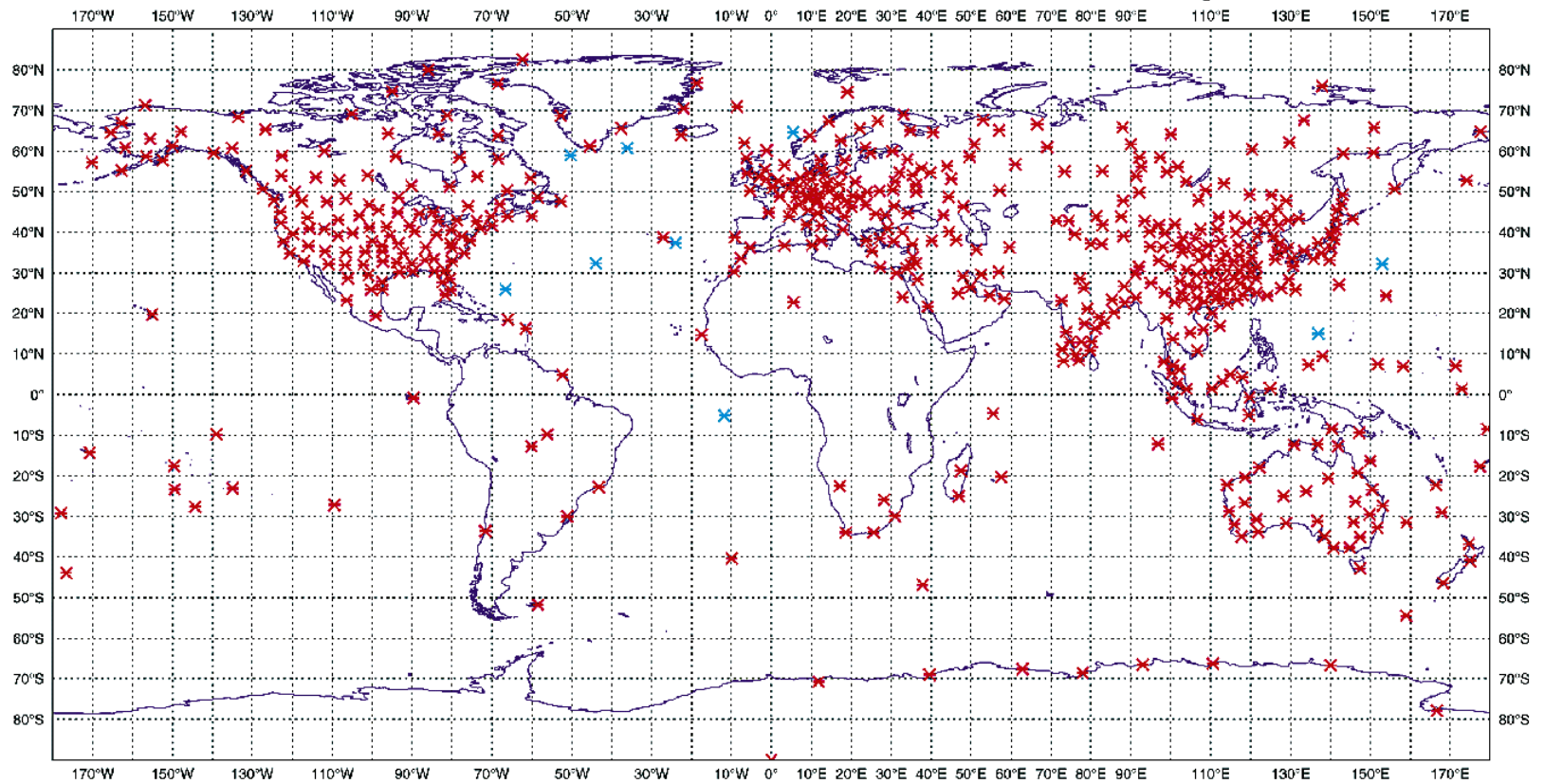
Line of heavy precipitation is well simulated in REFL run.

Error covariances : estimation and modelling (to weight and spatialise observed information)

How can we estimate error covariances ?

- The **true atmospheric state** is never (exactly) known.
- Use **observation-minus-background** departures to estimate some average variances and correlations of **R** and **B**, using assumptions on spatial structures of errors.
- Use an **ensemble** to simulate the error evolution and to estimate space- and time-dependent background error structures.
- Use **covariance modelling** to filter out sampling noise and other uncertainties in the ensemble.

Radiosonde observation network



Covariances of innovations

- Innovations = observation-background departures :

$$\begin{aligned}\mathbf{y}^o - H(\mathbf{x}^b) &= \mathbf{y}^o - H(\mathbf{x}^t) + H(\mathbf{x}^t) - H(\mathbf{x}^b) \\ &\sim \mathbf{e}^o - \mathbf{H}\mathbf{e}^b\end{aligned}$$

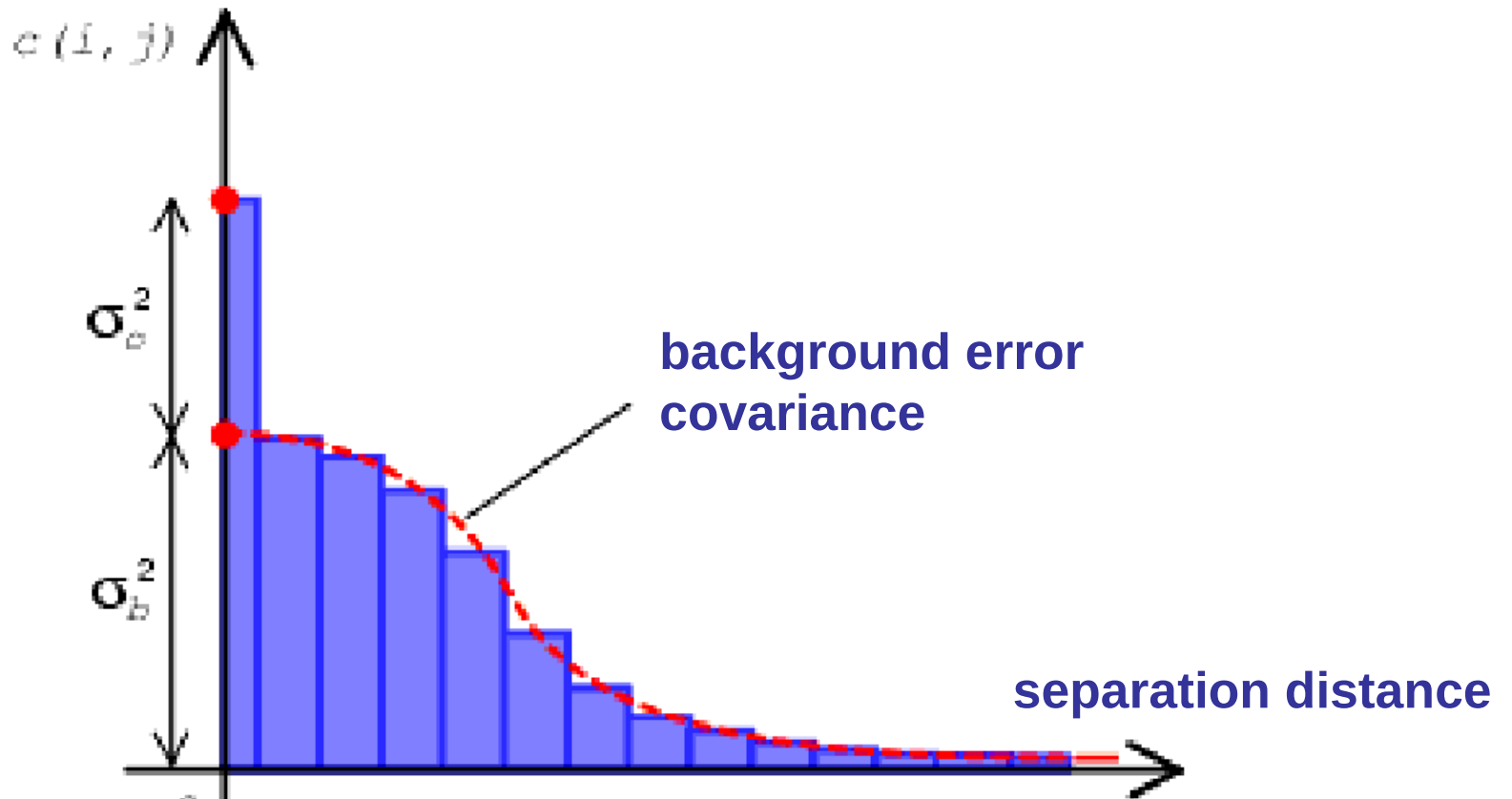
- Innovation covariances :

