Data assimilation in meteorology

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Plan of the talk

- Numerical Weather Prediction (NWP) and Data Assimilation (DA)
- Observations (in-situ and remote sensing)
- Error Covariances and Ensemble DA
- *A posteriori* diagnostics (observation-minus-background departures)

1. Numerical Weather Prediction

and Data Assimilation

The two main ingredients of weather forecasting

What will be the weather tomorrow ?

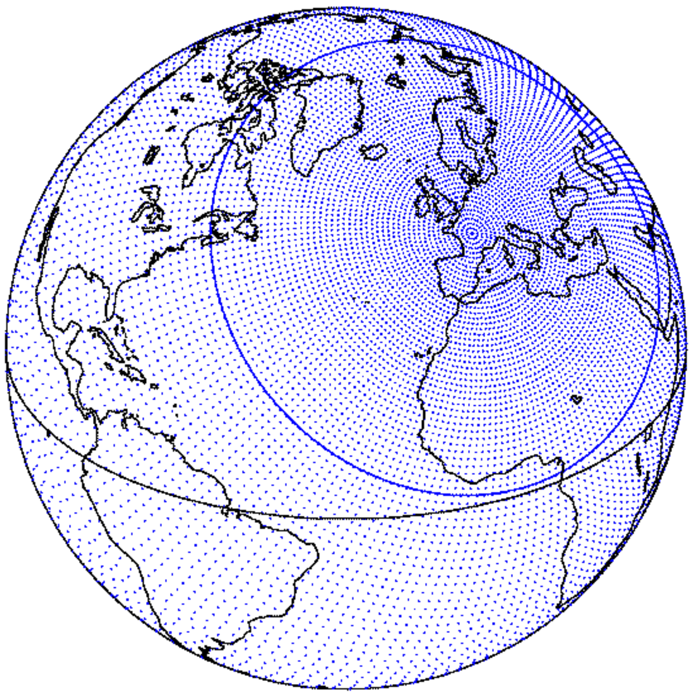
Bjerknes (1904) :

In order to do a good forecast, we need to :

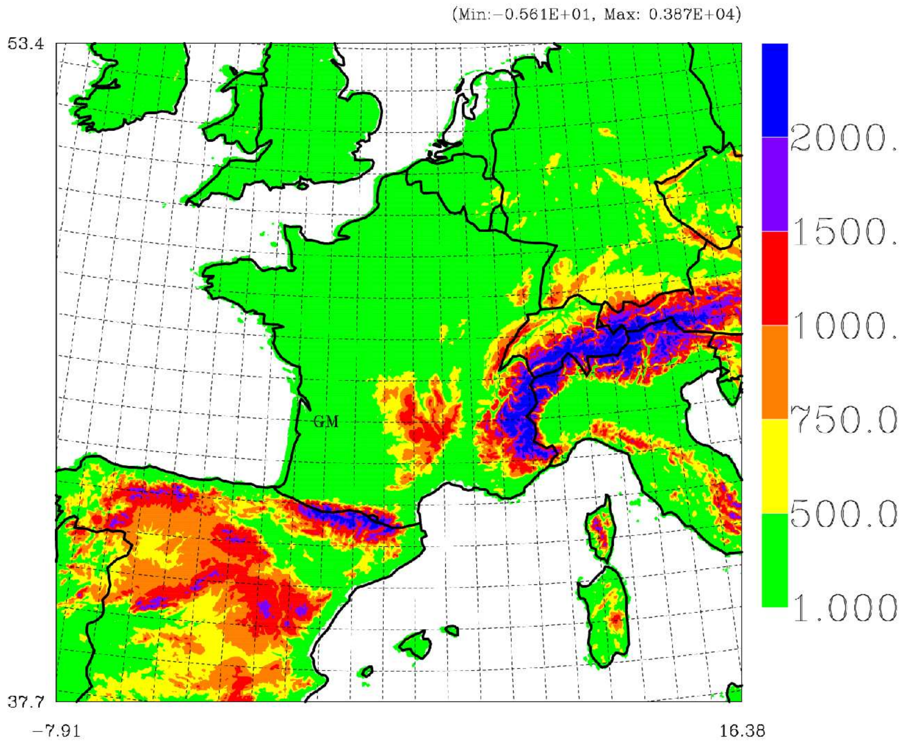
- know the atmospheric evolution laws
  (~ modeling) ;
- know the atmospheric state at initial time
  (~ data assimilation).

Numerical Weather Prediction at Météo-France
(in collaboration with e.g. ECMWF)

Global model (Arpège) : DX ~ 7-40 km

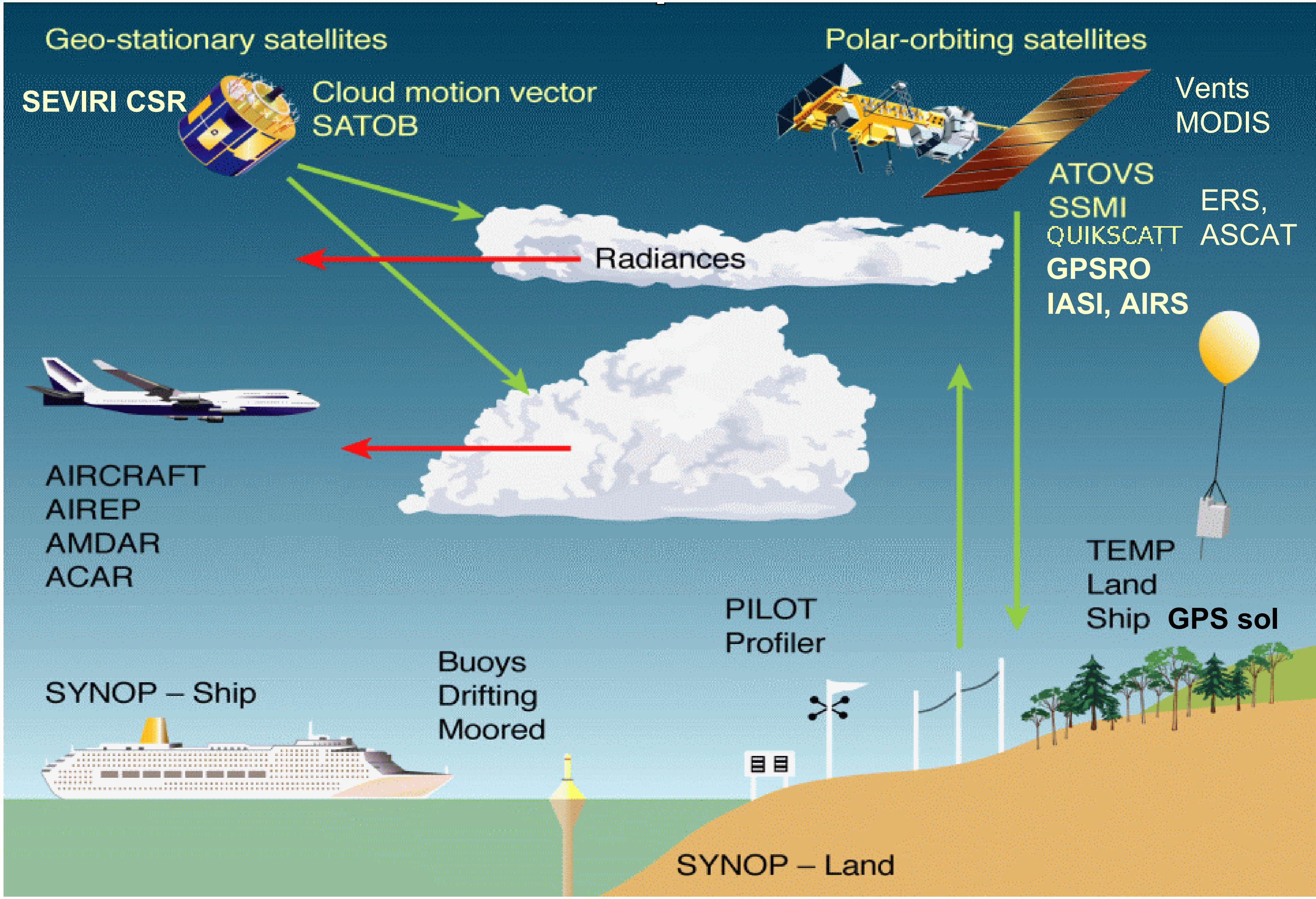


Arome : DX ~ 1.3 km



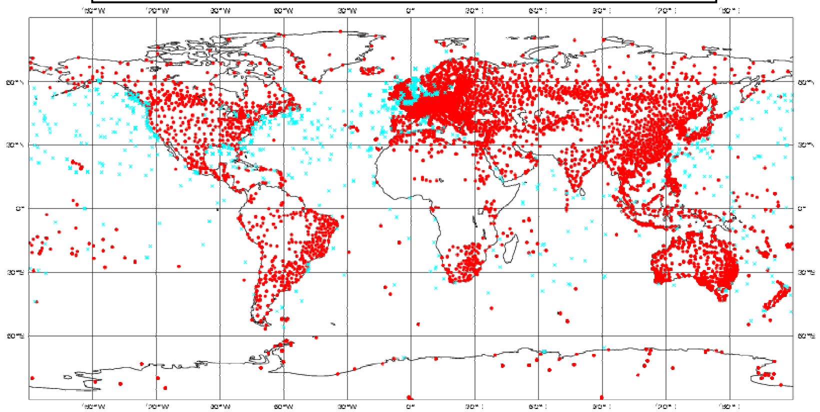
Equations of hydrodynamics and physical parametrizations (radiation, convection,…)
to predict the evolution of temperature, wind, humidity, …

Data that are assimilated in NWP models

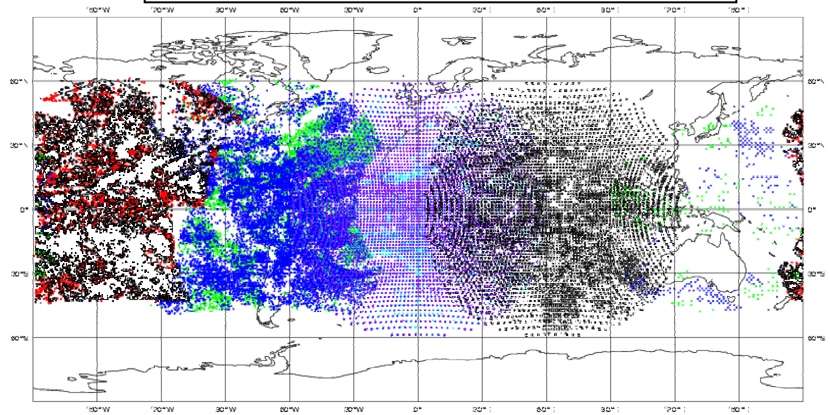


Spatial coverage and density of observations

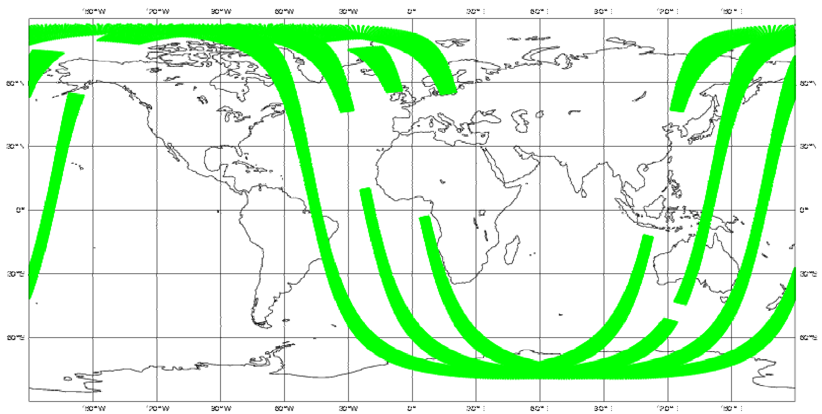
SURFACE DATA



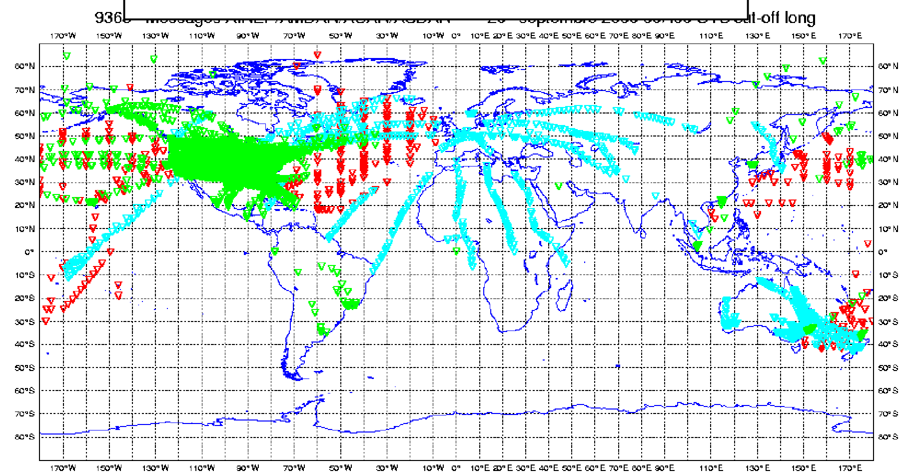
GEOSAT. WINDS



SCATTEROMETER

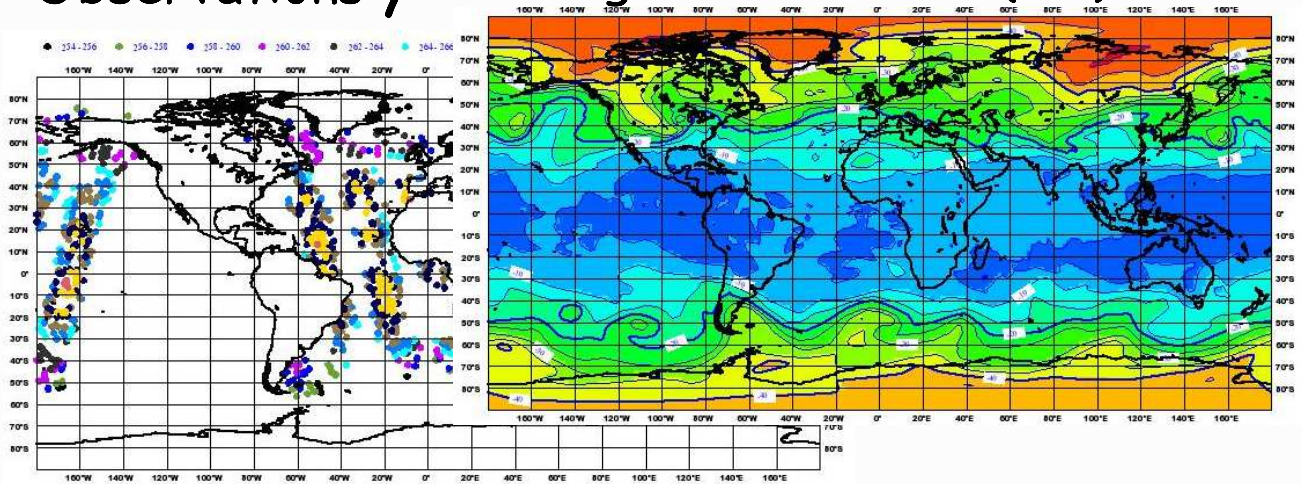


AIRCRAFT DATA

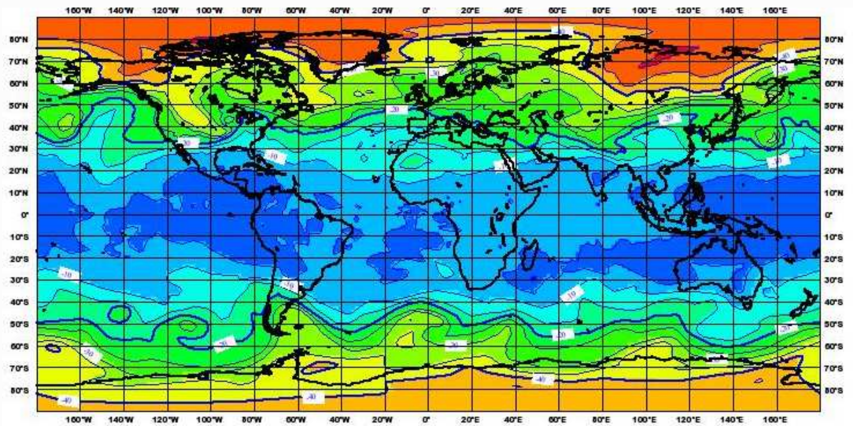


Data assimilation for NWP : illustration

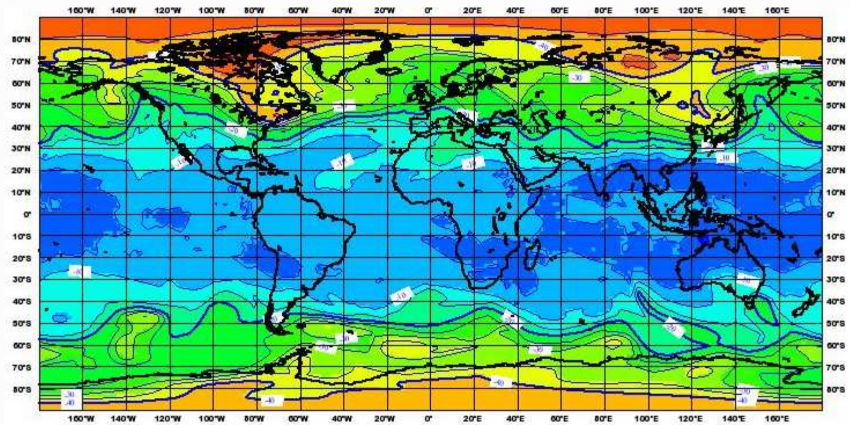
Observations $\mathbf{y}^o$ Background $\mathbf{x}^b = \boldsymbol{M}(\mathbf{x}^{a-})$



