Data assimilation in meteorology

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Plan of the talk

- Numerical Weather Prediction (NWP)
  and Data Assimilation (DA)
- Observations (in-situ and remote sensing)
- Error covariance estimation and modelling

1. Numerical Weather Prediction and Data Assimilation

The two main ingredients of weather forecasting

What will be the weather tomorrow ?

Bjerknes (1904) :

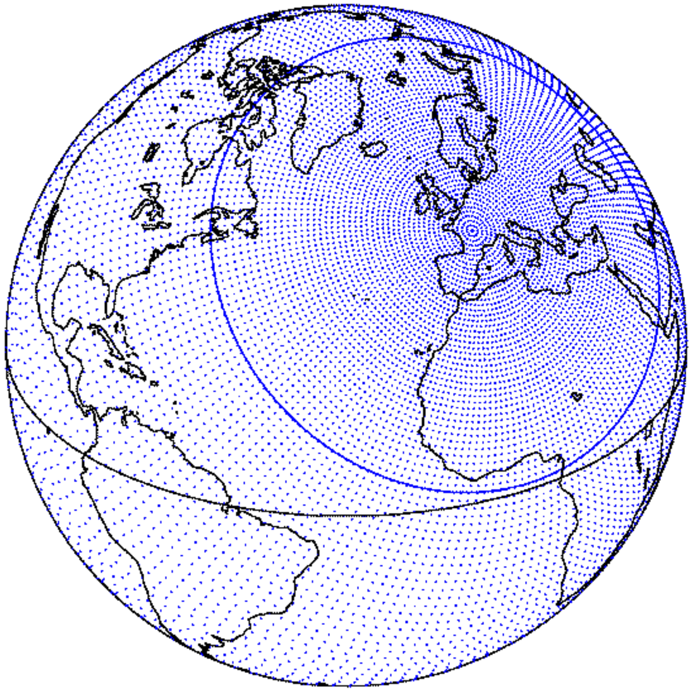
In order to do a good forecast, we need to know :

- the atmospheric evolution laws
  (~ modelling) ;

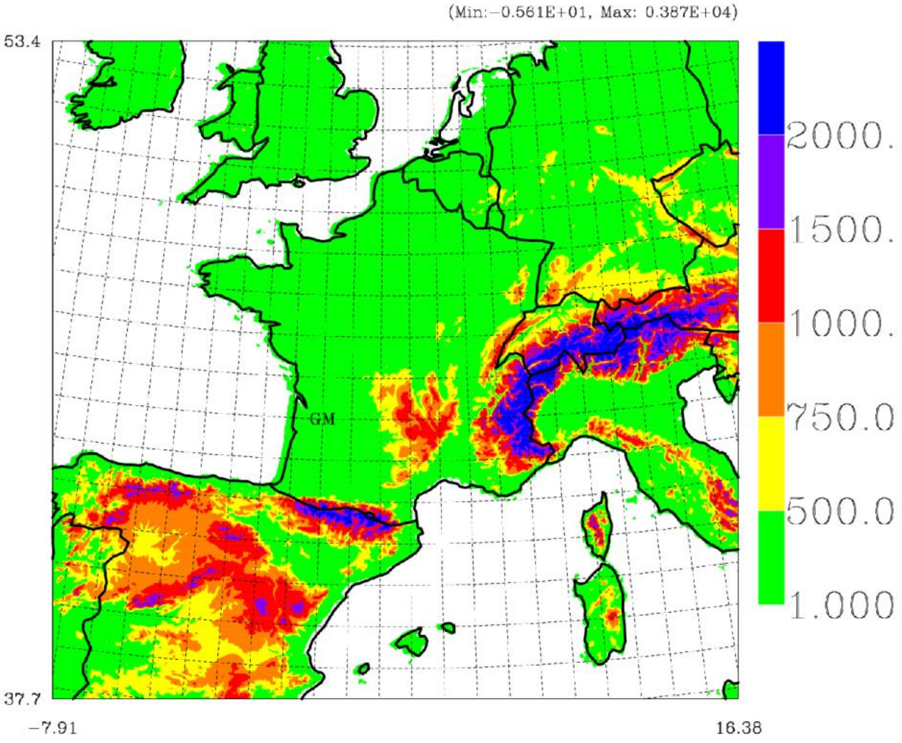
- the atmospheric state at initial time
  (~ data assimilation).