$$E[(\mathbf{y}^o - H(\mathbf{x}^b))(\mathbf{y}^o - H(\mathbf{x}^b))^T] = \mathbf{R} + \mathbf{H}\mathbf{B}\mathbf{H}^T$$

assuming that $E[\mathbf{e}^o (\mathbf{H}\mathbf{e}^b)^T] = \mathbf{0}$.

(e.g. Hollingsworth and Lönnberg 1986).

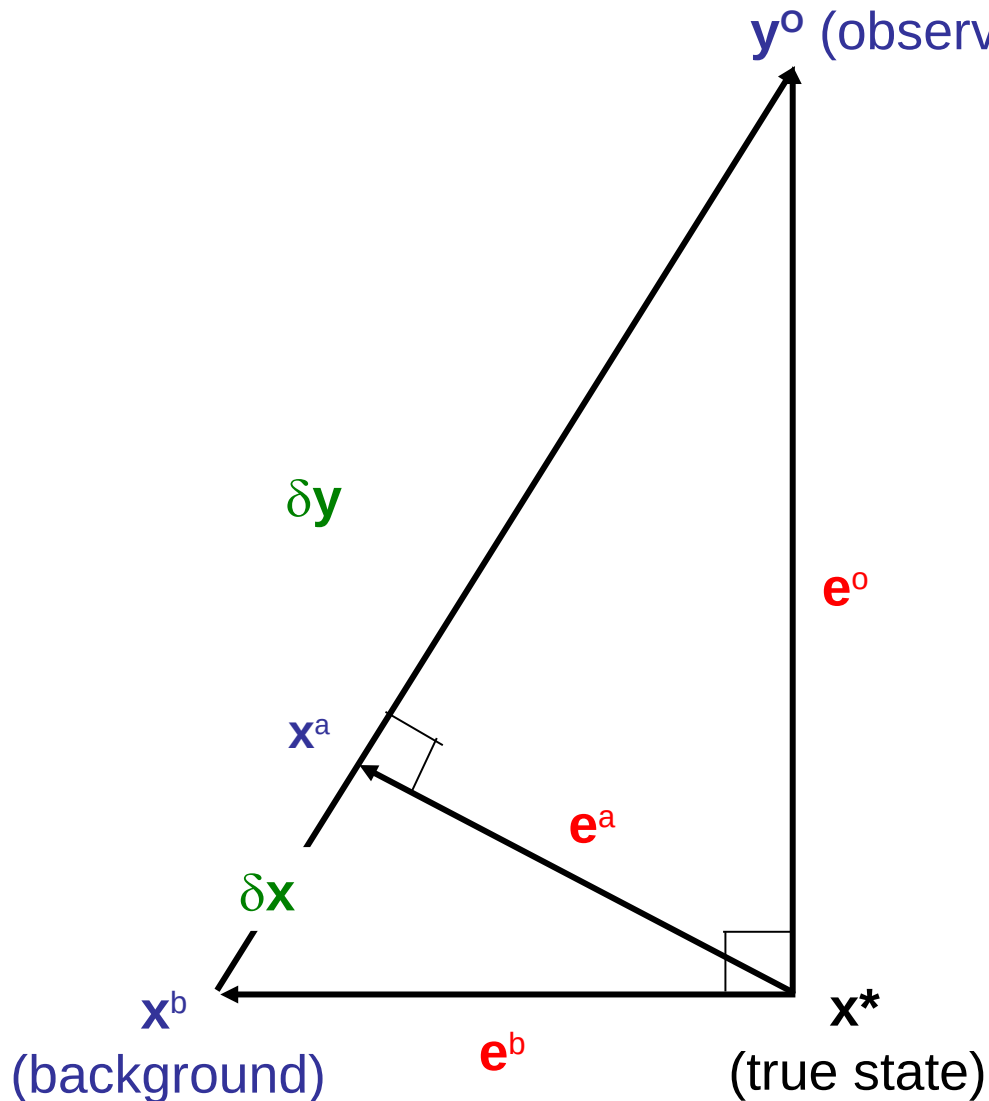
Covariances of innovations (with extrapolation to zero separation distance)



$$E[(\mathbf{y}^o - H(\mathbf{x}^b))(\mathbf{y}^o - H(\mathbf{x}^b))^T] = \mathbf{R} + \mathbf{H} \mathbf{B} \mathbf{H}^T$$

Extrapolation to zero separation distance allows different contributions to be estimated.