Analyzed state $\mathbf{x}^a$ at t_0



Forecast state $\mathbf{x}^f$ at t_0 + 96h



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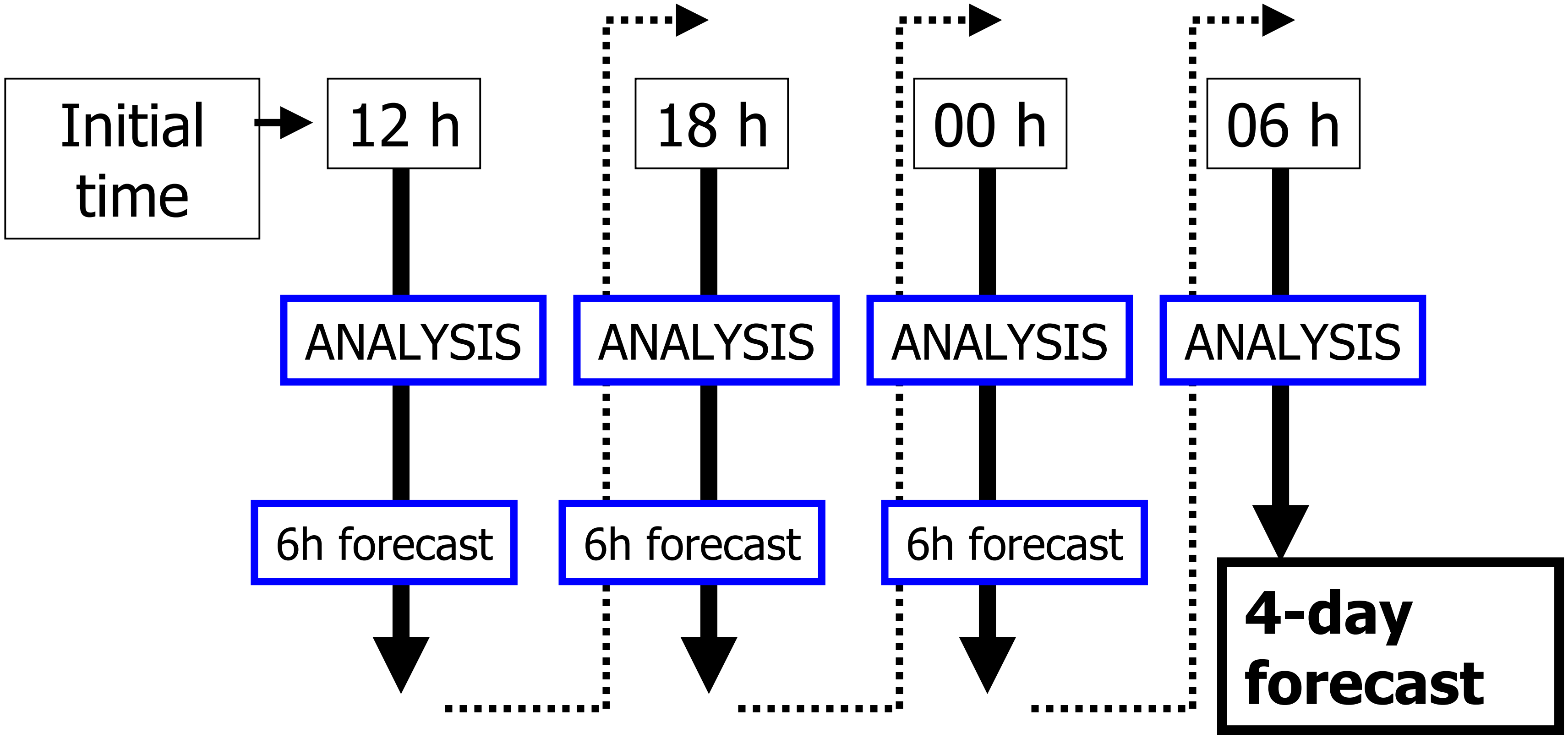
The data assimilation cycle



Memory of DA system is updated ~ continuously

Linear estimation of model state (1)

- BLUE analysis equation : $x^a = (I-KH)\, x^b + K\, y^o$
- H = observation operator = projection from model to observation space (e.g. spatial interpolation, radiative transfer, NWP model).
- K = observation weights :

$$K = BH^T \left(HBH^T + R \right)^{-1}$$

$$H\,K = \left(I + R\, (HBH^T)^{-1} \right)^{-1}$$

⇒ ~ ratio between background error covariances (matrix B)
and observation error covariances (matrix R).

⇒ Accounts for relative accuracy of observations,
and for spatial structures of background errors.

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Linear estimation of model state (2)

- Analysis increment equation :

$$x^a - x^b = K\,(y^o - H\, x^b)$$

$$\delta x = K\, \delta y$$

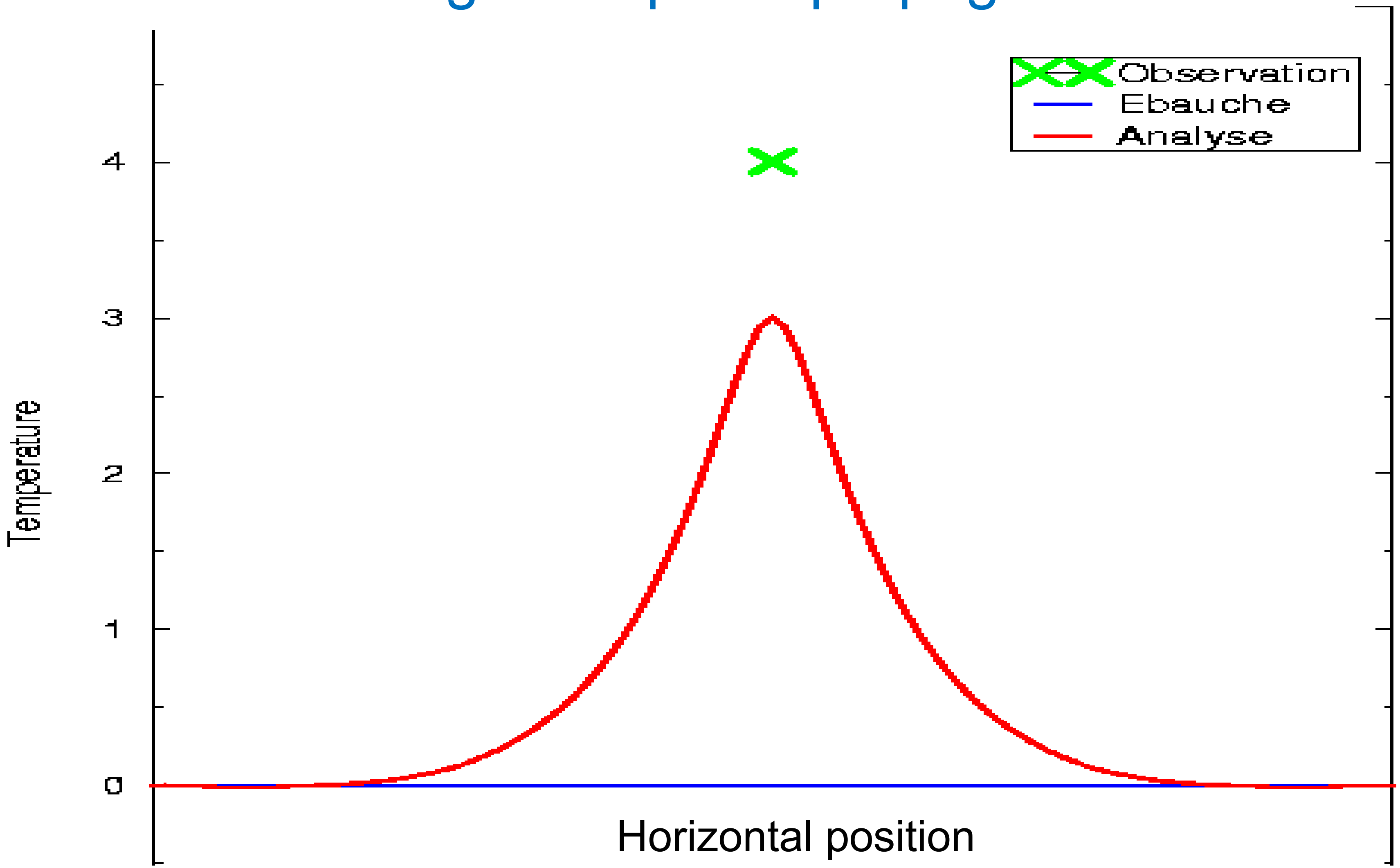
- Single-observation case (with uniform variances) :

$$\delta x(j) \;=\; cor^b(i,j)\;\; 1/(1+(\sigma^o/\sigma^b)^2)\;\; \delta y(i)$$

$\Rightarrow$ Filtering of observed information,
as a function of obs/bkd error variance ratios.

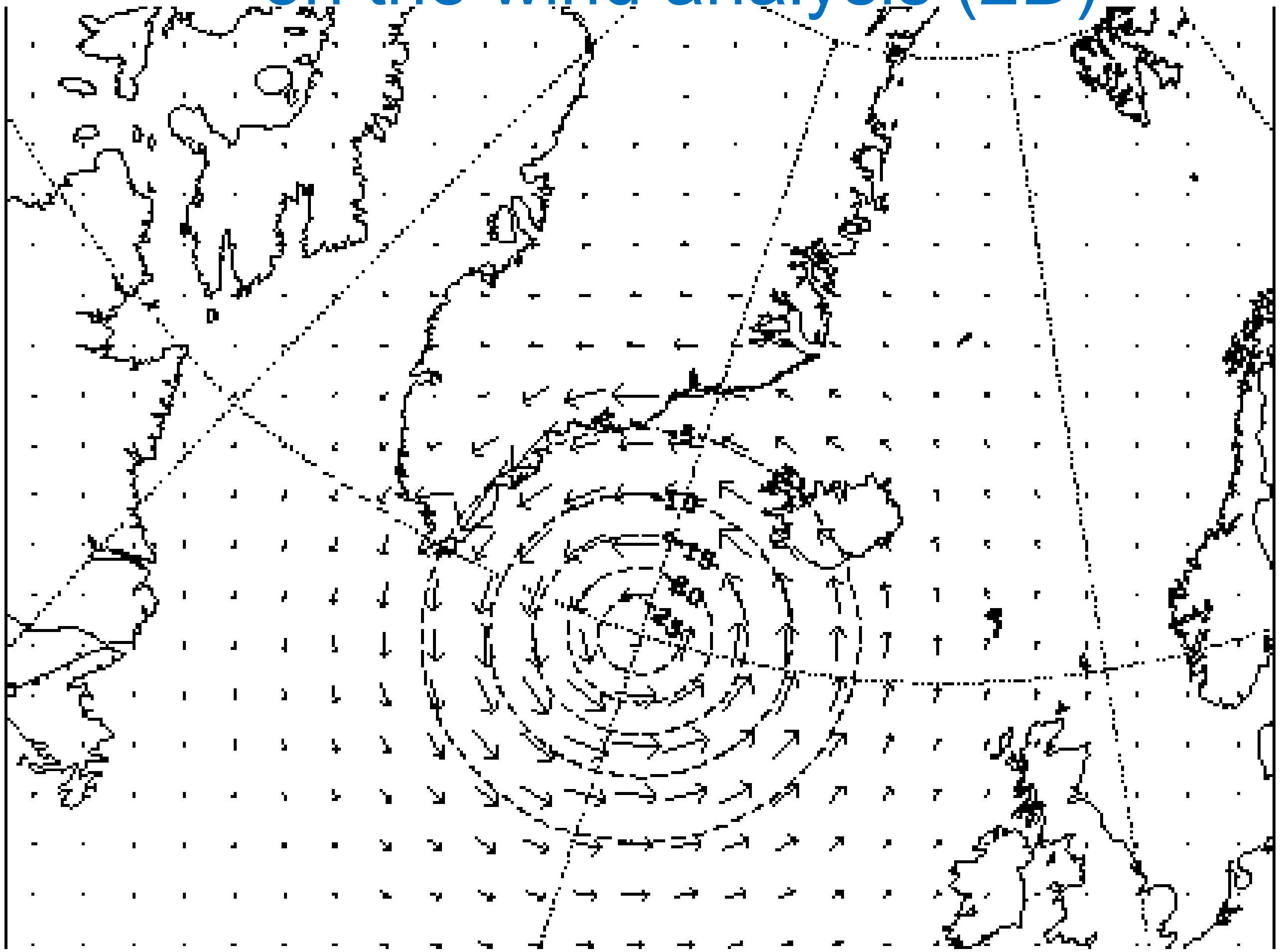
$\Rightarrow$ Spatial propagation of observed information,
as a function of background error correlations.

Impact of one observation (1D) : filtering and spatial propagation



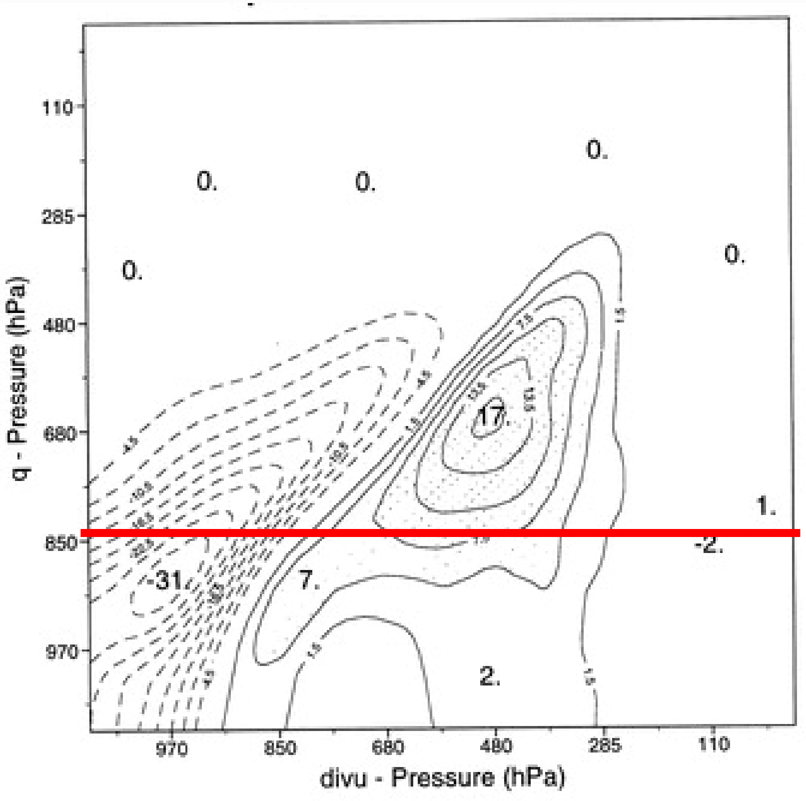
⇒ relative accuracies of observations and background, and characteristic spatial scales of bkd errors are accounted for.

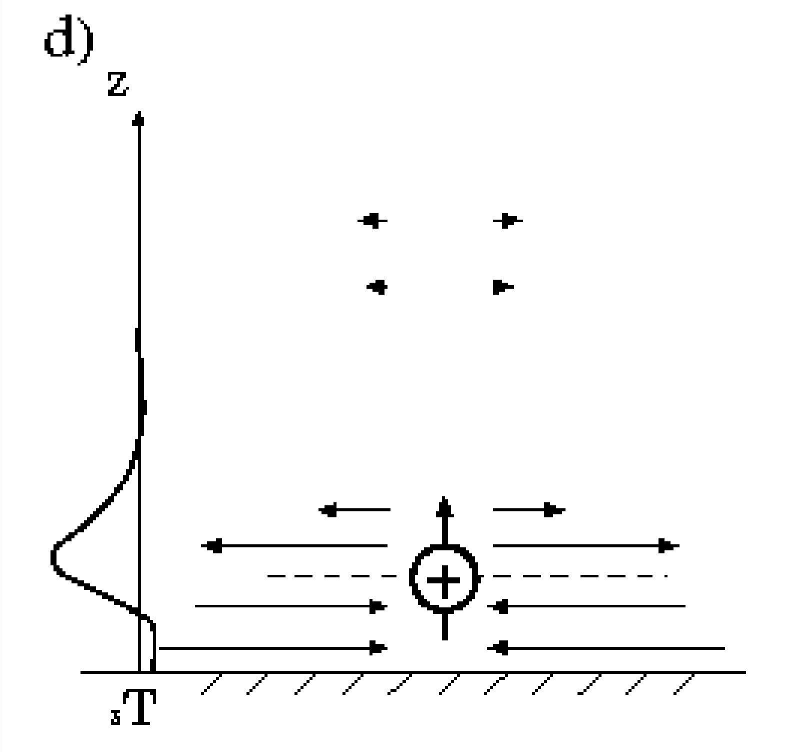
Impact of one surface pressure observation on the wind analysis (2D)



⇒ multivariate couplings (ex: pressure/wind) are also accounted for.

Divergence/humidity couplings





(Berre 2000, Montmerle et al 2006)

Linear estimation of model state (3)

- Size of B is huge : square of model size ~ $(10^8)^2 \sim 10^{16}$.