Numerical Weather Prediction at Météo-France
(in collaboration with e.g. ECMWF)

Global model (Arpège) : DX ~ 7-40 km

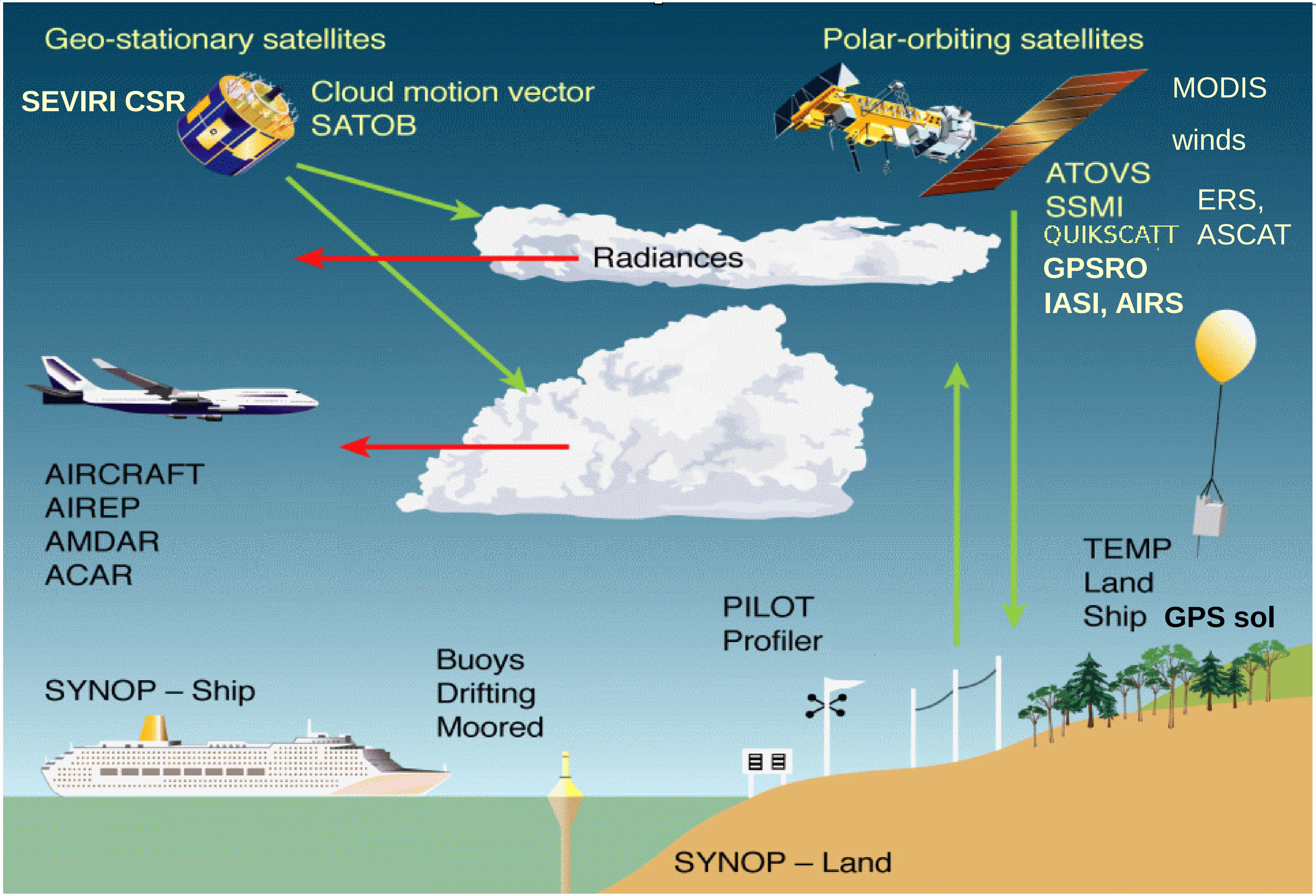


Arome : DX ~ 1.3 km



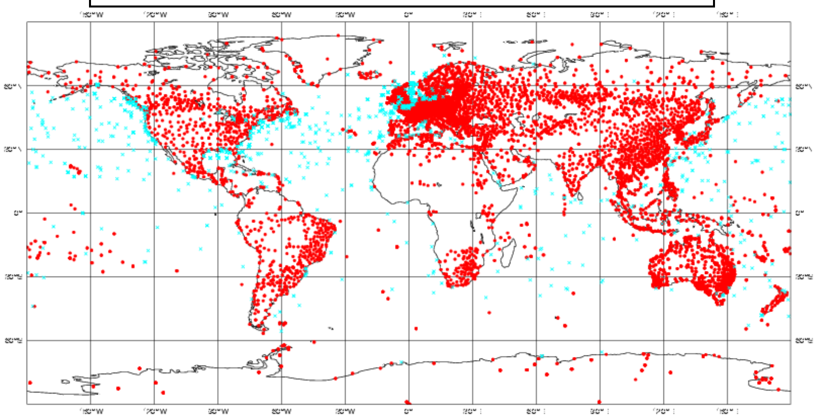
Equations of hydrodynamics and physical parametrizations (radiation, convection,…)
to predict the evolution of temperature, wind, humidity, …