Covariances of analysis residuals



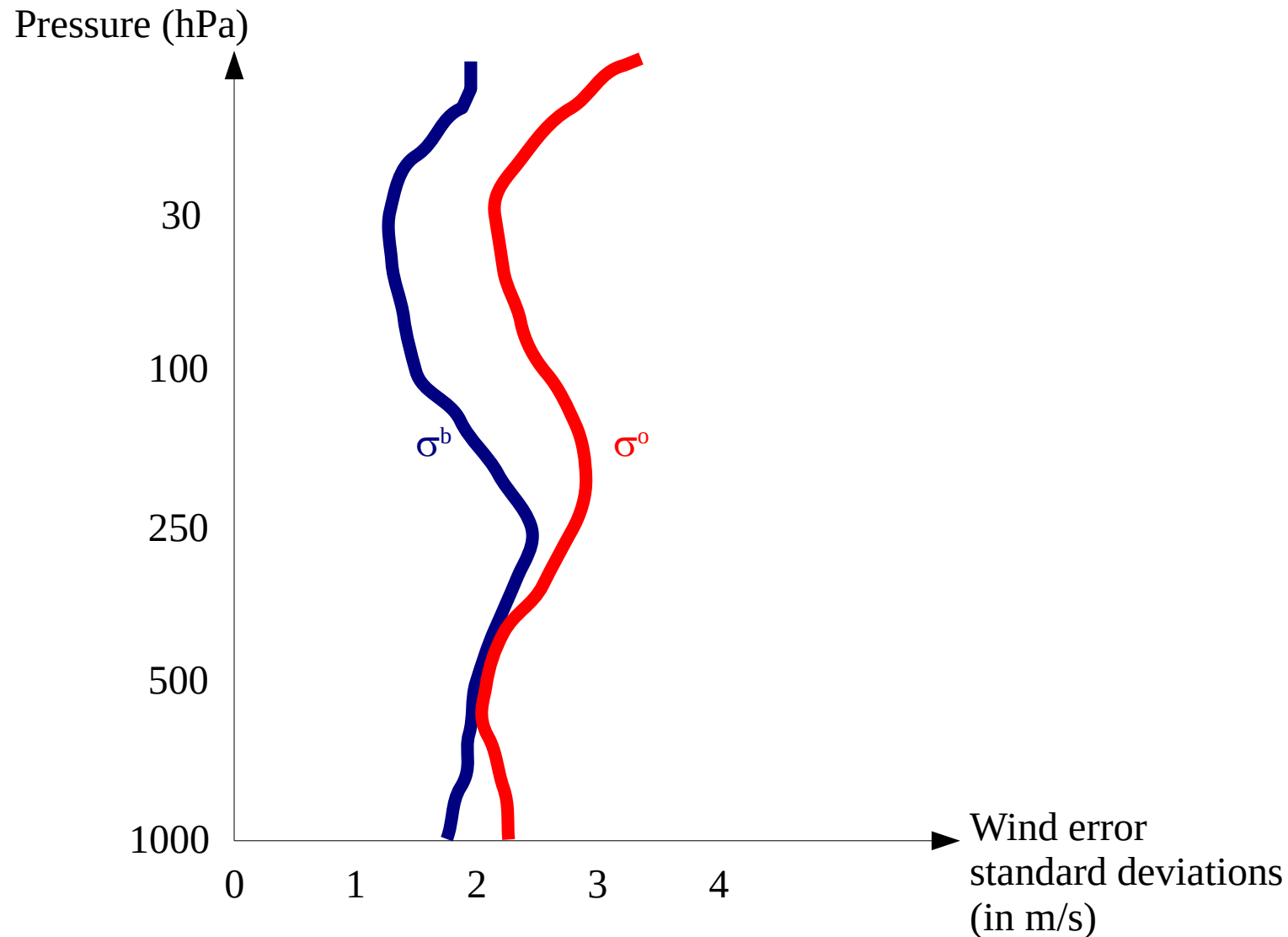
$$\delta \mathbf{y} = \mathbf{y}^o - H(\mathbf{x}^b) \quad (\text{innovation})$$

$$\mathbf{H} \delta \mathbf{x} = H(\mathbf{x}^a) - H(\mathbf{x}^b) \quad (\text{increment})$$

$$E[\mathbf{H} \delta \mathbf{x} \delta \mathbf{y}^T] = \mathbf{H} \mathbf{B} \mathbf{H}^T$$

$$E[(\mathbf{y}^o - H(\mathbf{x}^a)) \delta \mathbf{y}^T] = \mathbf{R}$$

Vertical profiles of standard deviations of background errors and observation errors



Space & time averages of innovation-based covariances

At a given location and time,
there is only 1 innovation value δy :
a single error realization is available locally
(e.g. for estimating background error variance).

$H(\mathbf{x}^b)$

y^o

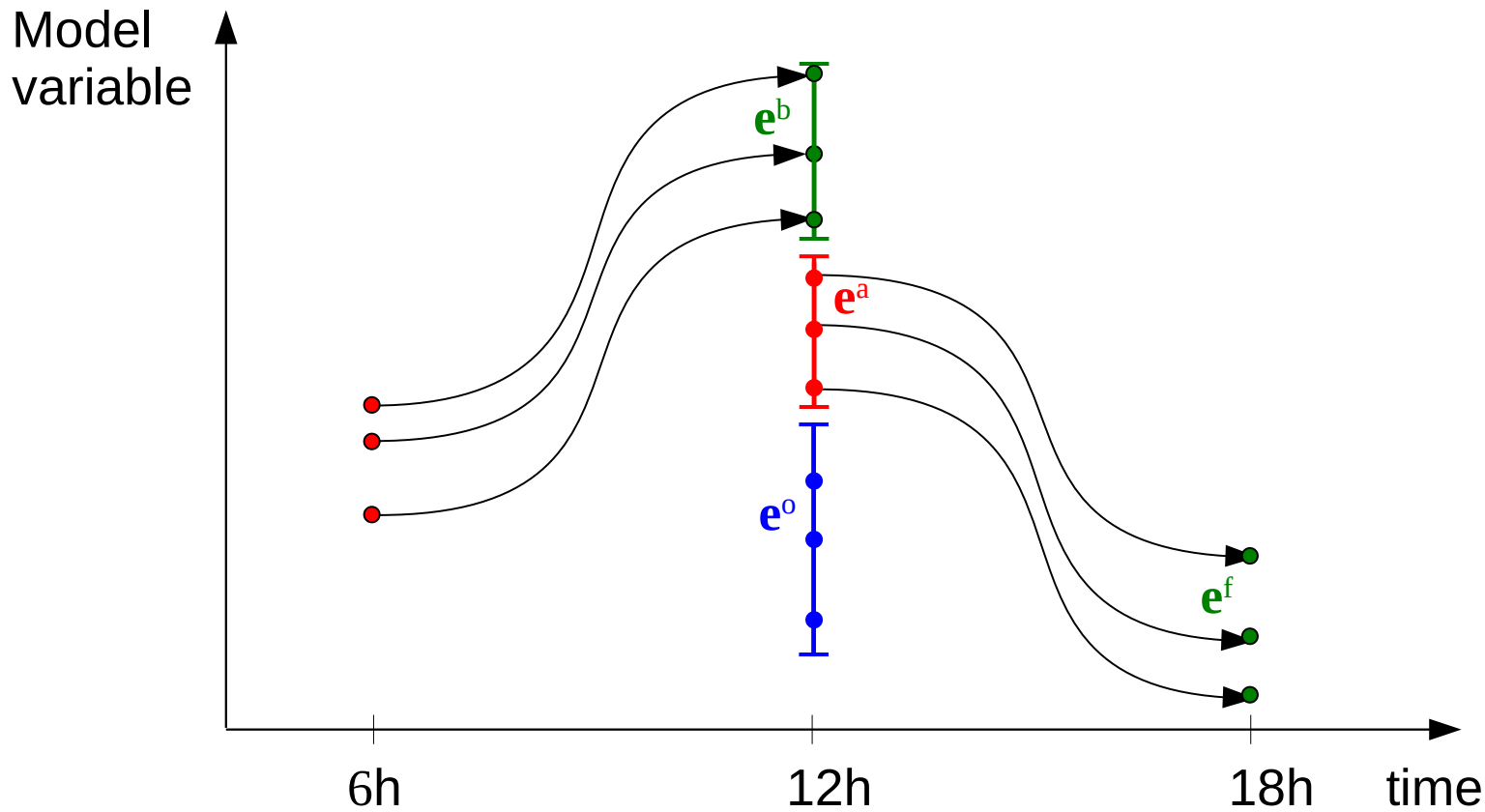
$\delta y \sim \mathbf{e}^o - \mathbf{H}\mathbf{e}^b$