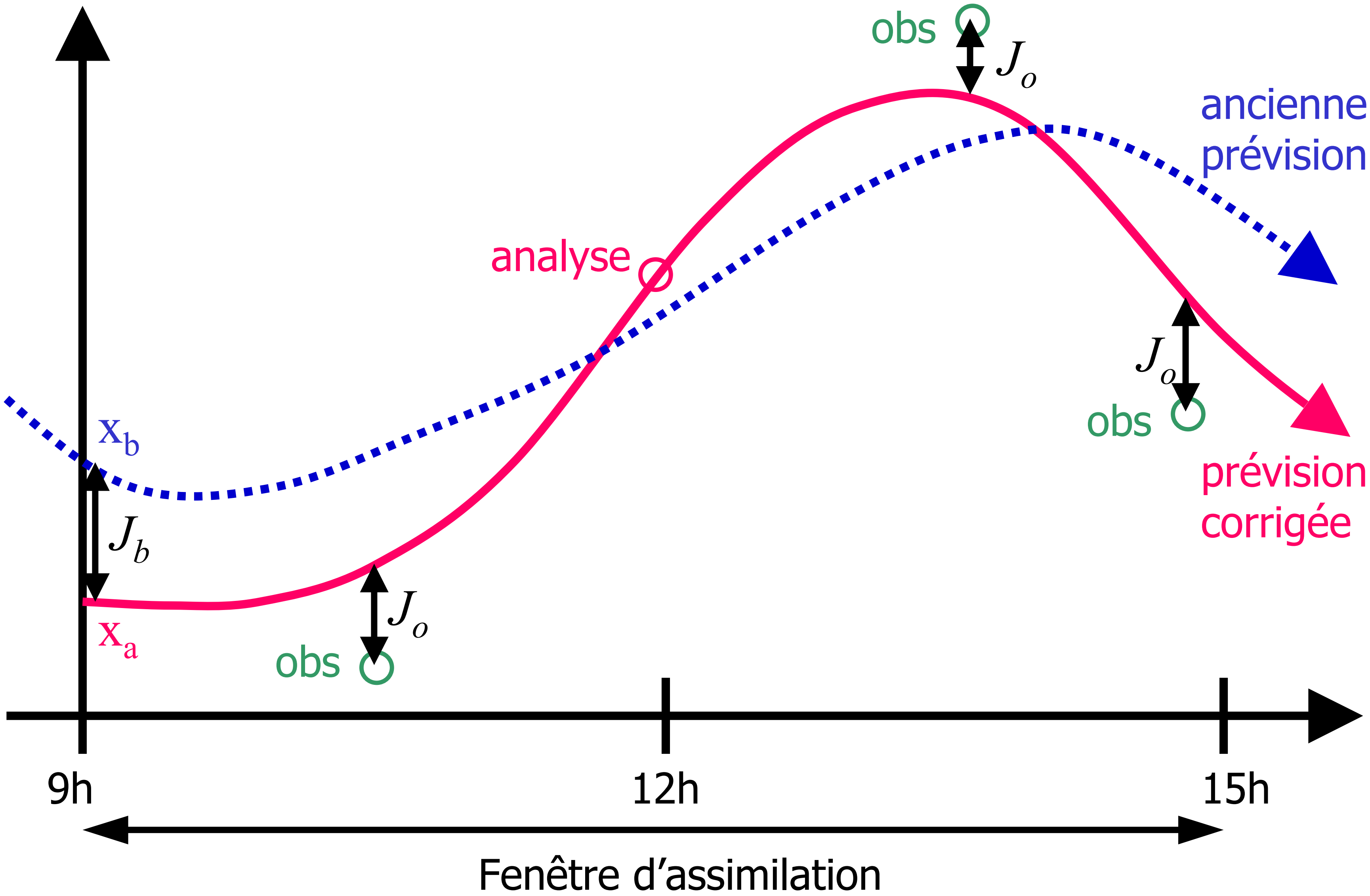
$\Rightarrow$ error covariances need to be estimated, simplified and modeled.

- Matrices too large to be inverted, but
  equivalent to minimize distance $J(x^a)$ to x^b and y^o (4D-Var)
  without explicit matrix inversions (e.g. Talagrand and Courtier 1987).

- Non linear features accounted for
  in calculation of departures between y^o and $H(x^b)$,
  and in iterative applications of 4D-Var.

Principle of 4D-VAR assimilation

(e.g. Talagrand and Courtier 1987, Rabier et al 2000)



Implementation of 4D-Var

- Analysis increment (BLUE equation) :

  $$\delta \mathbf{x} = \mathbf{x}^a - \mathbf{x}^b = \mathbf{K} (\mathbf{y}^o - \mathbf{H} \, \mathbf{x}^b) = \mathbf{K} \, \delta \mathbf{y}$$

  but **K** is difficult to handle explicitly in a real size system.

- Variational formulation :

  cost function : $J(\delta \mathbf{x}) = \delta \mathbf{x}^T \mathbf{B}^{-1} \delta \mathbf{x} + (\mathbf{d} - \mathbf{H} \, \delta \mathbf{x})^T \mathbf{R}^{-1} (\mathbf{d} - \mathbf{H} \, \delta \mathbf{x})$

  minimised when gradient $J'(\delta \mathbf{x})=0$ (equivalent to BLUE).

- Computation of J': development and use of adjoint operators (transpose).

- Generalized observation operator *H* : includes NWP model *M*.

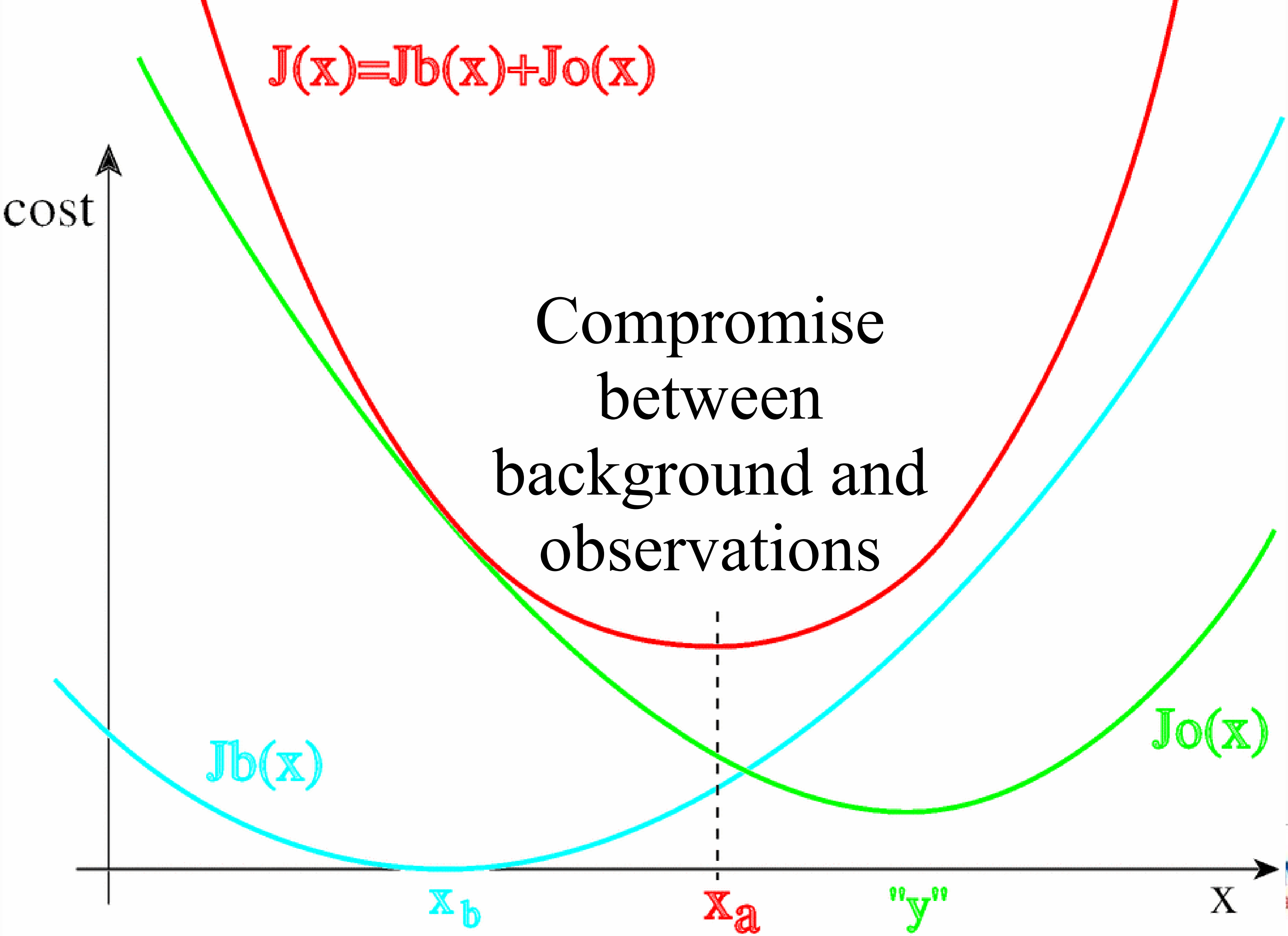
- Cost reduction :

  analysis increment $\delta \mathbf{x}$ can be computed at low resolution

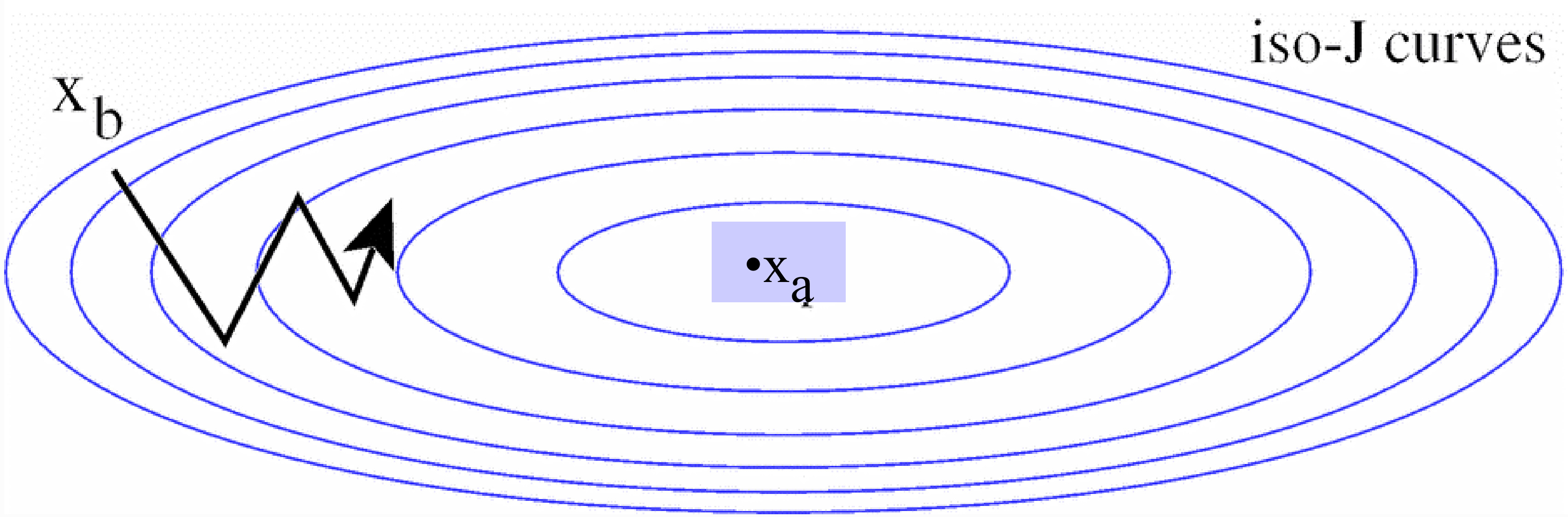
  (Courtier, Thépaut et Hollingsworth, 1994)

Schematic representation of

$$J(x) = (x - x_b)^T B^{-1} (x - x_b) + (y - H[x])^T R^{-1} (y - H[x])$$



Importance of preconditioning



• Some gradient directions have much larger amplitudes than others : problem of “narrow valley" linked to the metric of x.

• Use a change of variable such as J becomes nearly “circular”: much faster convergence.

2. In-situ observations and remote sensing data

Observation networks in meteorology: in situ measurements

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Observation networks in meteorology: satellite data

Constellation of polar orbiting or geostationary satellites

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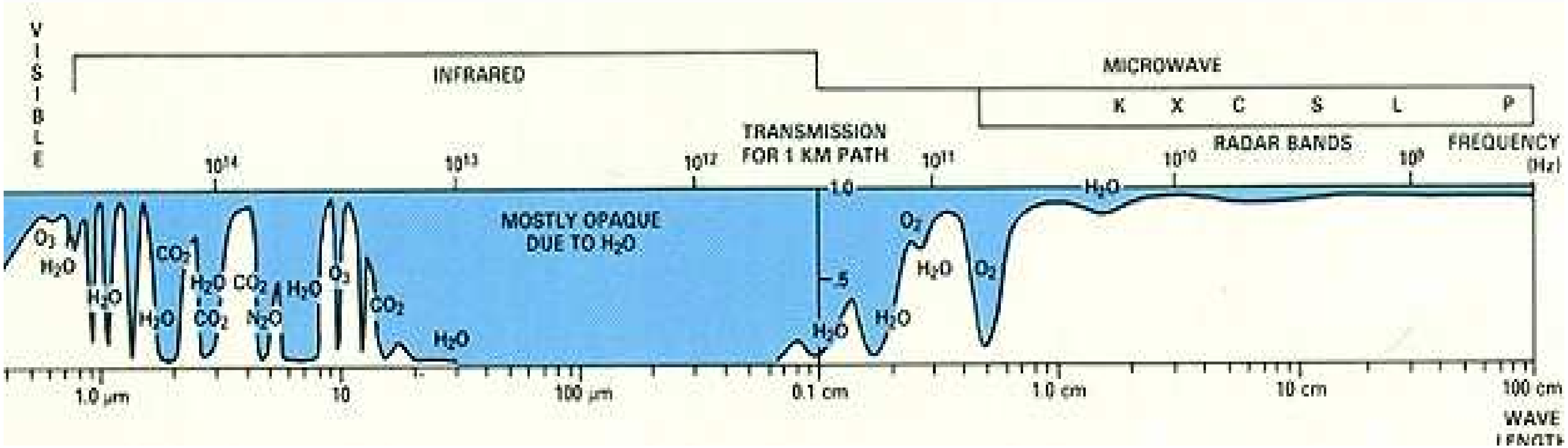
What is measured by satellite sensors ?

- Sensors do not measure directly atmospheric temperature and humidity, but electromagnetic radiation : brightness temperature or radiance.

- Depending on wave length (or frequency), information on gas concentration or physical properties (temperature or pressure or humidity) of atmosphere.

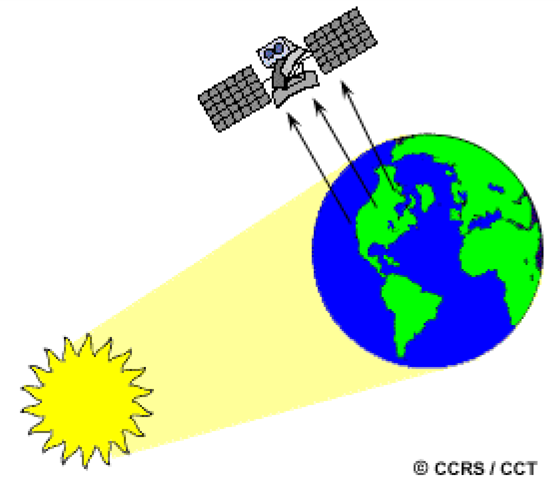
- Observations in atmospheric windows ➔ information on surface.



What is measured by satellite sensors ?

Passive measures

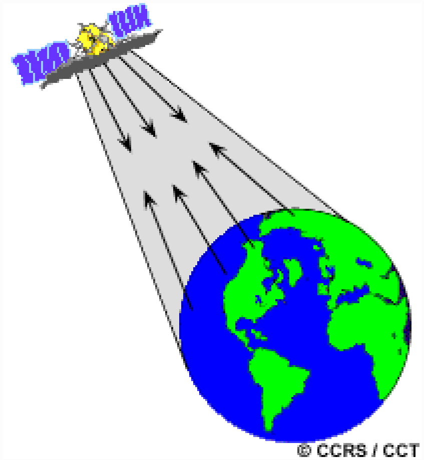
(no energy emitted from instrument)



Measures natural radiation emitted by Earth/Atmosphere from Sun origin

Active measures

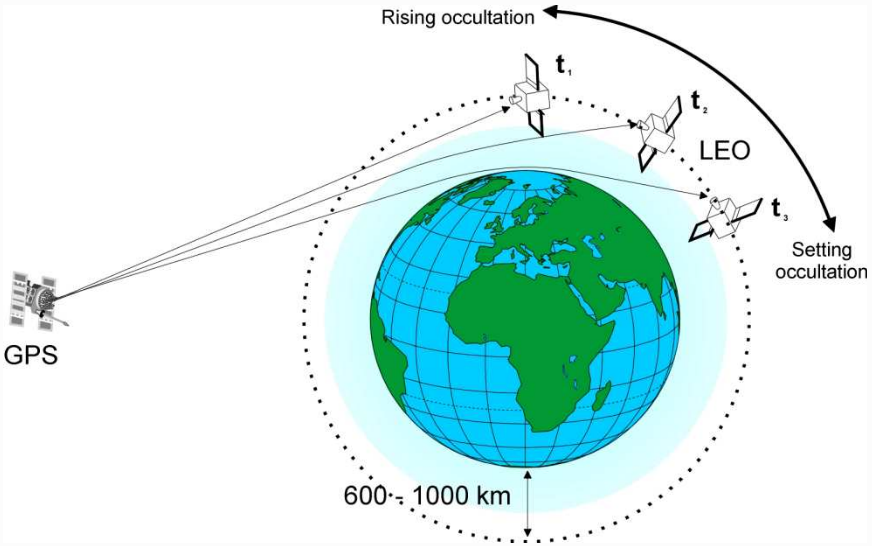
(energy emitted from instrument)



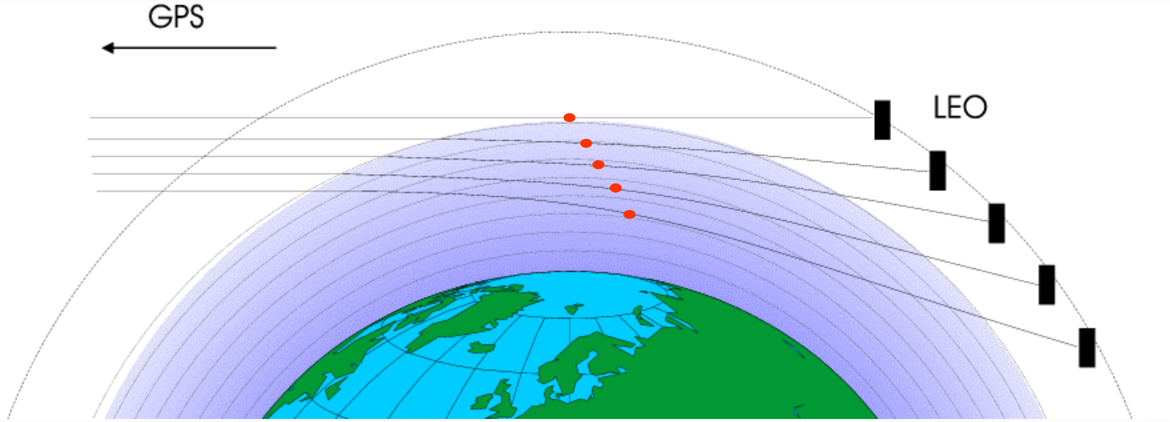
Radiation emitted by satellite and then reflected or diffused by Earth/Atmosphere

Example of active remote sensing

GPS radio occultation:



- Low-Earth Orbit satellites receive a signal from a GPS satellite.
- The signal passes through the atmosphere and gets refracted along the way.
- The magnitude of the refraction depends on temperature, moisture and pressure.
- The relative position of GPS and LEO changes over time => vertical scanning of the atmosphere.



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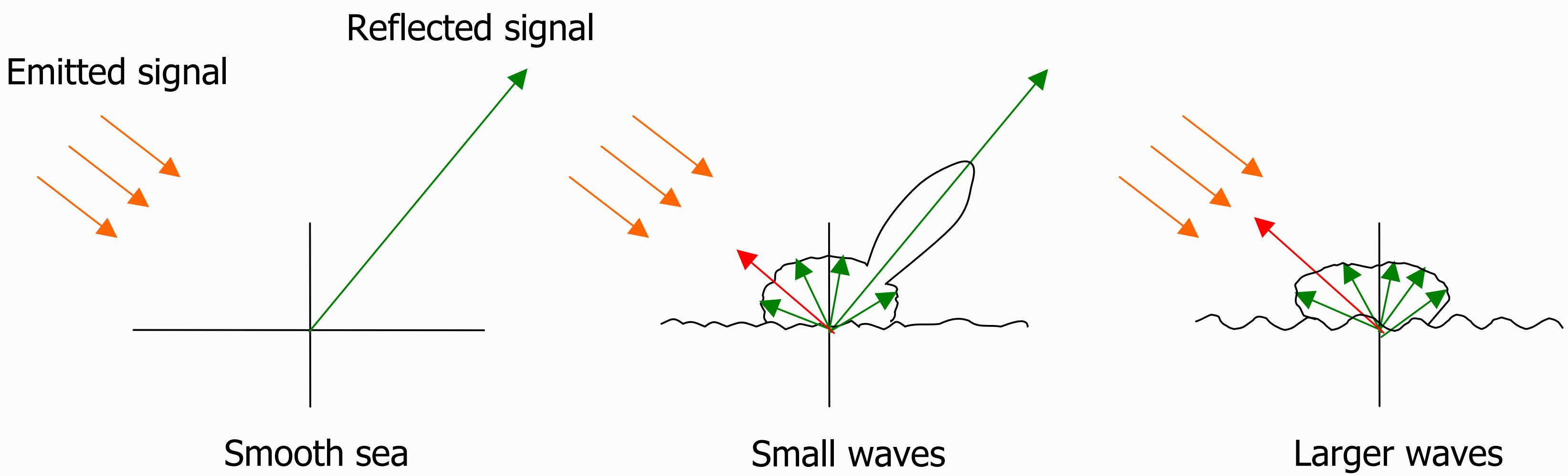
GPS stations of Météo France: Toulouse and Guipavas

- Propagation of GPS signal is slowed by atmosphere (dry air and water vapour)
- More than 500 GPS stations over Europe provide an estimation of Zenith Total Delay (ZTD) in real time to weather centres.
  - All weather instrument
  - High temporal resolution

Scatterometers

They send out a microwave signal towards a sea target.
The fraction of energy returned to the satellite depends on wind speed and direction.



=> Measurements of near surface wind over the ocean,
through backscattering of microwave signal reflected by waves.

Passive remote sensing

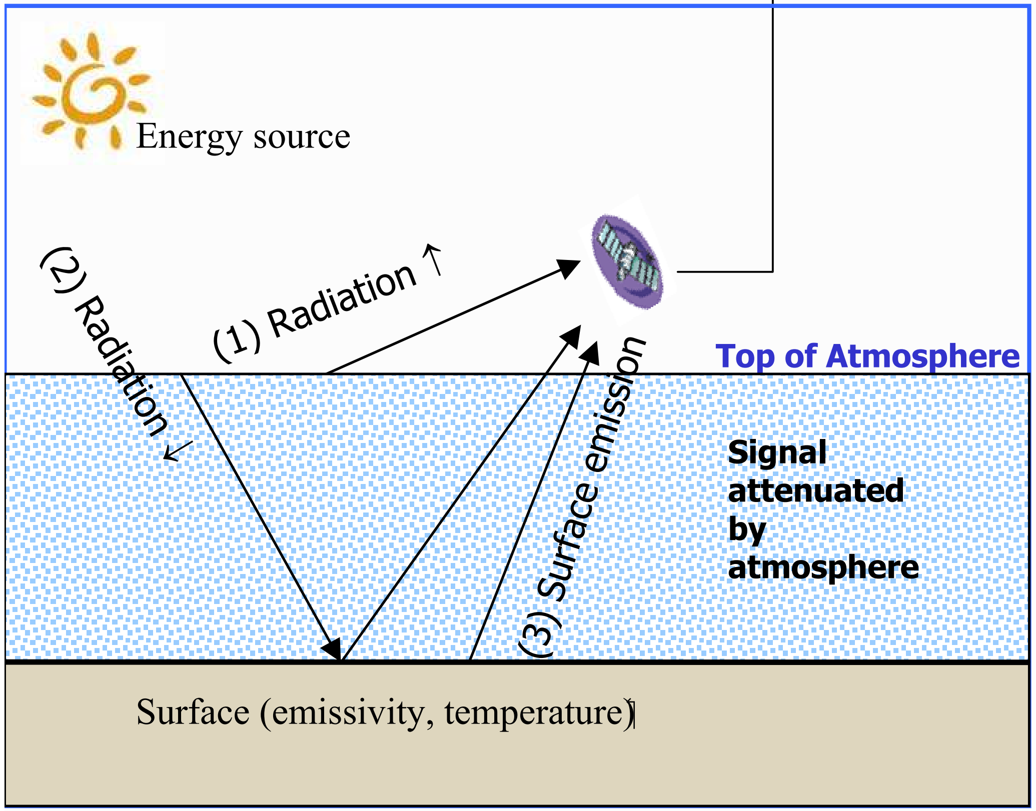
Only natural sources of radiation (sun, earth…) are involved, and the sensor is a simple receiver, « passive ».

Atmosphere in Parallel Plan, no diffusion, specular surface

$$T(p,\upsilon)=\varepsilon(p,\upsilon)Ts\tau+(1-\varepsilon(p,\upsilon))\tau T(\upsilon,\downarrow)+T(\upsilon,\uparrow)$$

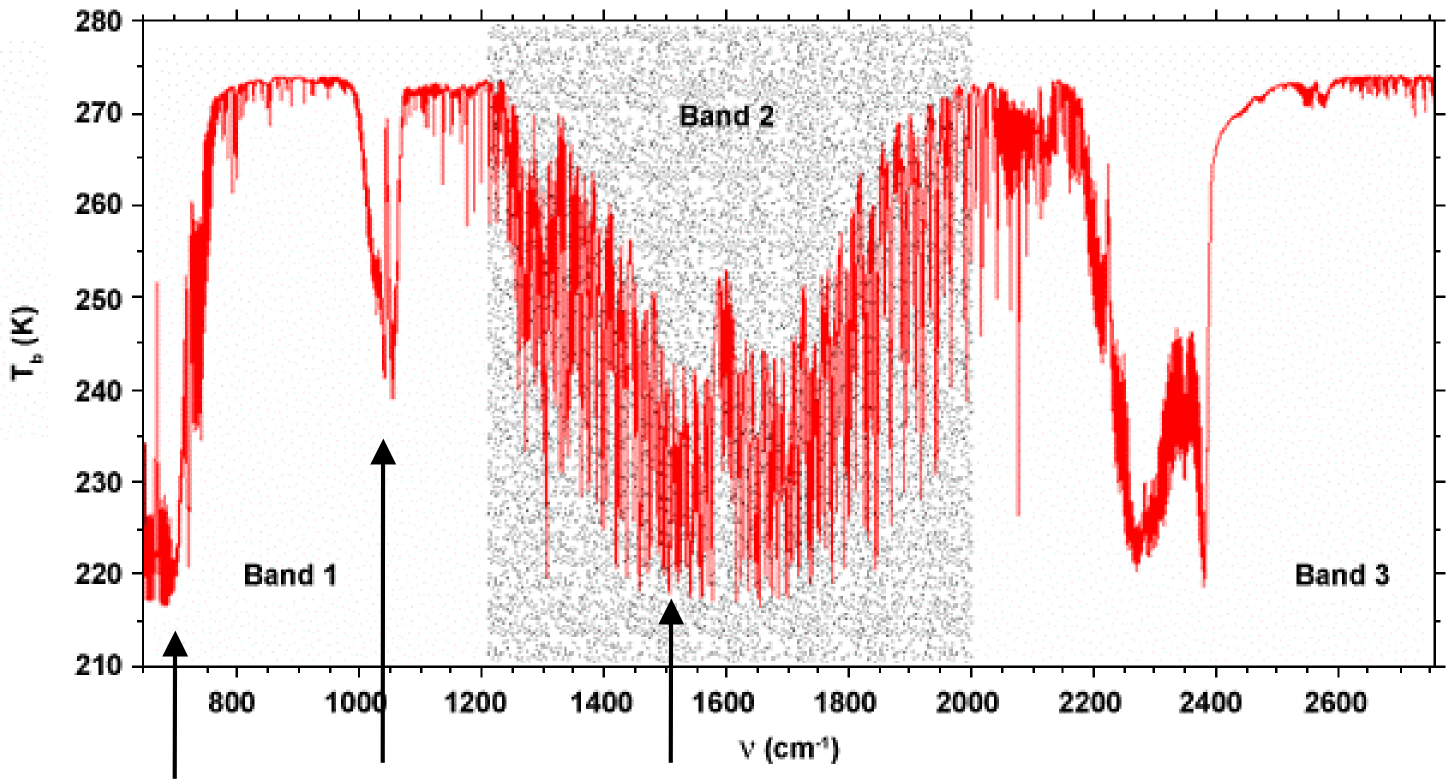
Emissivity

Model outputs for RT: T, Q forecast or radiosondes or reanalyses



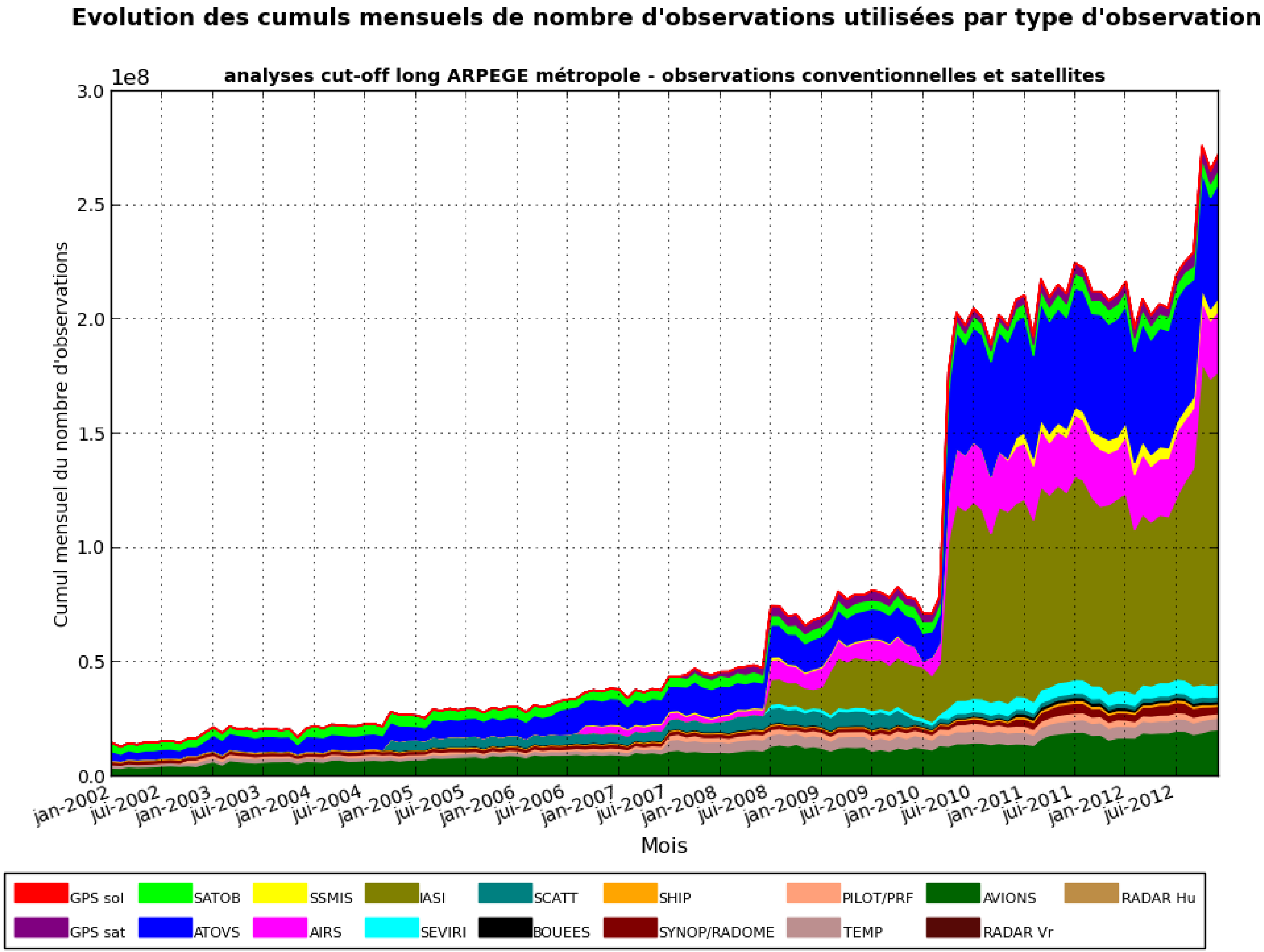
IASI, infra-red interferometer developed by CNES and EUMETSAT

IASI offers a very high spectral resolution



Temperature ozone Water vapor

Number of observations used in ARPEGE
(global DA at Météo-France)



Radar network in France

- 24 radars (17 Doppler C-Band, every 15 minutes).

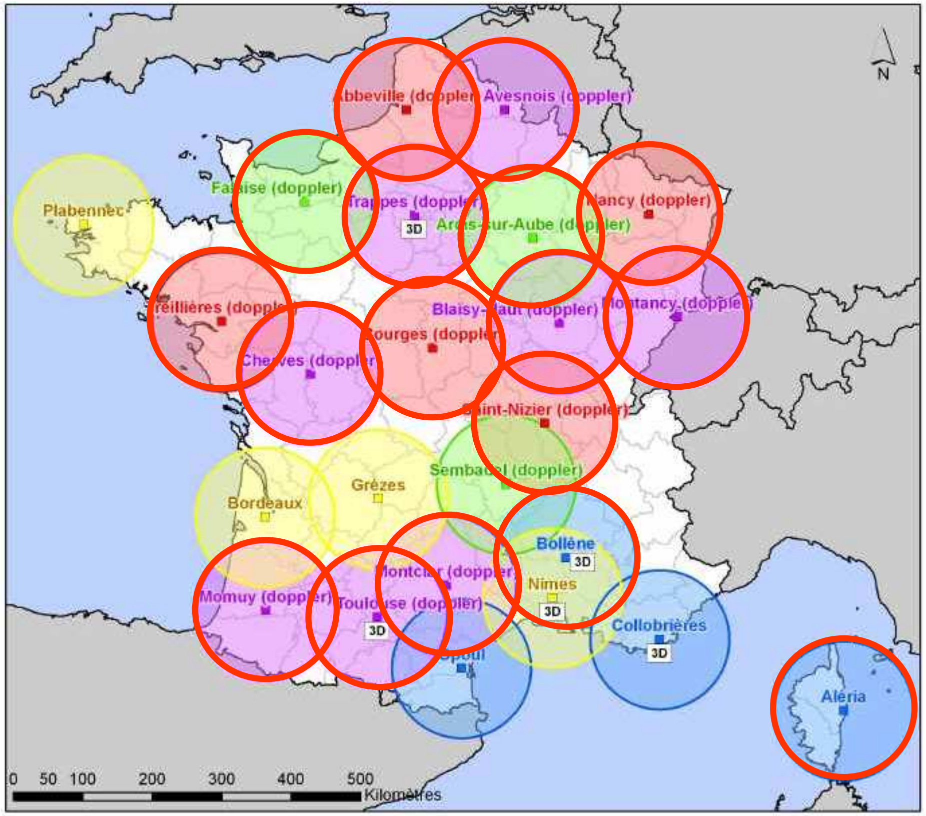
Doppler Radar

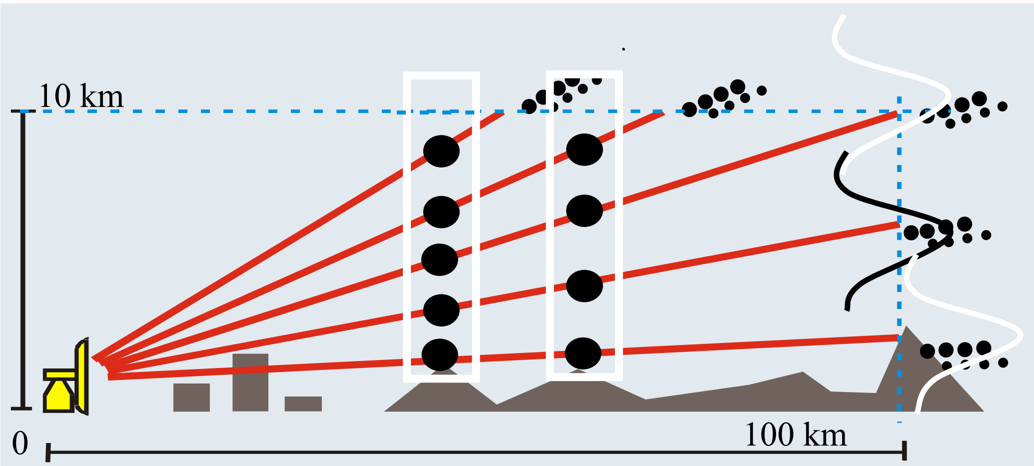
- Observations :

reflectivities Z (related to precipitation),

radial winds Vr (doppler effect : modified frequency of signal, when the target is moving => wind observation),

archived at 1km resolution.