=> Statistical averages (mathematical expectations)
need to be replaced by
space and time averages (ergodic assumption).

=> only space or time averages of **B** and **R**
can be estimated from innovation data.

=> consider other approaches, such as ensemble methods.

Ensemble Data Assimilation (EDA) : simulation of error cycling, by adding & propagating perturbations



$$e^a = (I - KH)e^b + Ke^0$$

$$e^f = Me^a + e^m$$

with $e^b = e^{f-}$

and $e^0 = R^{1/2} \eta$ (random draws of R)

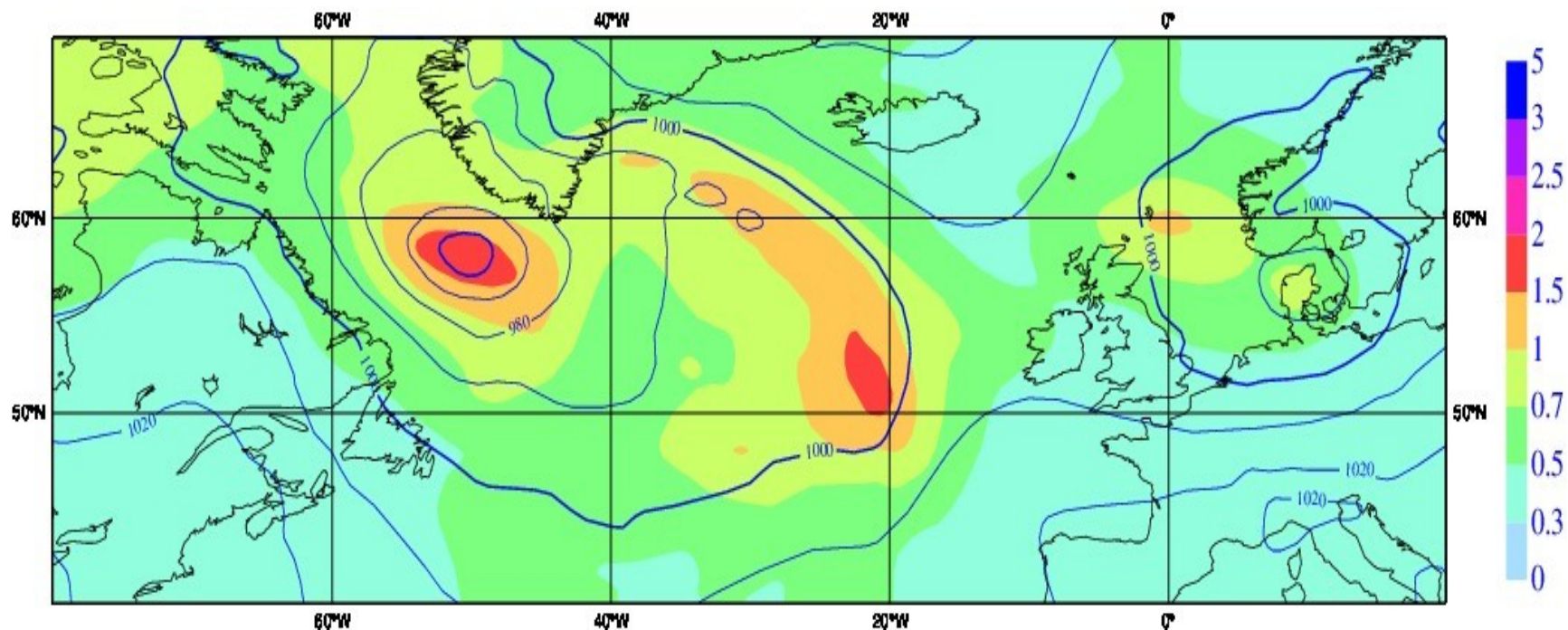


(e.g. Houtekamer et al 1996, Fisher 2003, Berre et al 2006 ;
ARPEGE : 50 members to estimate flow-dependent B and to initialise ensemble predictions)

Simulation and propagation of observation errors and model errors

- Observation errors can be simulated by adding random draws of \mathbf{R} : $\mathbf{e}^o = \mathbf{R}^{1/2} \boldsymbol{\eta}^o$.
- Model errors can be simulated in different ways, e.g. by :
 - adding random draws of \mathbf{Q} : $\mathbf{e}^m = \mathbf{Q}^{1/2} \boldsymbol{\eta}^m$ (additive or mult. inflation);
 - using a multi-model approach (or multi-physics) ;
 - perturbing physical tendencies of the model ;
 - perturbing model parameters. (...)
- Observation and model perturbations are propagated during the successive analysis/forecast steps of DA cycling.
- Flow-dependent background error covariances can be estimated from the ensemble spread.

Dynamics of background error variances



Standard deviations of surface pressure errors (hPa)
(superimposed with MSLP analysis (hPa)).