Observations assimilated as profiles in the model

Pixel altitude is computed using a constant refractivity index along the path (effective radius approximation)

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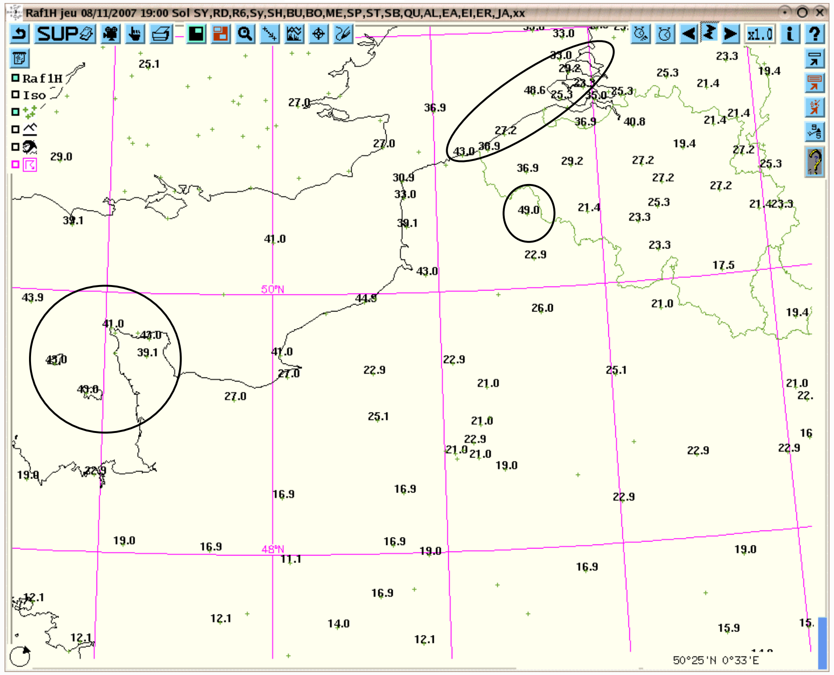
Assimilation of radar radial winds

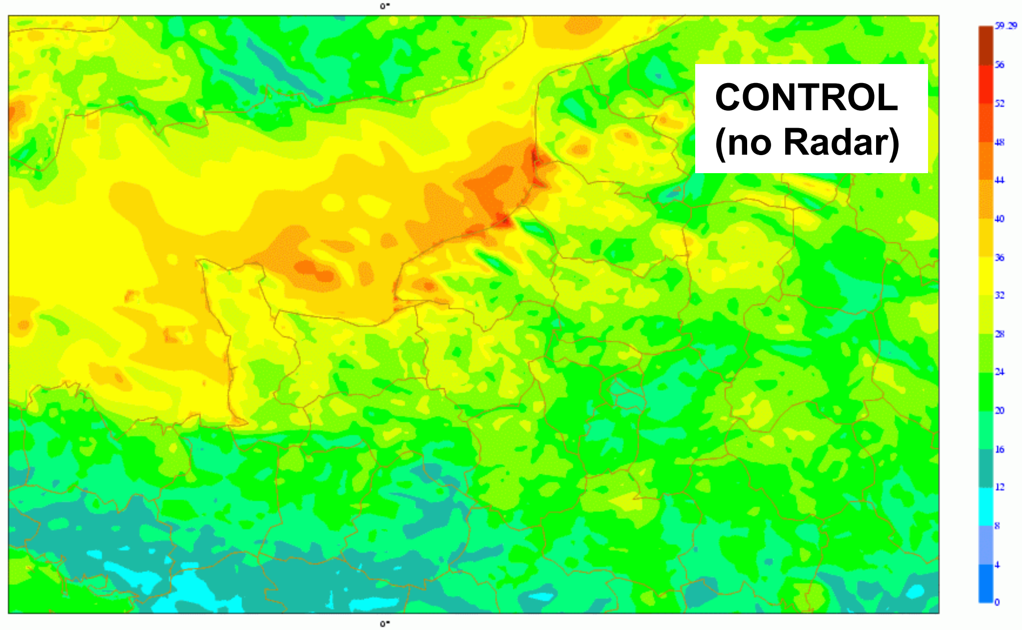
Wind gust at 10 m (kt)

Forecast +1h (19 UTC)

OBS



Thursday 8 November 2007 18UTC PARIS Forecast t+1 VT: Thursday 8 November 2007 19UTC 10m **



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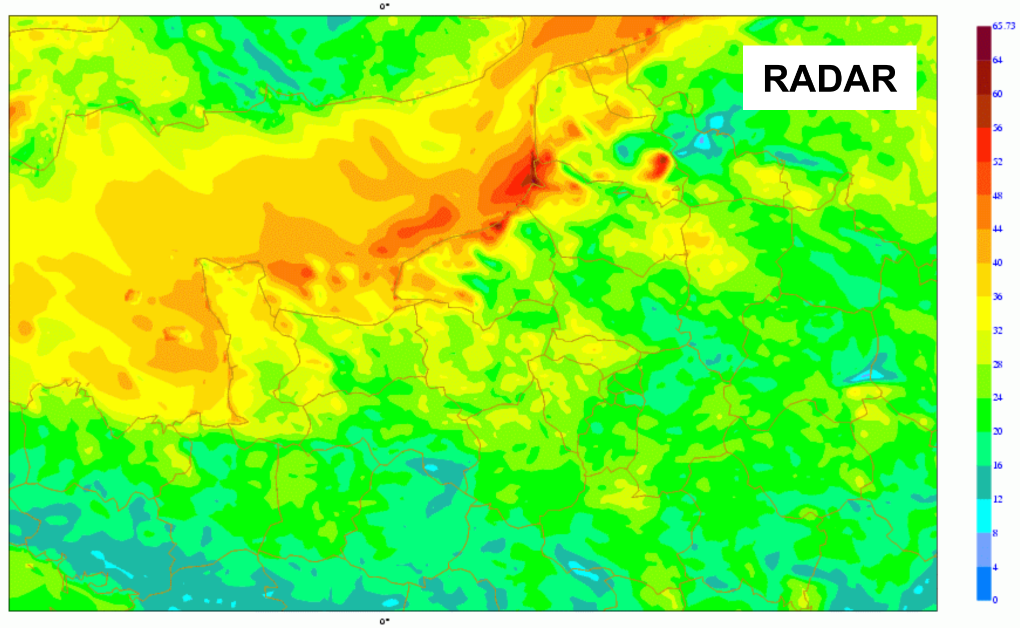
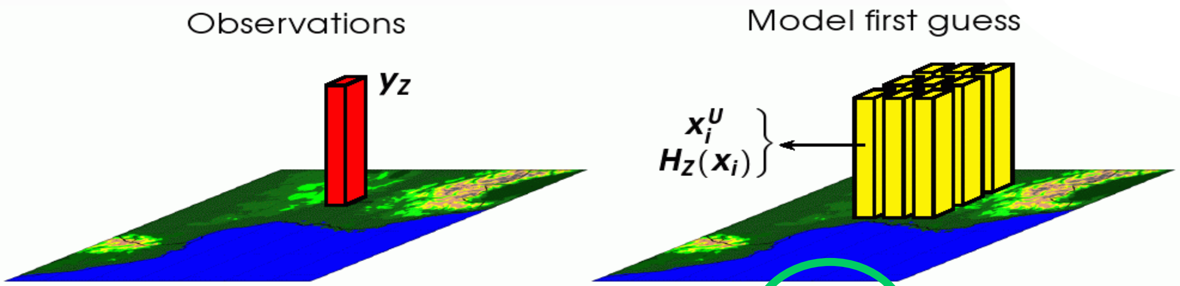


Inversion method of reflectivity profiles

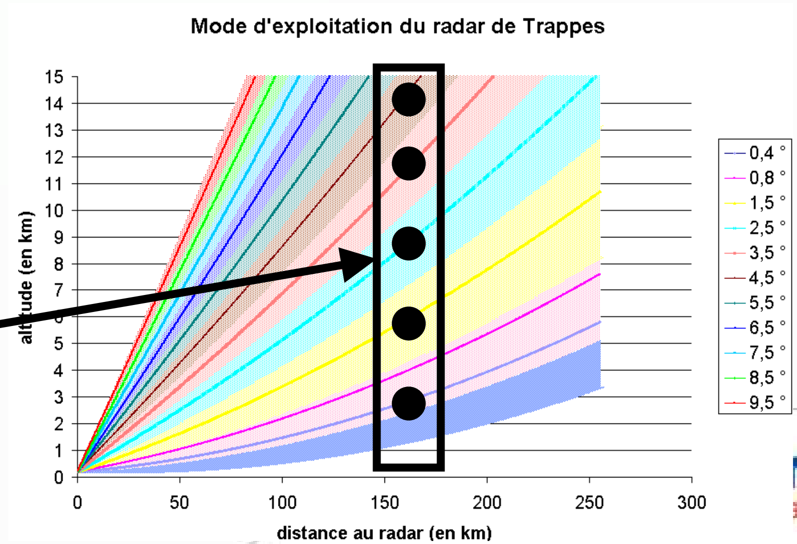
Caumont, 2006: use model profiles in the neighborhood of observations



$$y^U_{po} = \sum_{i \in \text{neighbours}} x_i^U \frac{\exp\left(-\frac{1}{2}\|y_Z - H_Z(x_i)\|^2\right)}{\sum_{j \in \text{neighbours}} \exp\left(-\frac{1}{2}\|y_Z - H_Z(x_j)\|^2\right)}$$

Observation operator for reflectivities

y^U_{po}: column of pseudo-observed relative humidity,
y_Z: column of observed reflectivities,
x_i^U: column of relative humidity,
$H_Z(x_i)$: column of simulated reflectivities.



Coherence between the inverted profile and the precipitating cloud that the model is able to create y_{po}^U

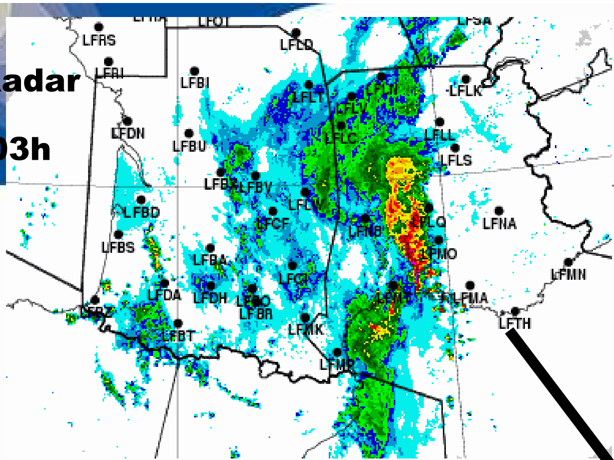
Case 7/8 october: South-East

Comparison of 3h FORECASTS
between REFL runs and CONTROL runs:

line of heavy precipitation is well analysed in REFL.

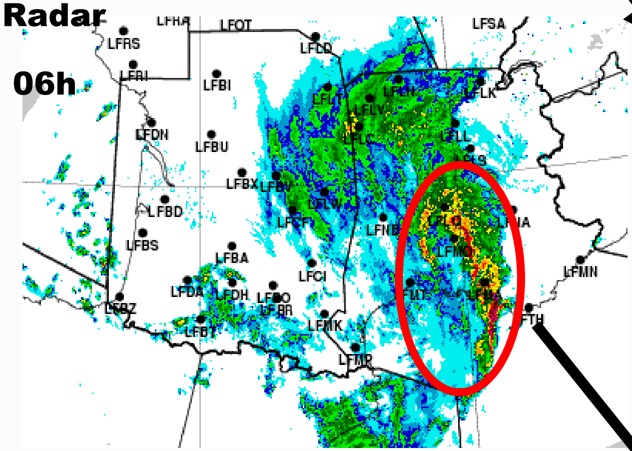
Radar

03h



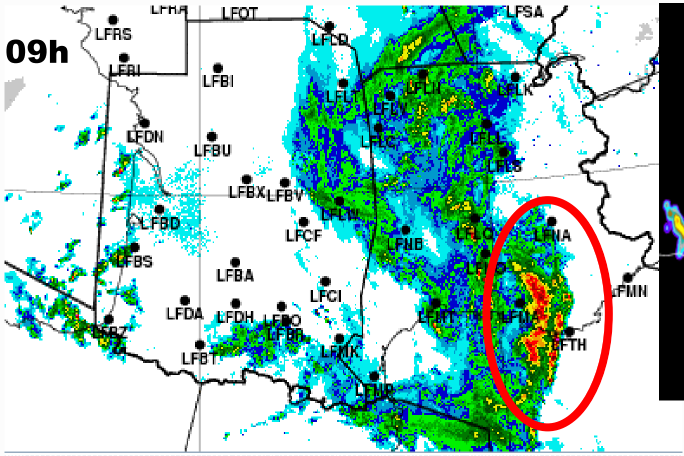
Radar

06h

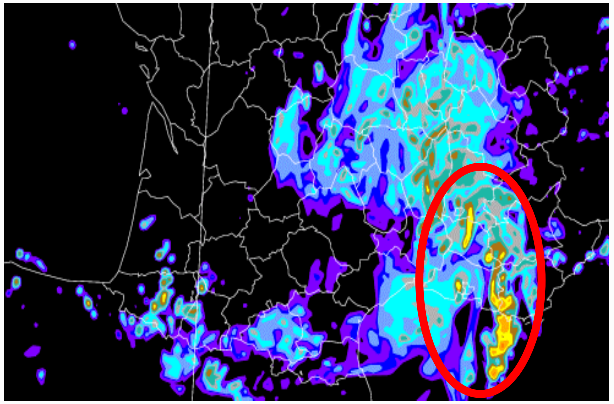


Radar

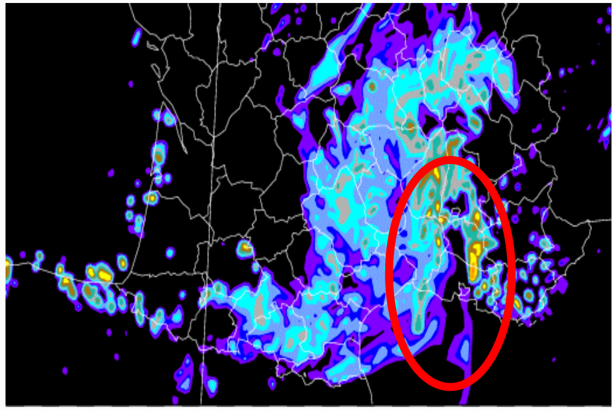
09h



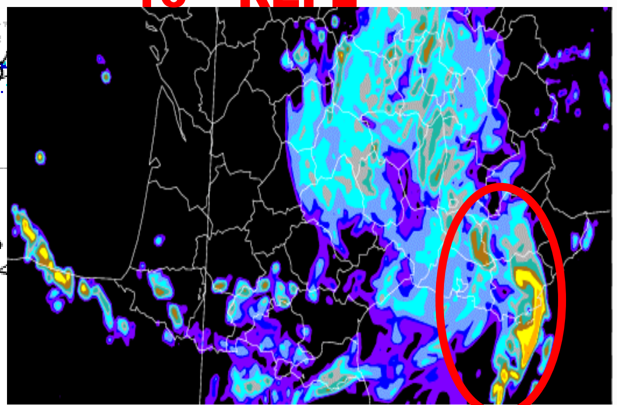
r3 – REFL



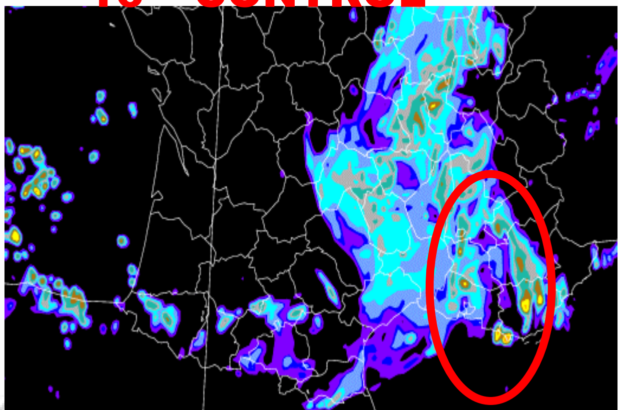
r3 – CONTROL



r6 – REFL



r6 – CONTROL



3. Error Covariances and Ensemble Data Assimilation

Observation weights and Error covariances

- BLUE analysis equation :

$$x^a = (I-KH)\, x^b + K\, y^o$$

- K = observation weights :

$$K = BH^T\, (\, HBH^T + R\,)^{-1}$$

$\Rightarrow$ Need to estimate B and R, before specifying them.

How can we estimate error covariances ?

- The true atmospheric state is never exactly known.