=> larger weight given to observations in regions where
the background is particularly uncertain (intense weather events)

Modelling and filtering covariances

- Huge size of **B** : model it with operators which are sparse and/or of small size.
- Sampling noise, and other uncertainties. => Spatio-temporal filtering.
- Factorisation : $\mathbf{B} = \mathbf{B}^{1/2} \mathbf{B}^{\top/2}$
 $\mathbf{B}^{1/2} = \mathbf{L} \mathbf{S} \mathbf{C}^{1/2}$

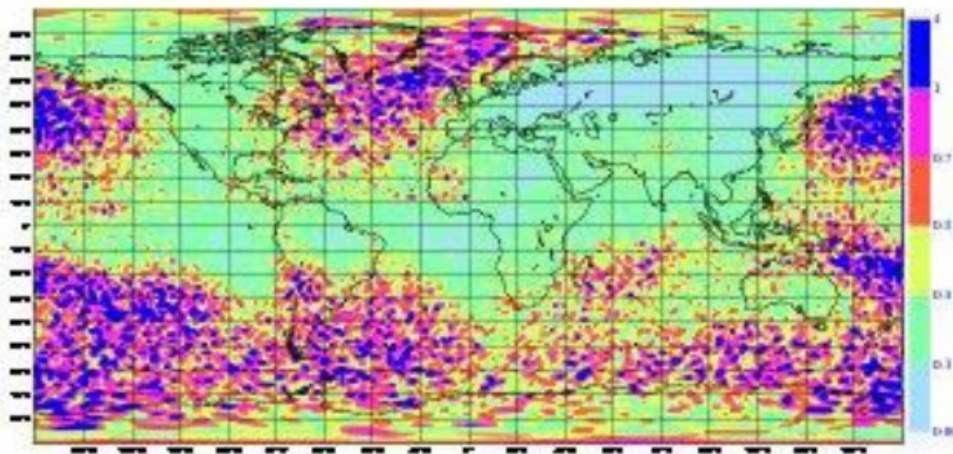
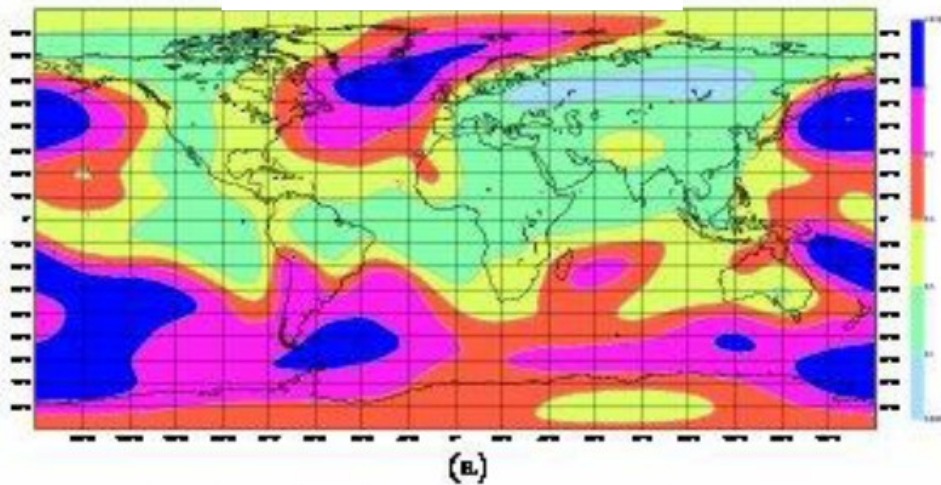
L ~ mass/wind **cross-covariances** (related to geostrophy),
including flow-dependence (non linear balances).

S flow-dependent **standard deviations** (~ expected error amplitudes),
filtered spatially.

C matrix of **3D spatial correlations** (~ spatial structures of errors),
filtered in wavelet space (block-diagonal model).

Spatial filtering of variance field

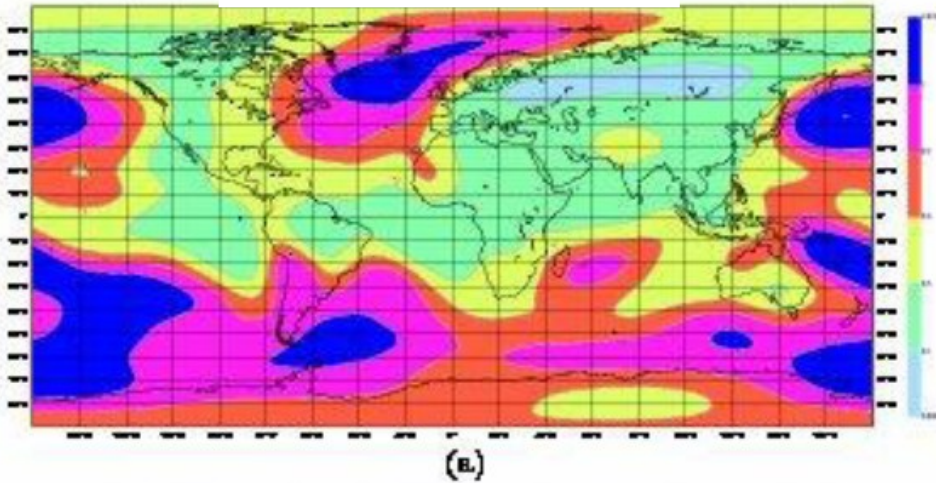
EXACT
VARIANCES



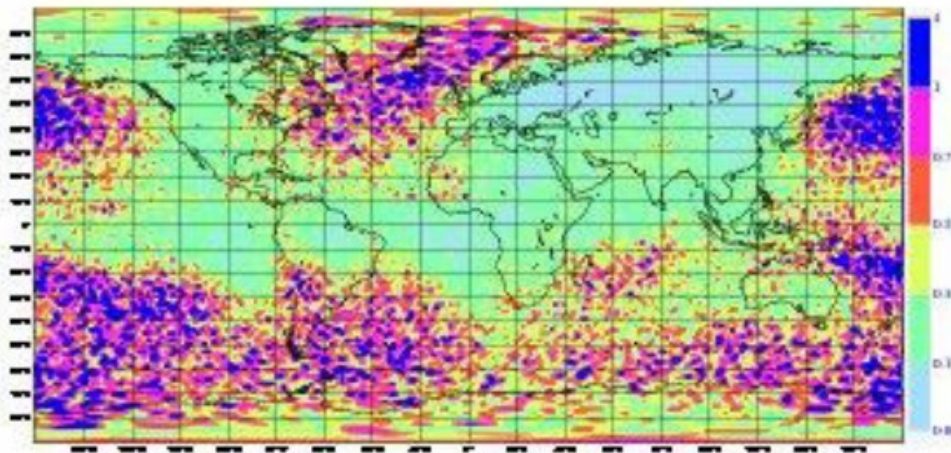
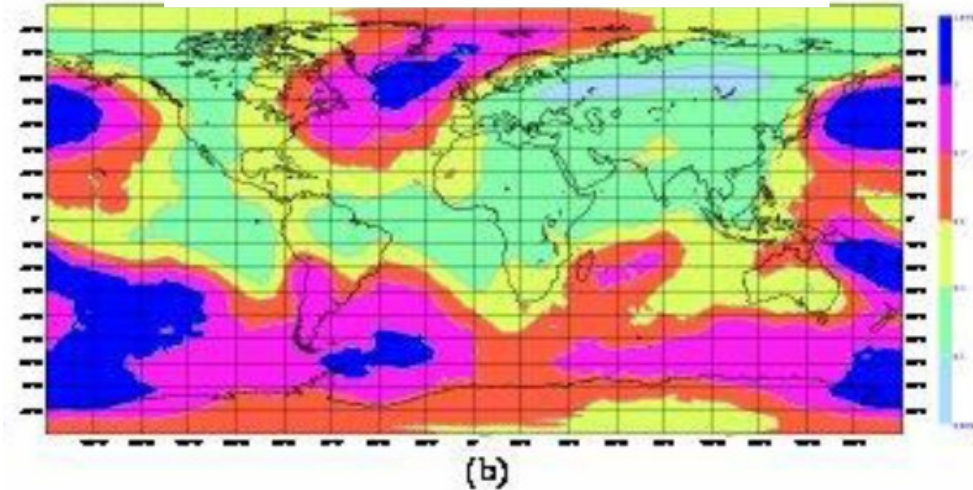
RAW ENSEMBLE
VARIANCES (N=6)

Spatial filtering of variance field

EXACT
VARIANCES



FILTERED ENSEMBLE
VARIANCES (N=6)



RAW ENSEMBLE
VARIANCES (N=6)

Low-pass filter applied to raw variances :

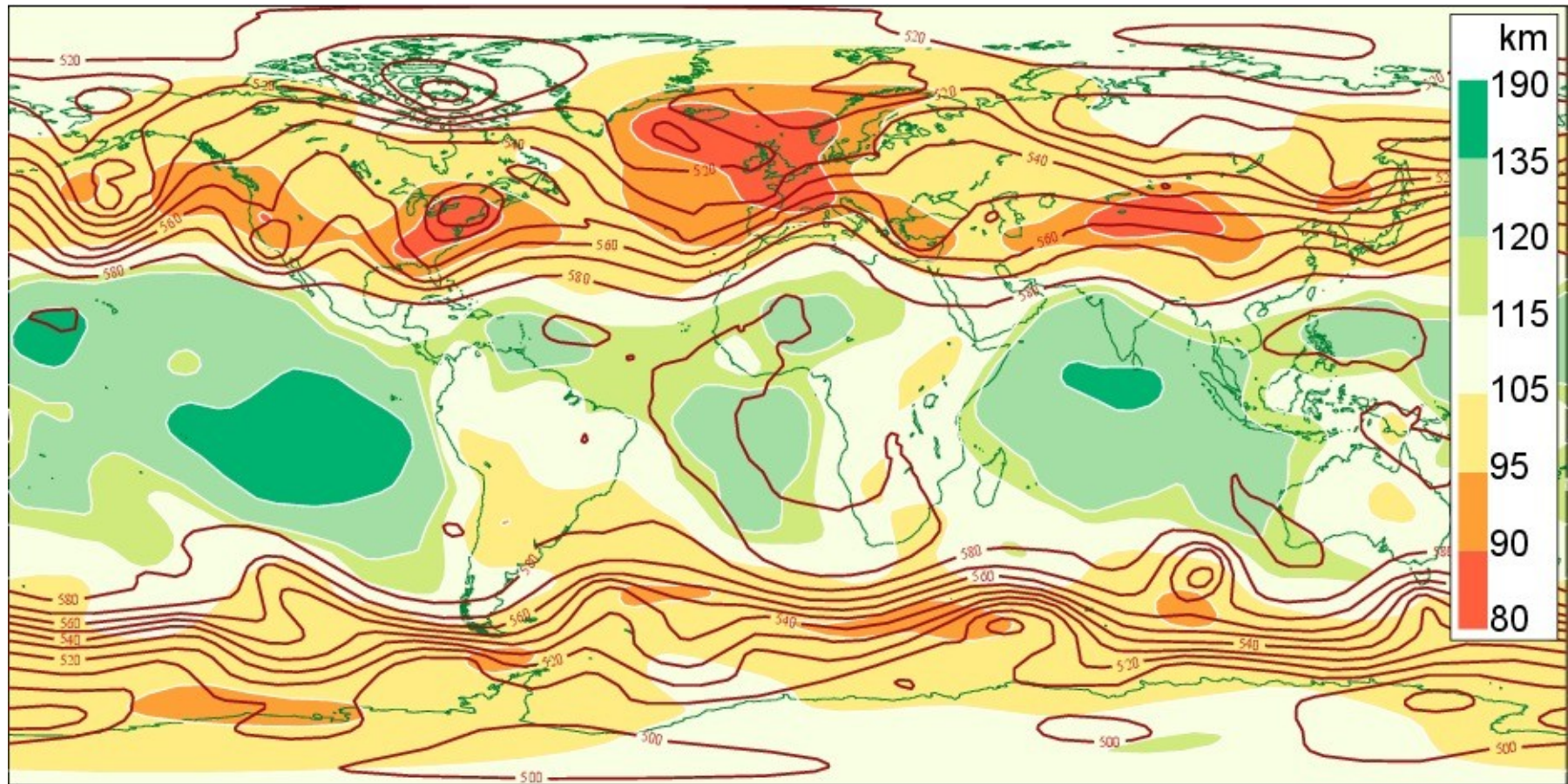
$$\mathbf{v}'_b = \mathbf{F} \mathbf{v}_b$$

with \mathbf{F} optimized in spectral space /
spatial structures of signal & noise :