- Use observation-minus-background departures :

$$y_o - H x_b \sim (y_o - H x_t) + (H x_t - H x_b)$$

$$\sim e_o - H e_b$$

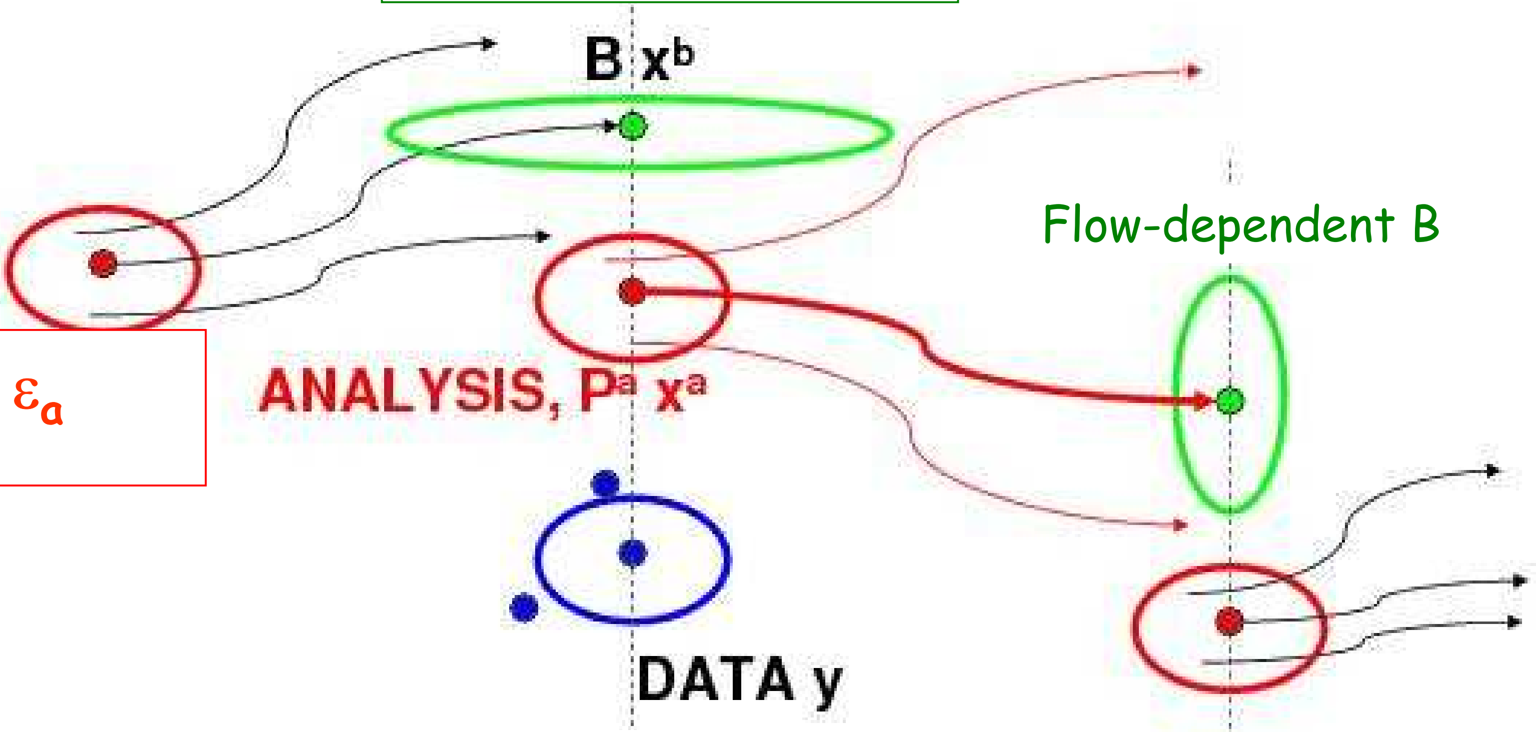
to estimate some average features (e.g. variances, correlations) of R and B, using assumptions on spatial structures of errors.

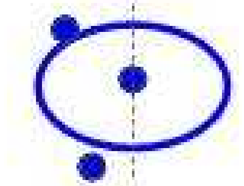
- Use ensemble to simulate the error evolution and to estimate complex background error structures.

Ensemble assimilation (EnDA = EnVar, EnKF, …) : simulation of the error evolution

$$\boxed{\varepsilon_f = \mathbf{M}\,\varepsilon_a + \varepsilon_m}$$

$$\boxed{\varepsilon_a}$$





Explicit observation perturbations, and
background perturbations (cycling + model error).

(Houtekamer et al 1996; Fisher 2003 ;
Ehrendorfer 2006 ; Berre et al 2006)

Analysis error equation

- Analysis state (BLUE, K = 4D-Var gain matrix) :

$$x_a = (I-KH)\, x_b + K\, y_o$$

- True state :

$$x_t = (I-KH)\, x_t + K\, Hx_t$$

- Analysis error :

$$e_a = x_a - x_t$$

i.e.

$$e_a = (I-KH)\, e_b + K\, e_o$$

Analysis perturbation equation

- Perturbed analysis :

$$x'_a = (I-KH)\, x'_b + K\, y'_o$$

- Unperturbed analysis :

$$x_a = (I-KH)\, x_b + K\, y_o$$

- Analysis perturbation :

$$\varepsilon_a = x'_a - x_a$$

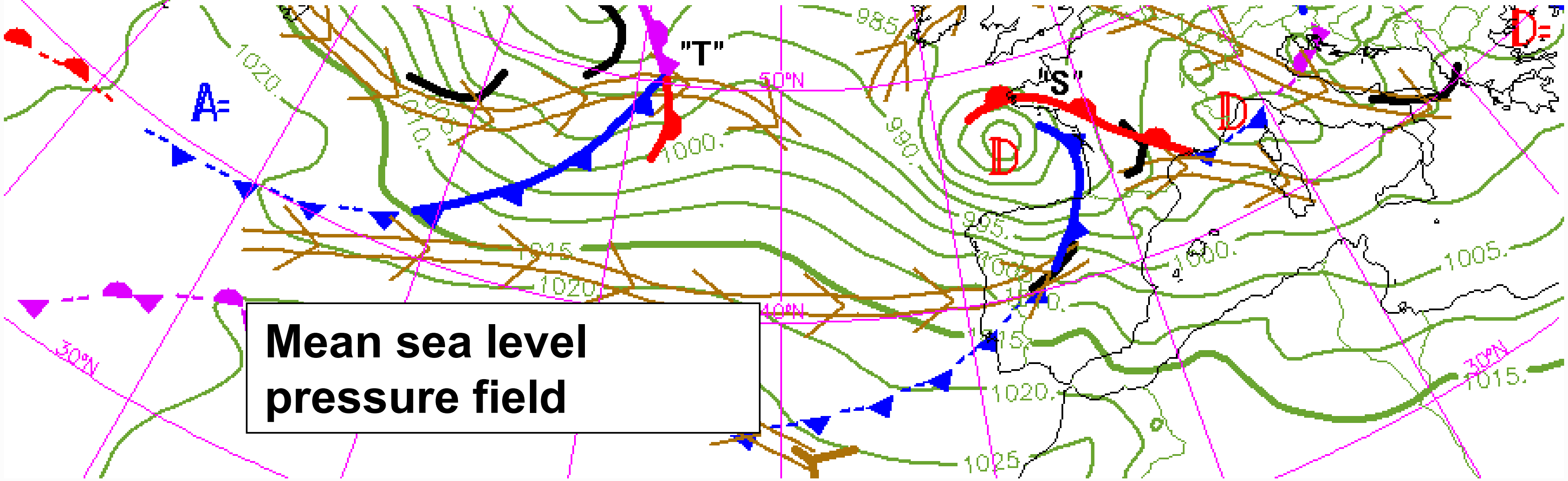
i.e. $$\varepsilon_a = (I-KH)\, \varepsilon_b + K\, \varepsilon_o$$

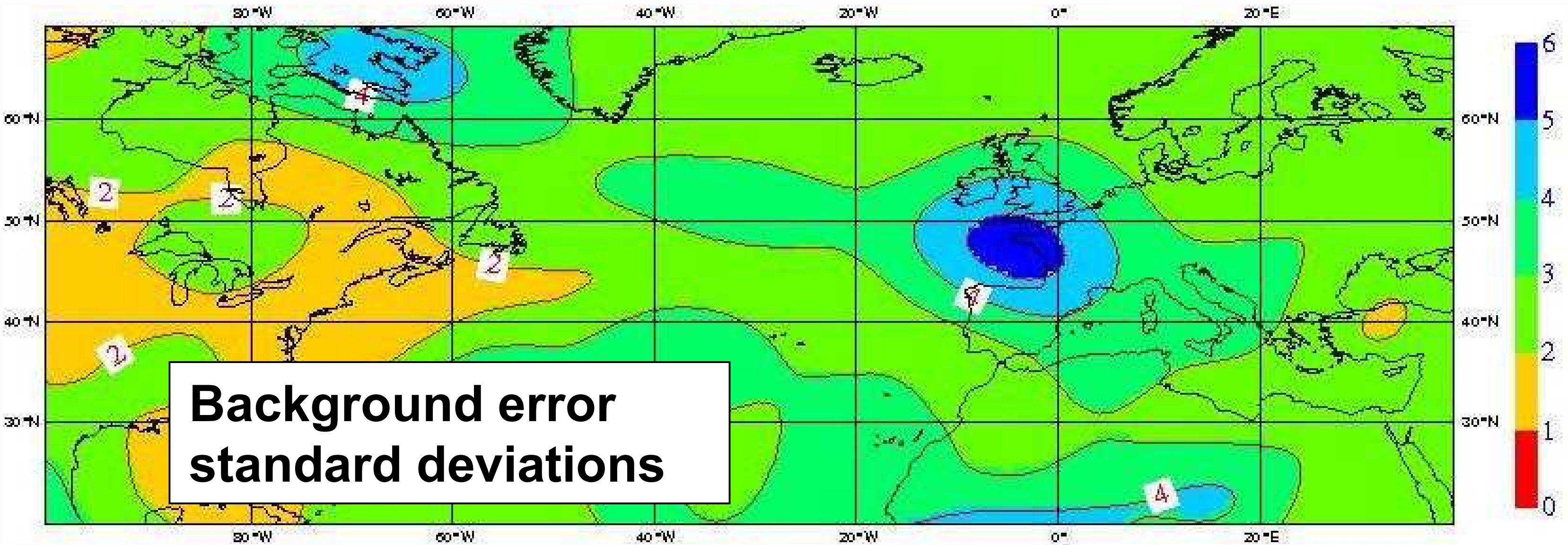
=> Estimate 4D-Var errors by using perturbed inputs.

Estimation of background error variances from ensemble spread

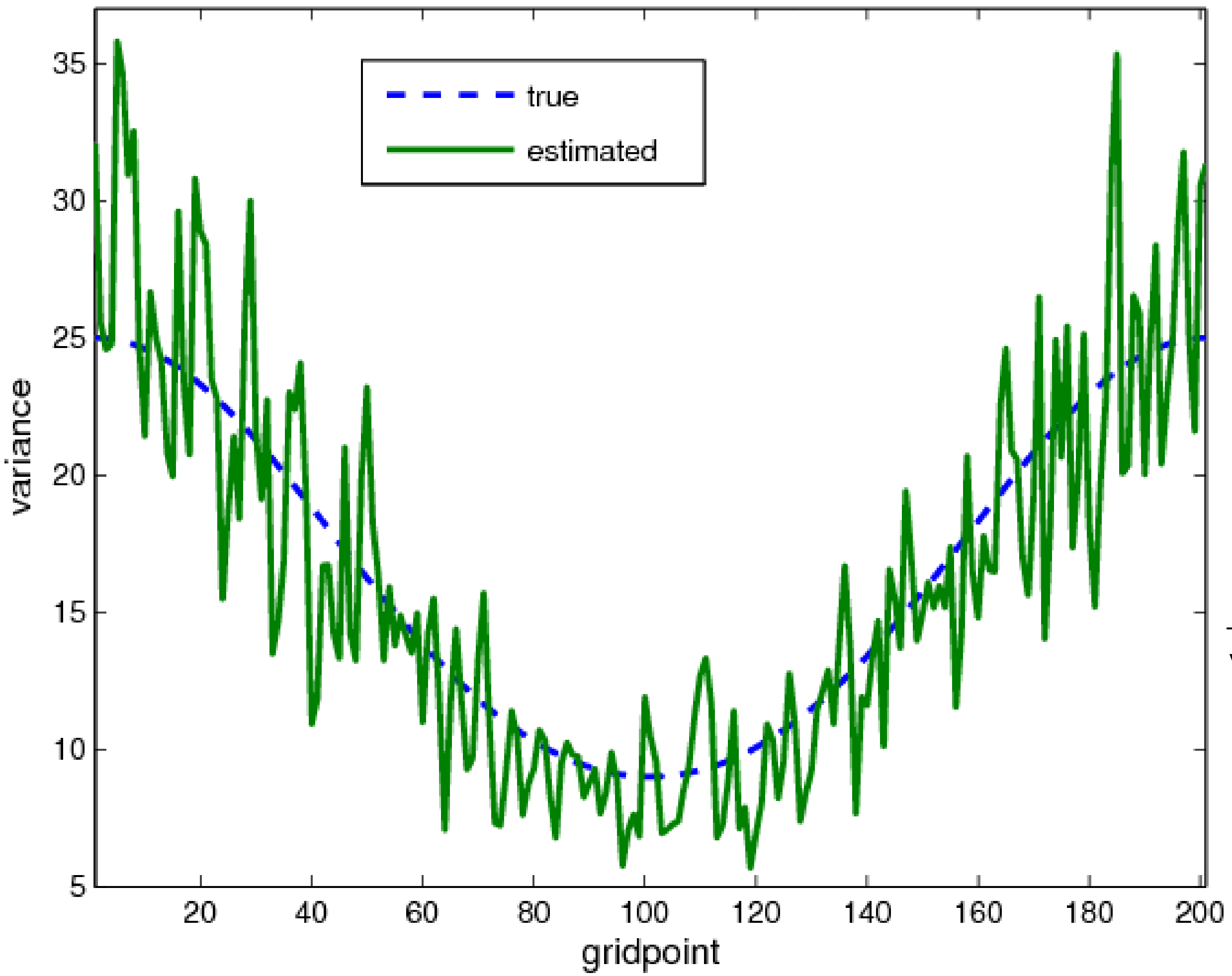
$$Var(e_b) = 1/(N-1) \sum_n (x'_b(n) - x'_b(mean))^2$$

Connexion between large errors and intense weather
(Klaus storm, 24/01/2009, 00/03 UTC)





Spatial structure of sampling noise for variances (Raynaud et al 2009, Berre and Desroziers 2010)



$\varepsilon_b = \mathbf{B}^{1/2}\,\eta$

$\eta \sim \mathcal{N}(0,\mathbf{I})$

N = 50 members

L(ε_b) = 200 km

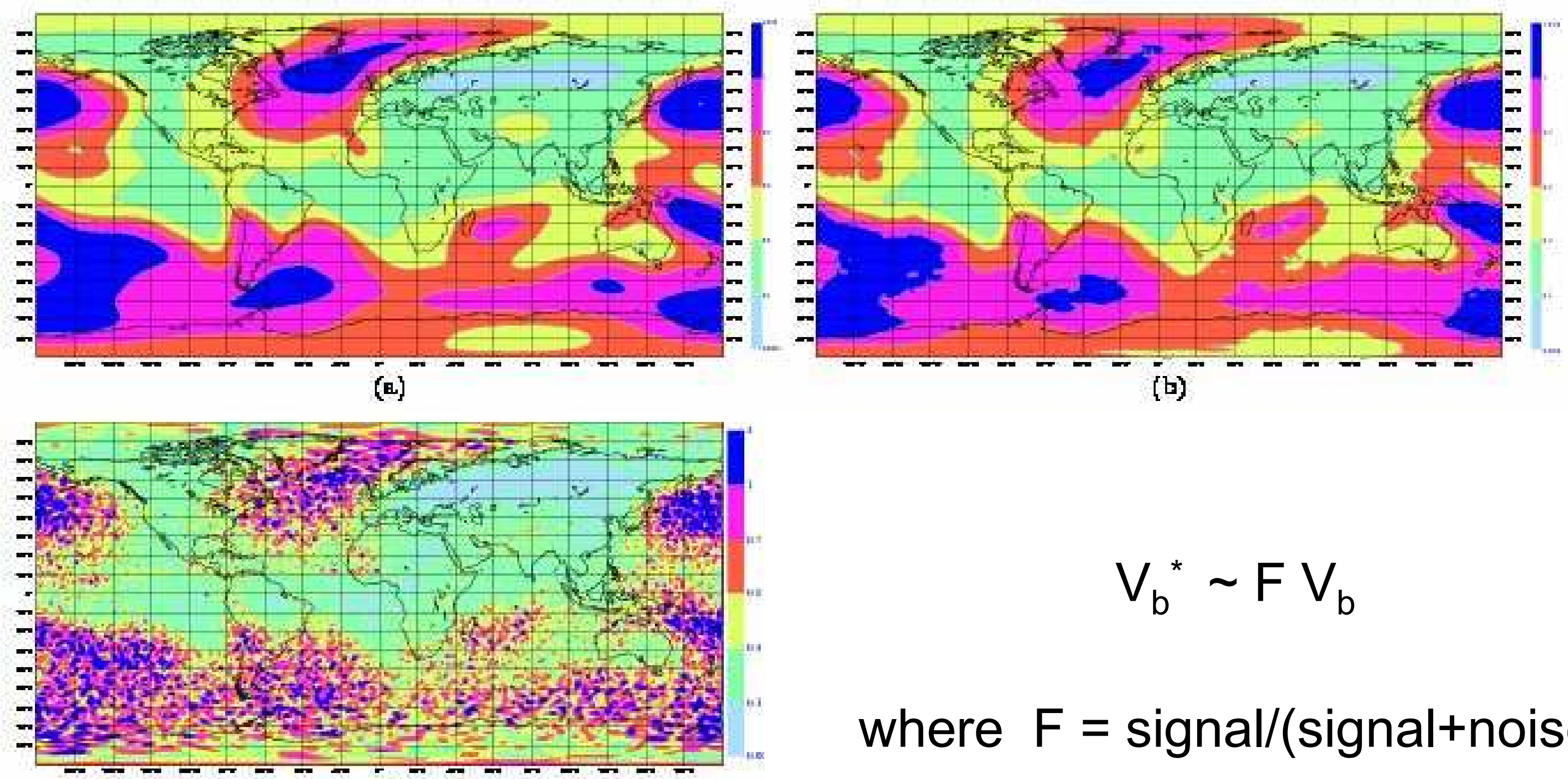
$$\overline{V^e\,(V^e)^T} = 2/(N-1)\ \mathbf{B}^* \circ \mathbf{B}^*$$

$\Rightarrow$ Employ filtering in order to extract large scale signal, and remove small scale sampling noise.