$$F = 1 / (1 + E[\text{noise}^2]/\text{signal}^2)$$

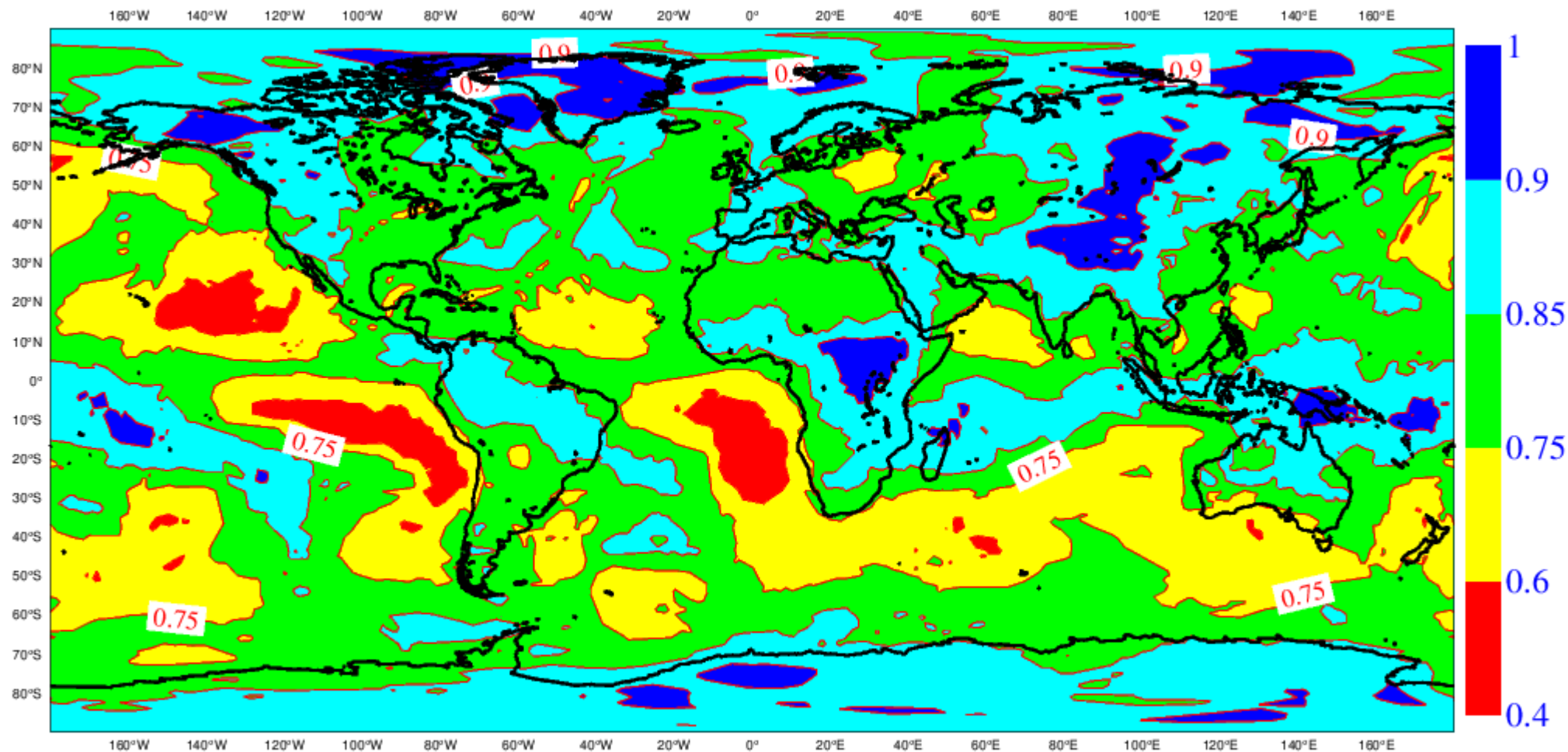
$$\text{and } E[v_b^2] = \text{signal}^2 + E[\text{noise}^2]$$

Dynamics of horizontal correlations



Horizontal length-scales (in km) of wind errors near 500 hPa,
superimposed with geopotential

Dynamics of vertical correlations

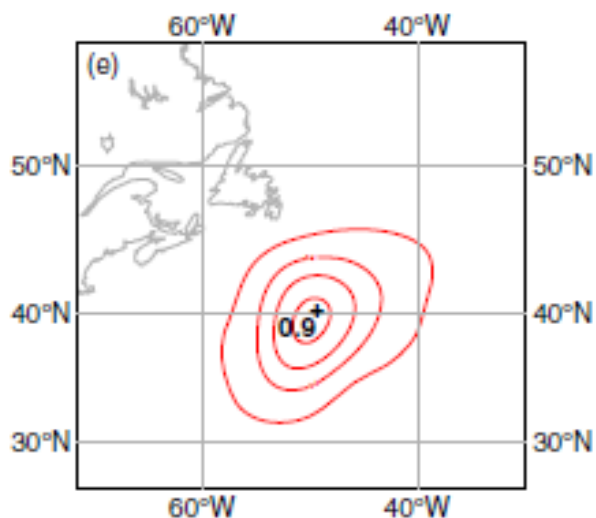


Vertical correlations of temperature errors
between 850 & 870 hPa

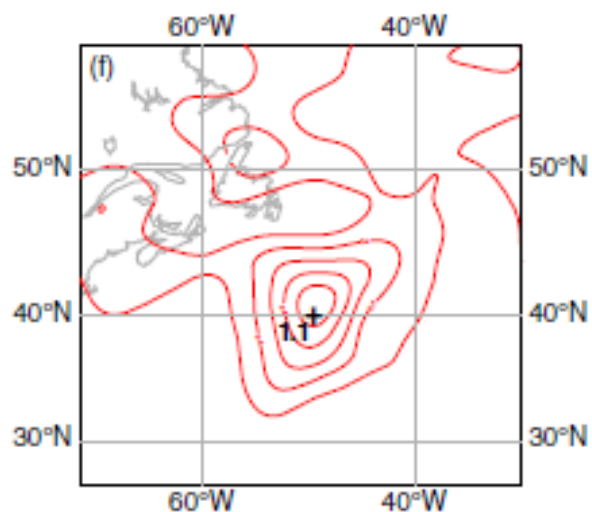
=> lesser amount of vertical propagation in stable areas.

Covariance anisotropy and localisation

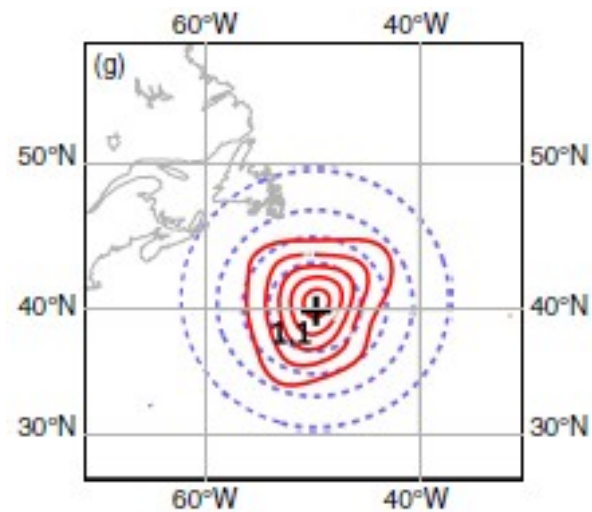
Ensemble to get information on anisotropy,
but requires filtering = localisation.