"OPTIMIZED" SPATIAL FILTERING

OF THE VARIANCE FIELD

« TRUE » VARIANCES **FILTERED VARIANCES (N = 6)**



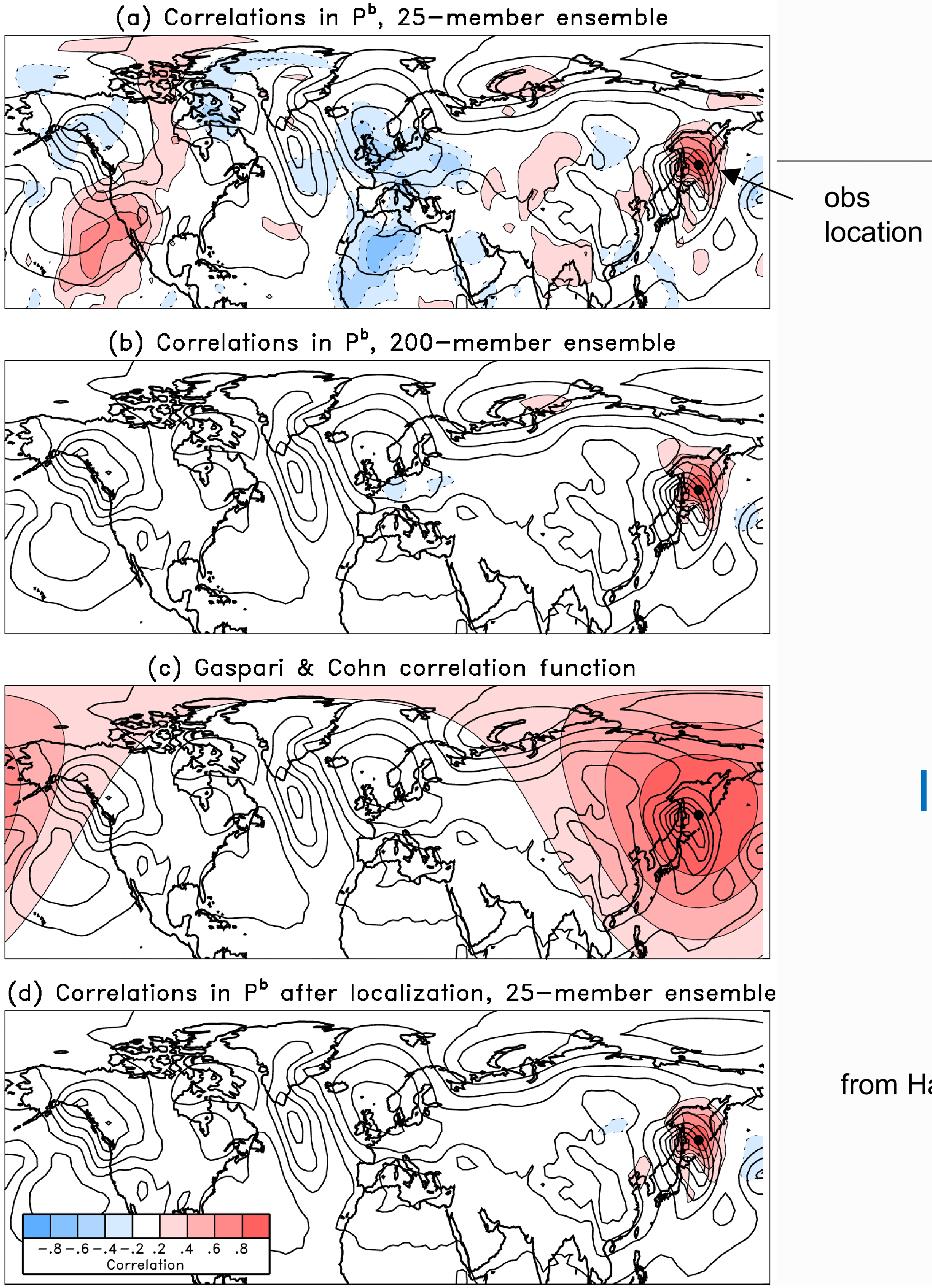
(a) (b) (c)

RAW VARIANCES (N = 6)

$$V_b^* \sim F\, V_b$$

where F = signal/(signal+noise)

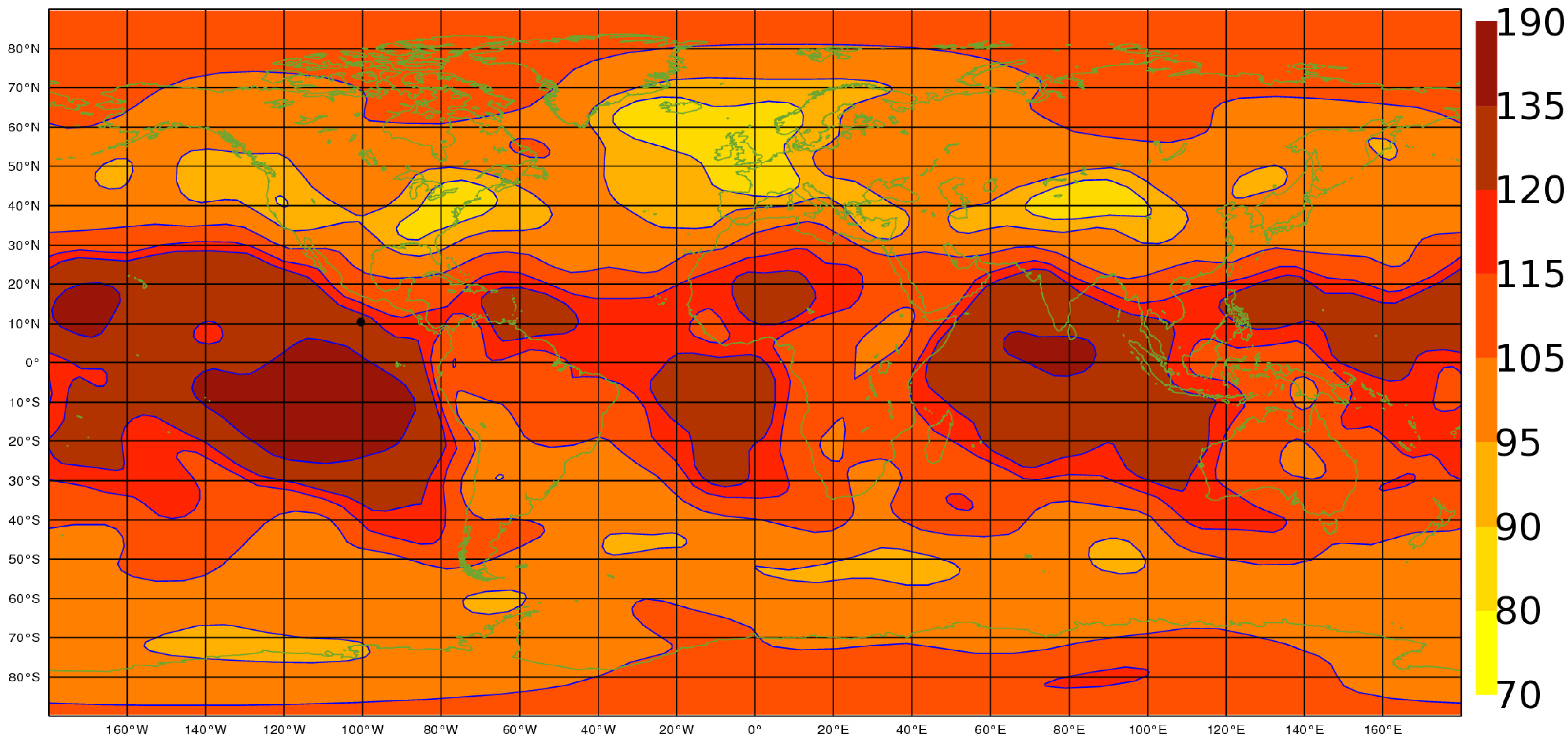
(Berre et al 2007,2010, Raynaud et al 2008,2009)



Schur filtering of long-distance correlations

from Hamill, Chapter 6 of "*Predictability of Weather and Climate*"

Flow-dependent background error correlations using EnDA and wavelets



Wavelet-implied horizontal length-scales (in km), for wind near 500 hPa, averaged over a 4-day period.

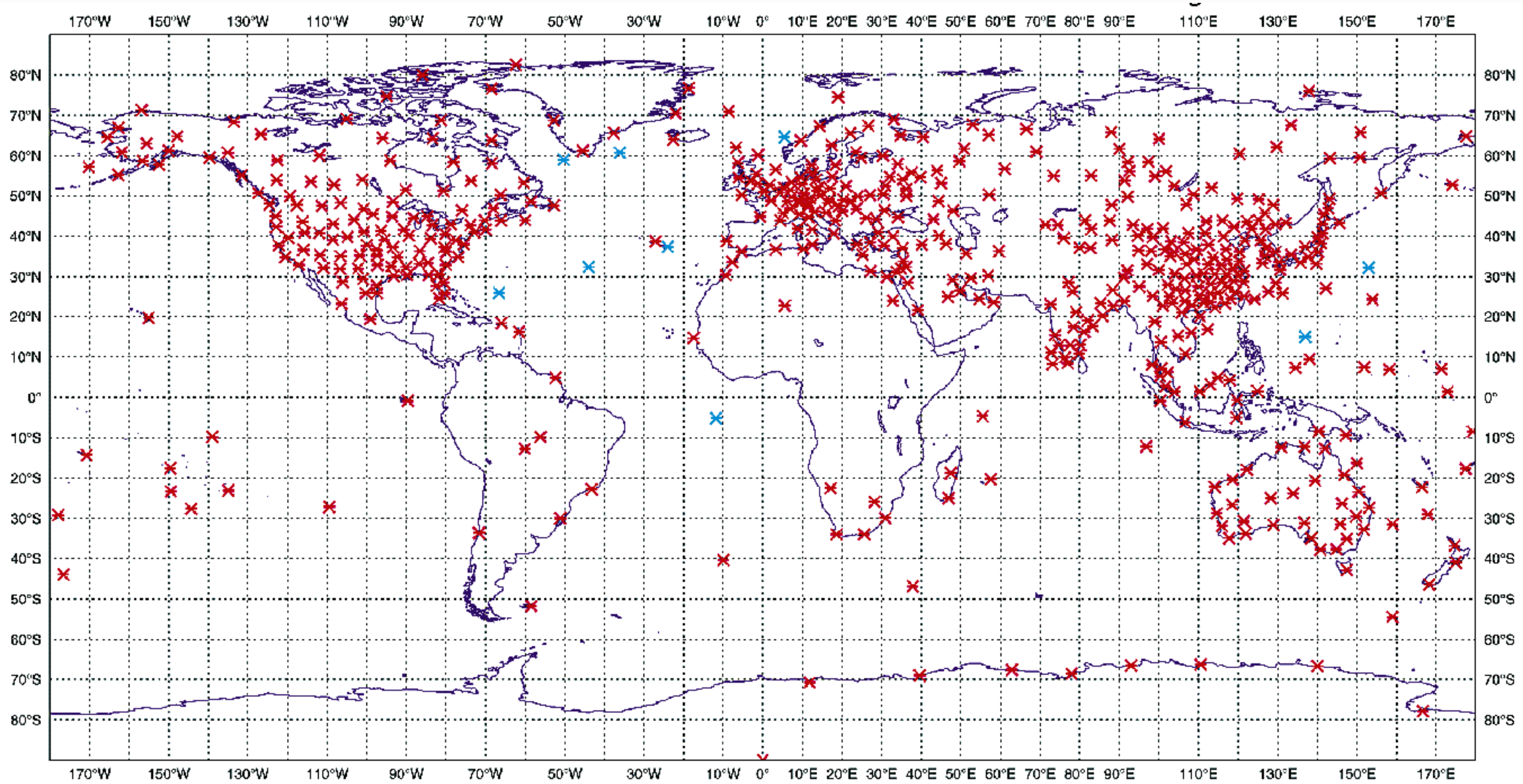
(Varella et al 2013)

METEO FRANCE
Toujours un temps d'avance

4. A posteriori diagnostics

(observation-minus-background departures)

RADIOSONDE OBSERVATIONS



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Covariances of innovations

- Innovation = observation-minus-background :

$$y_o - H\,x_b = y_o - H\,x_t + H\,x_t - H\,x_b$$
$$= e_o - H\,e_b$$

- Innovation covariances :

$$E[(y_o-Hx_b)(y_o-Hx_b)^T] = R + HBH^T$$

assuming that $E[(e_o)(He_b)^T]=0$.

(e.g. Hollingsworth and Lönnberg 1986)

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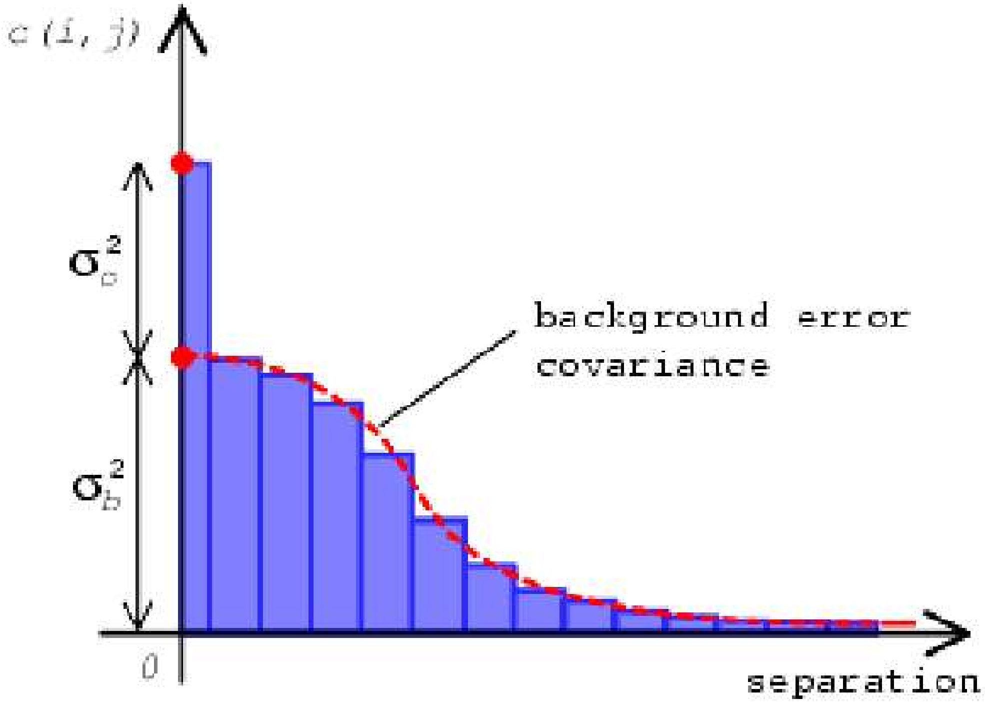
Toujours un temps d'avance

Hollingsworth and Lönnberg method

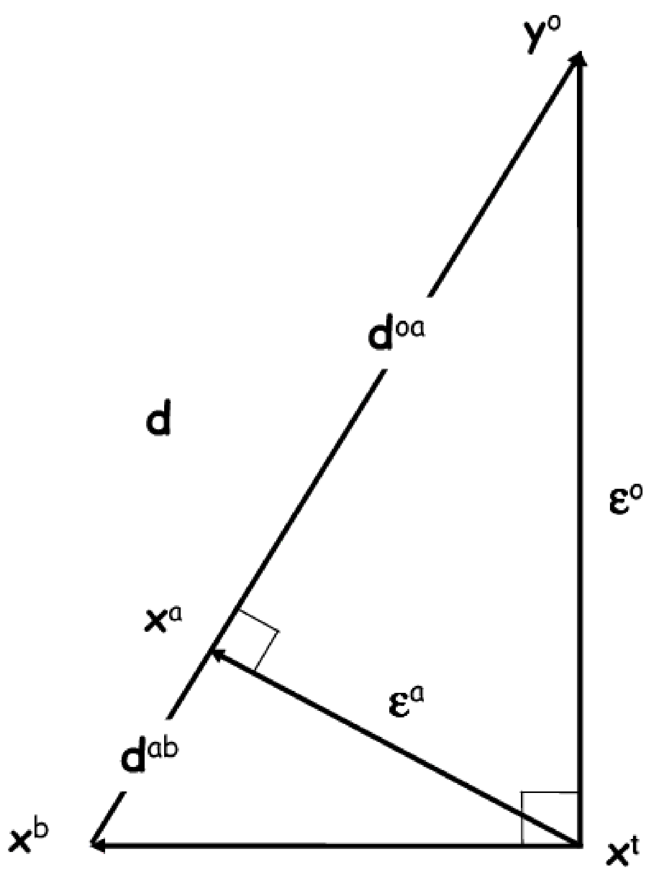


(From Bouttier and Courtier, ECMWF)

Innovation method : properties

- Provides estimates in observation space only.
- A good quality data dense network is needed.
- Assumption that observation errors are « white ».
- An objective source of information on B and R.

Diagnostics in observation space



(Desroziers et al, 2005)

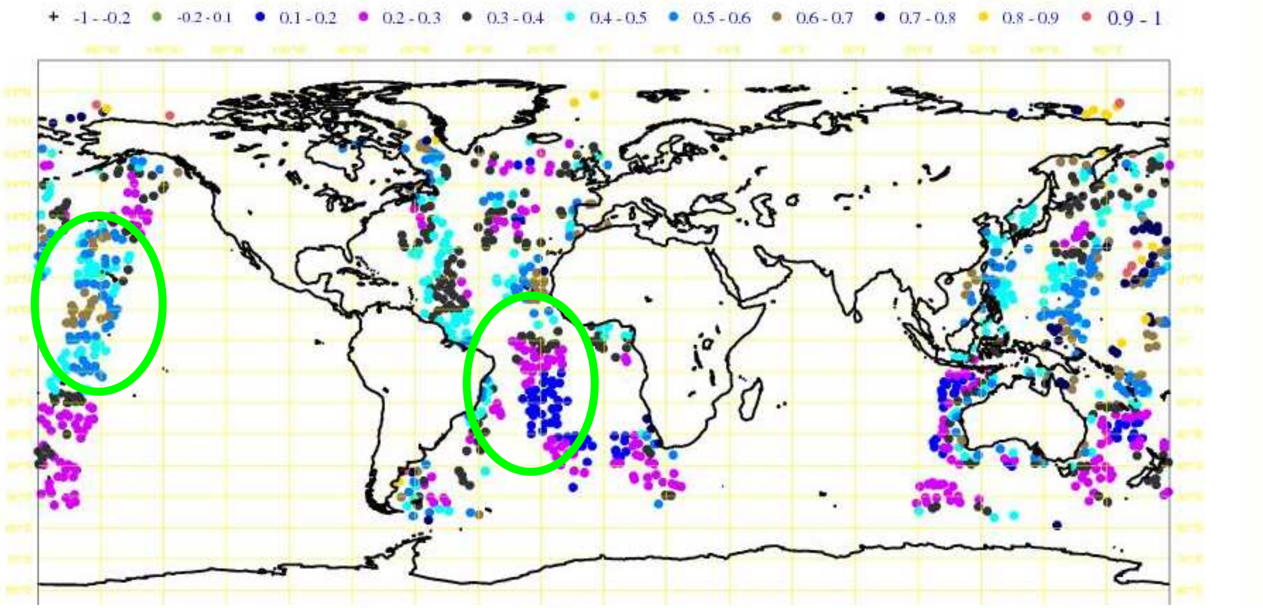
- $\mathbf{d} = \mathbf{y}^o - H(\mathbf{x}^b)$
- $\mathbf{d}^{oa} = \mathbf{y}^o - H(\mathbf{x}^a)$
- $\mathbf{d}^{ab} = H(\mathbf{x}^a) - H(\mathbf{x}^b)$
- $E[\mathbf{d}^{oa}\ \mathbf{d}^T] = \mathbf{R}$
- $E[\mathbf{d}^{ab}\ \mathbf{d}^T] = \mathbf{HBH}^T$
- $E[\mathbf{d}^{ab}\ \mathbf{d}^{oaT}] = \mathbf{HAH}^T$
- $\langle \varepsilon, \varepsilon' \rangle = E[\varepsilon\ \ \varepsilon'^T]$

Toujours un temps d'avance

Validation of flow-dependent estimates of errors in HIRS 7 space (28/08/2006 00h) (Berre et al 2007, 2010)

Ensemble estimate

of error std-devs

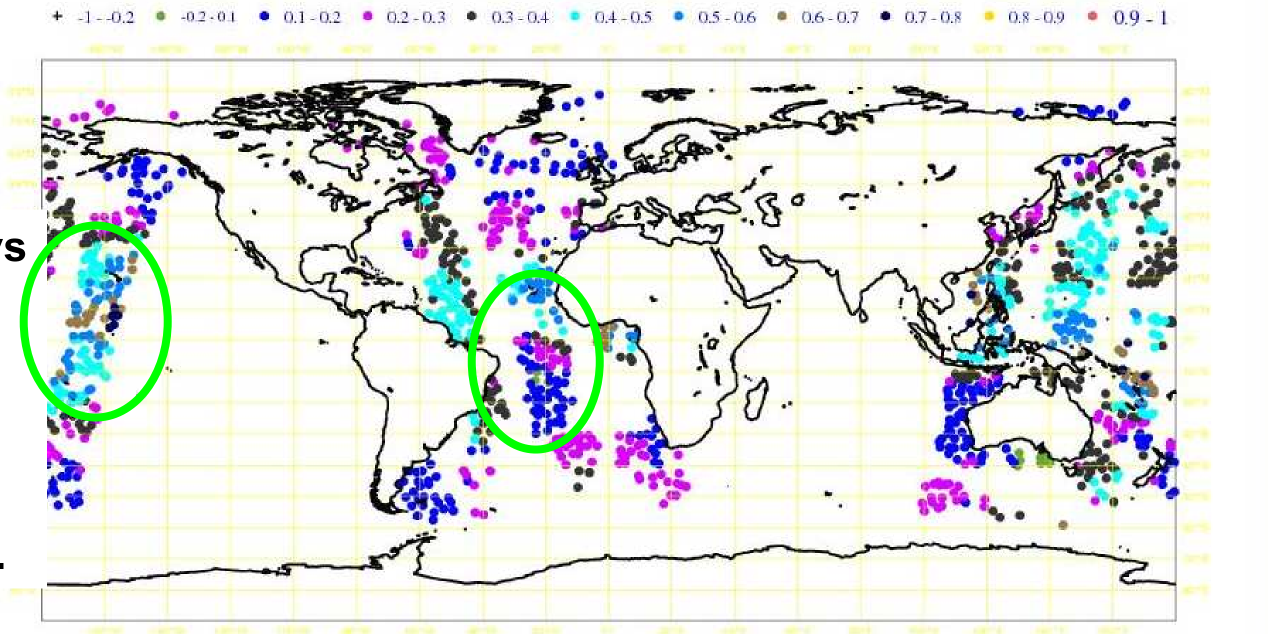


« Observed » error std-devs

$\text{cov}(H\,dx\,,\,dy) \sim H\,B\,H^T$

(Desroziers et al 2005)

=> model error estimation.



Use of innovations to estimate model error covariances $Q=cov(e_m)$

- Forecast error equation :

$$e_f = \mathbf{M}\, e_a + e_m$$

$$\mathbf{B} = \mathbf{MAM^T} + \mathbf{Q} \qquad \text{(if } e_m \text{ uncorrelated with } e_a\text{)}$$

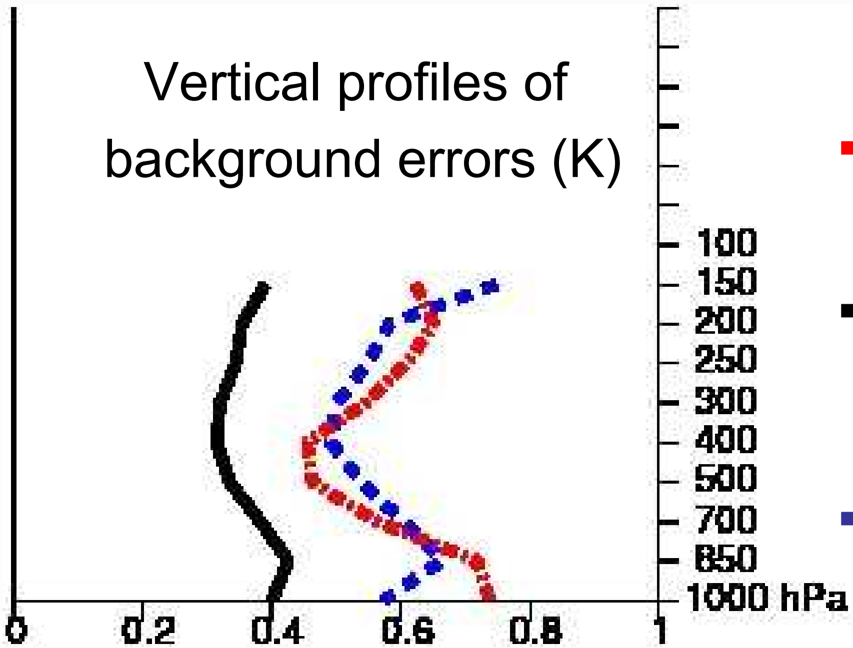
- Use ensemble assimilation (before adding model perturbations) to estimate evolved analysis error covariances (MAM^T).

- Use innovation diagnostics to estimate « B » (or at least HBH^T) (forecast error covariances).

- Estimate Q by comparing B and MAM^T (e.g. Daley 1992).

- Represent model error by inflating forecast perturbations in accordance with Q estimate.

Model error in M.F. ensemble 4D-Var
(Raynaud et al 2012, QJRMS)



Observation-based estimate

Ensemble-based estimate, model error neglected

Ensemble-based estimate, model error accounted for

Conclusions

- Data Assimilation (DA) is vital for weather forecasting (NWP).
- Observations are very diverse in type, density and quality.
- 4D-Var for temporal and non linear aspects.
- Ensemble DA methods for error simulation and covariance estimation.
- Sampling noise issues and filtering techniques.
- A posteriori diagnostics for validation of error covariances, and for estimation of model errors.

Some references

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- Berre, L., Ştefănescu, S., Belo Pereira, M.. The representation of the analysis effect in three error simulation techniques. Tellus A, 58A, pp 196-209.

Thank you

for your attention

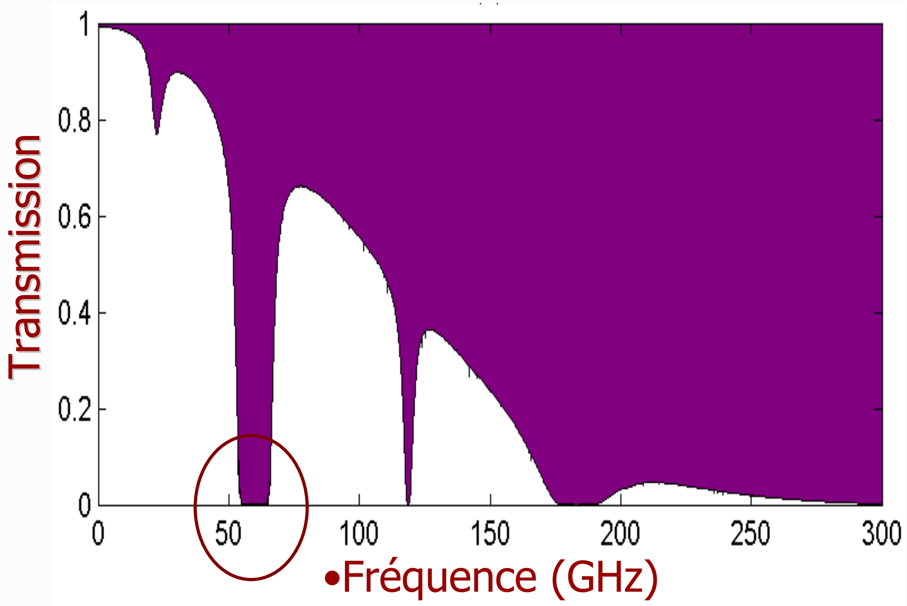
What is measured by satellite sensors ?

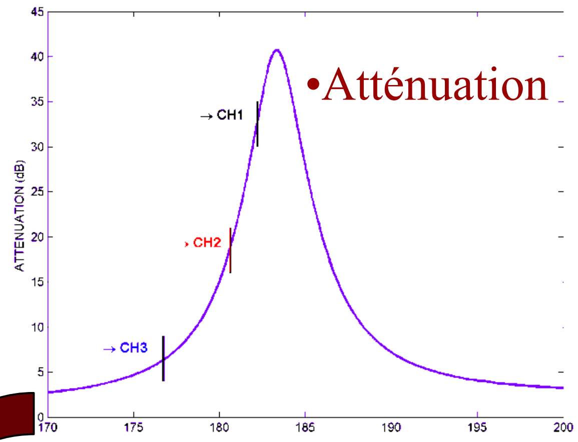
Soundings of atmosphere ?

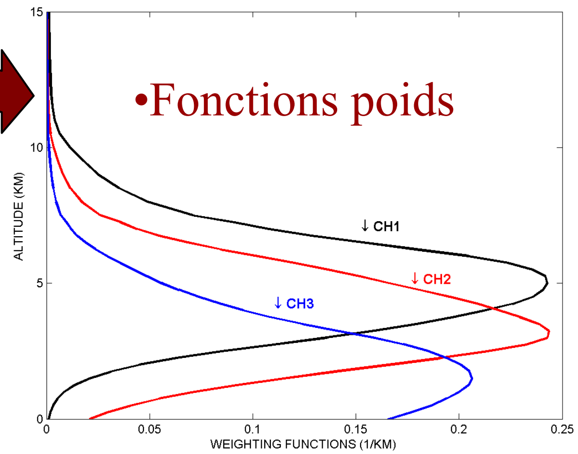
- In micro-waves: absorption par by water vapor, oxygen
- Largeur des bandes d'absorption: Pression (altitude) (< 60km): les bandes d'absorption plus larges quand la pression augmente

<u>Les mesures loin (proches) d'une bande d'absorption: information sur les basses (hautes) couches atmosphériques</u>

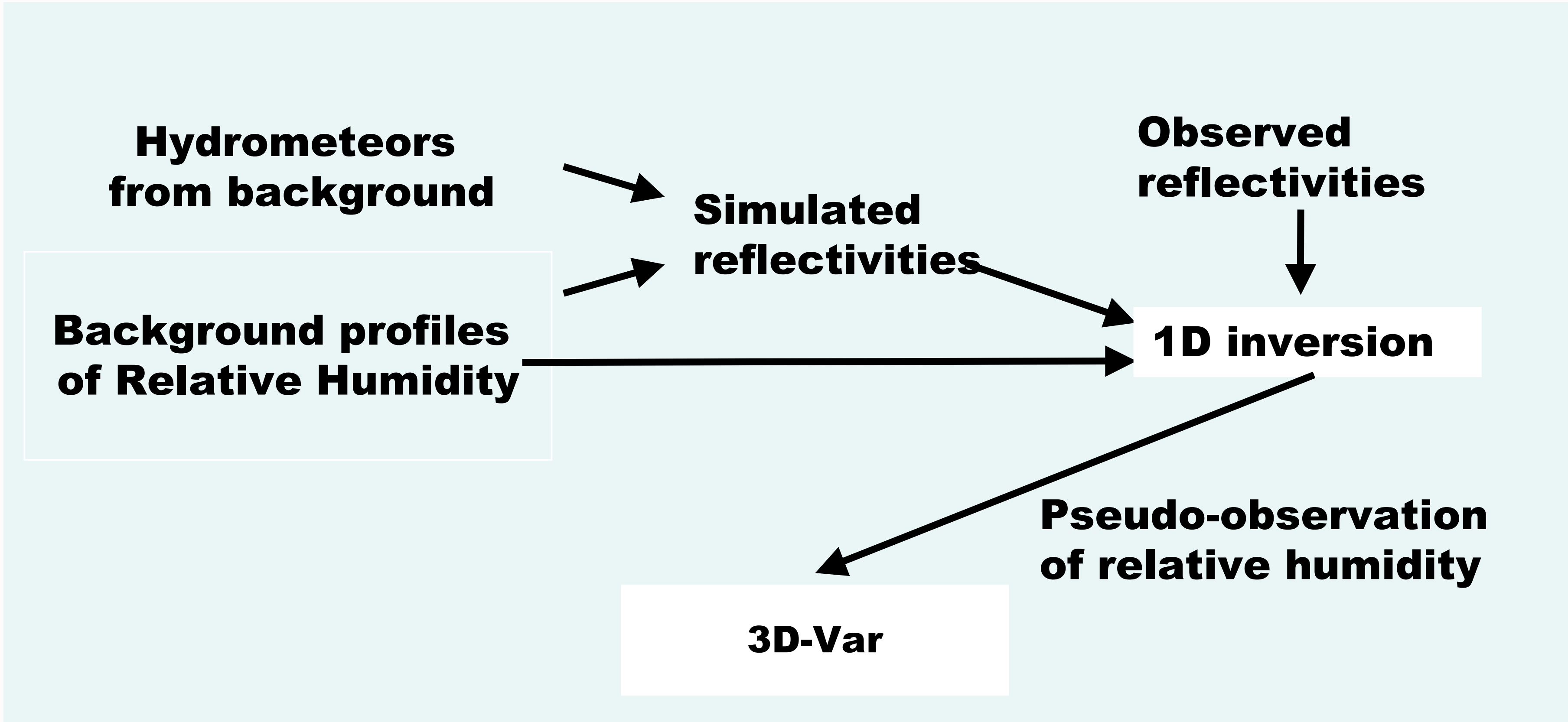
$$attenuation(dB) = e^{\frac{-transmission}{\cos(\theta)}}$$







Assimilation of reflectivities in AROME : Method 1D + 3D-Var : general algorithm



Spatial filtering of raw ensemble variances

- Expansion of the raw variance field V_{raw} :

$$V_{raw} = V_{signal} + V_{noise}$$

with V_{signal} assumed uncorrelated with true signal V_{signal}

- Filtering V_{raw} through linear regression formalism :

$$\begin{aligned} V_{signal} \sim V_{filtered} &= F \quad V_{raw} \\ &= cov(V_{signal},V_{raw})/var(V_{raw}) \quad V_{raw} \\ &= 1/(1+var(V_{signal})/var(V_{noise})) \quad V_{raw} \end{aligned}$$

- Estimation of signal and noise variances (in spectral space) :

$$var(V_{noise}) = 2/(N-1)\ B^* \circ B^*$$

$$var(V_{signal}) = var(V_{raw}\ ) - var(V_{noise})$$

=> F = low-pass spectral filter, equivalent to local spatial averaging.

Modelling of background error covariances

- Size of B is far too large.
- Can't be computed explicitly (nor stored in memory).

$\Rightarrow$ Model B as product of sparse operators.

B as product of sparse operators

$$B^{1/2} = L\ S\ C_u^{1/2}$$

L : ~ cross-covariances (~sparse regressions),

S : diagonal matrix of standard deviations.

C_u : sparse model of auto-correlations

(e.g. diagonal matrix in spectral space).

$$B = L\ S\ C_u\ S\ L^T$$

Covariances of residuals

- Analysis increment : $H\,\delta x = HK\,(y_o - Hx_b)$

  with $HK = HBH^T\,(HBH^T + R)^{-1}$

- Covariances between $H\delta x$ and omb :

$$E[(H\,\delta x)(y_o - Hx_b)^T] = HK\,E[(y_o - Hx_b)(y_o - Hx_b)^T]$$

$$\sim HK\,(HB_tH^T + R_t)$$

$$\sim HBH^T\,(HBH^T + R)^{-1}\,(HB_tH^T + R_t)$$

$$\sim HB_tH^T$$

either assuming K ~ optimal,

or, for averaged σ_b, assuming that structures

in B,R are much different. (Desroziers et al 2005)

Model error representations

- Additive inflation (temporally uncorrelated) :
  random draws from estimated model error covariances.

- Multiplicative inflation (temporally correlated) :
  mult. amplification of forecast perturbations.

- Multi-model ensembles (difficult to maintain ?):
  use different models to reflect model uncertainties.

- Stochastic physics : perturbations with
  amplitudes proportional to physical tendencies.

- SKEB : backscattering of small scale energy
  dissipated by horizontal diffusion.

⇒ Comparison by Houtekamer et al 2009 :
inflation is the most « efficient » approach.