« Exact » covariances



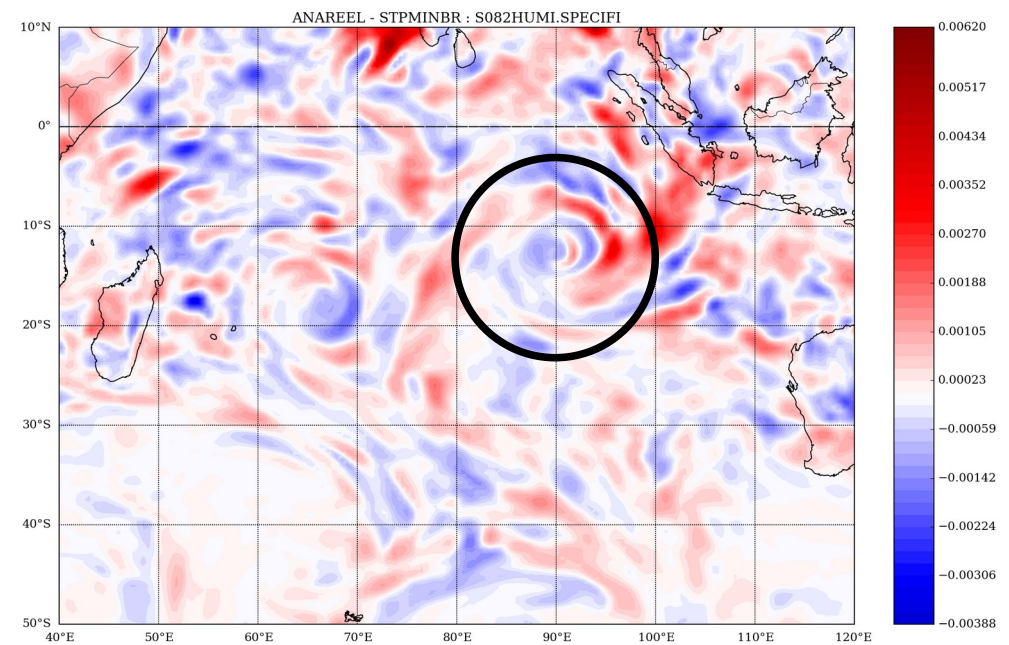
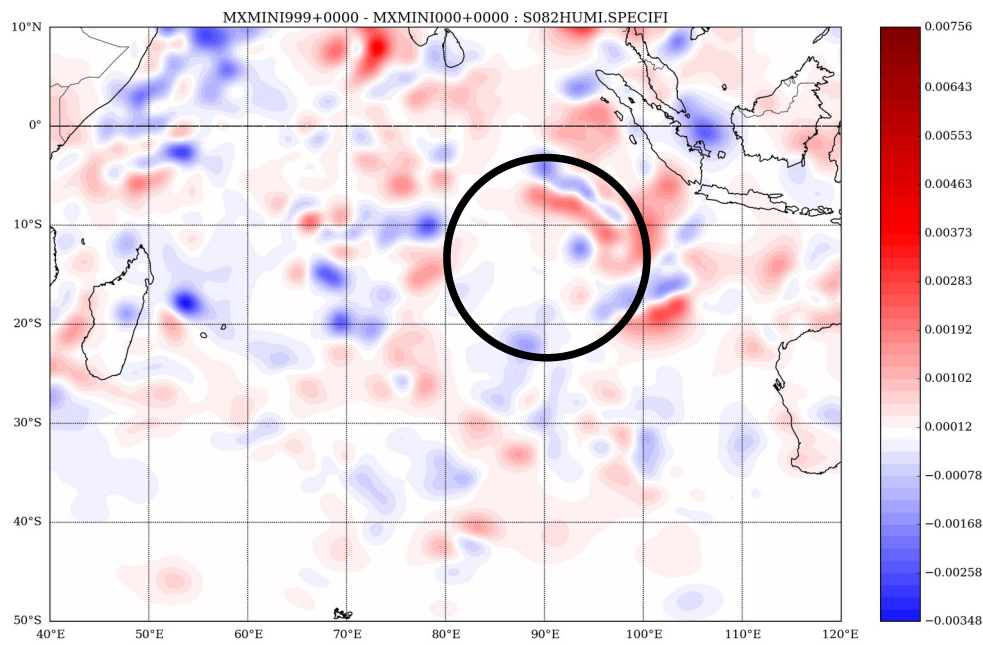
Raw covariances
(200 members)



Localised covariances
(200 members)

Flow-dependent anisotropic increments

Humidity analysis increments (near 850 hPa)



With isotropic correlations

With anisotropic correlations,
filtered by localisation



Conclusions

Conclusions

- Data assimilation is vital for weather forecasting.
- Observations are very diverse in type, density and quality.
- 4D-Var for temporal and non linear aspects.

- Observation-background departures for estimation of average variances and correlations in **R** and **B**.
- Ensemble DA for error simulation and for covariance dynamics.

- Sampling noise issues and filtering methods.
- Towards 4DEnVar (variational assimilation based on a 4D ensemble).



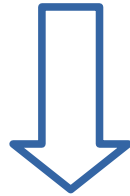
Thanks for your attention

Spécification des incertitudes

(observations, modèle) du système d'analyse/prévision



Assimilation d'ensemble : simulation de la propagation des erreurs au cours du "cyclage" de l'assimilation



Spécification des covariances d'erreur d'ébauche, formulations **EnVar** de l'assimilation

Prévision d'ensemble : simulation de la propagation / amplification des erreurs au cours de la prévision



Prévision probabiliste : traitement statistique des prévisions de l'ensemble

Properties of innovation methods

- Provides estimates in observation space.
- A good quality data dense network is needed.
- Assumption that observation errors are spatially uncorrelated.
- An objective source of information on **B** and **R**.
- At a given location and time, only 1 innovation value : only a single error realization is available.

=> Statistical averages (expectations) are replaced by space and time averages (ergodic assumption).

4DEnVar

Variational analysis based on a 4D Ensemble

Minimisation of $J(\underline{\delta\mathbf{x}})$ where $\underline{\delta\mathbf{x}}$ is a 4D analysis increment :

$$J(\underline{\delta\mathbf{x}}) = \underline{\delta\mathbf{x}}^T \underline{\mathbf{B}}^{-1} \underline{\delta\mathbf{x}} + (\underline{\mathbf{d}} - \underline{\mathbf{H}} \underline{\delta\mathbf{x}})^T \underline{\mathbf{R}}^{-1} (\underline{\mathbf{d}} - \underline{\mathbf{H}} \underline{\delta\mathbf{x}})$$

with $\underline{\mathbf{B}} = \underline{\mathbf{X}}^{b'} \underline{\mathbf{X}}^{b'T} \circ \underline{\mathbf{L}}$, where $\underline{\mathbf{L}}$ is a localization matrix,

$$\underline{\mathbf{X}}^{b'} = (\underline{\mathbf{x}}^{b'}_1, \dots, \underline{\mathbf{x}}^{b'}_{N^e}),$$

$$\underline{\mathbf{x}}^{b'}_{ne} = \underline{\mathbf{x}}^b_{ne} - \langle \underline{\mathbf{x}}^b \rangle / (N^e - 1)^{1/2}, \quad ne = 1, N^e.$$

$\underline{\mathbf{x}}^{b'}$ of dimension $K+1$ (time) \times M (3D variables) \times N (dim 3D).

(Liu et al, 2008, 2009 ; Buehner et al, 2010 ; Lorenc, 2012 ;

Desroziers et al 2014).

4DEnVar

Variational analysis based on a 4D Ensemble

- 4D covariances from an ensemble of trajectories.
- Improved realism of 4D background error covariances (anisotropies, non linear evolution including all physical processes).
- Lesser need to develop and maintain an adjoint model in this case.
 - Especially important for AROME.
- Pursue within the variational framework
 - Global assimilation of all available observations, distributed in space and in time.
- Introduces additional levels of parallelism (space, time, ensemble).