Data that are assimilated in NWP models

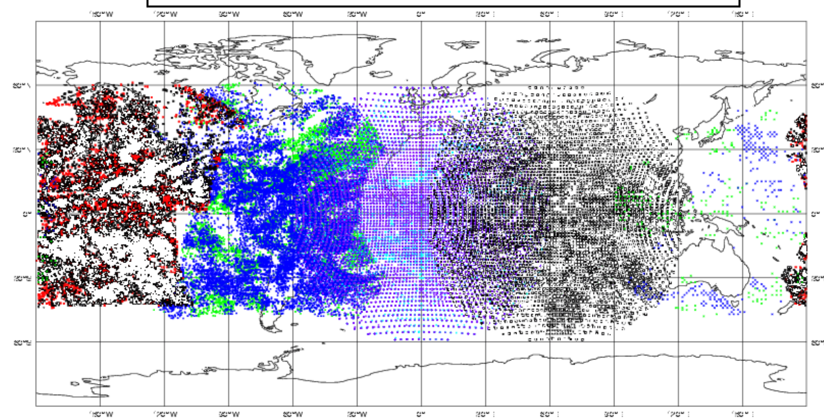


Spatial coverage and density of observations

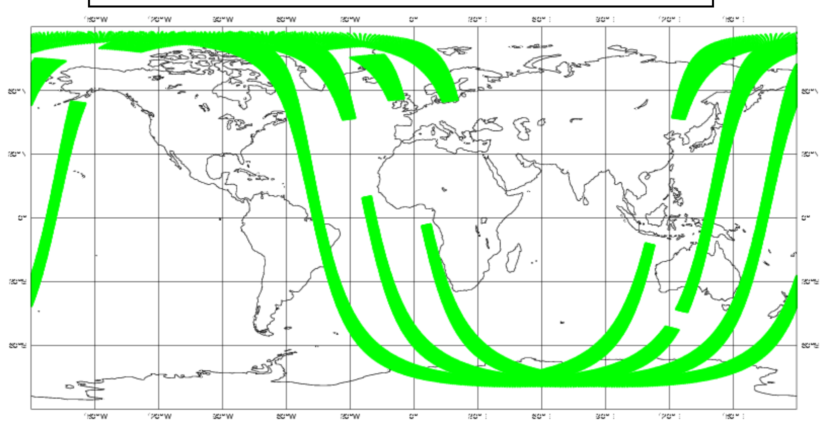
SURFACE DATA



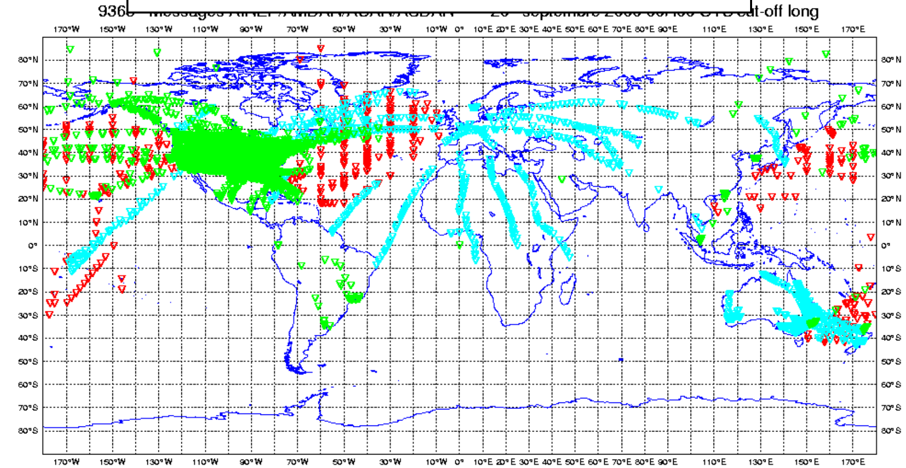
GEOSAT. WINDS



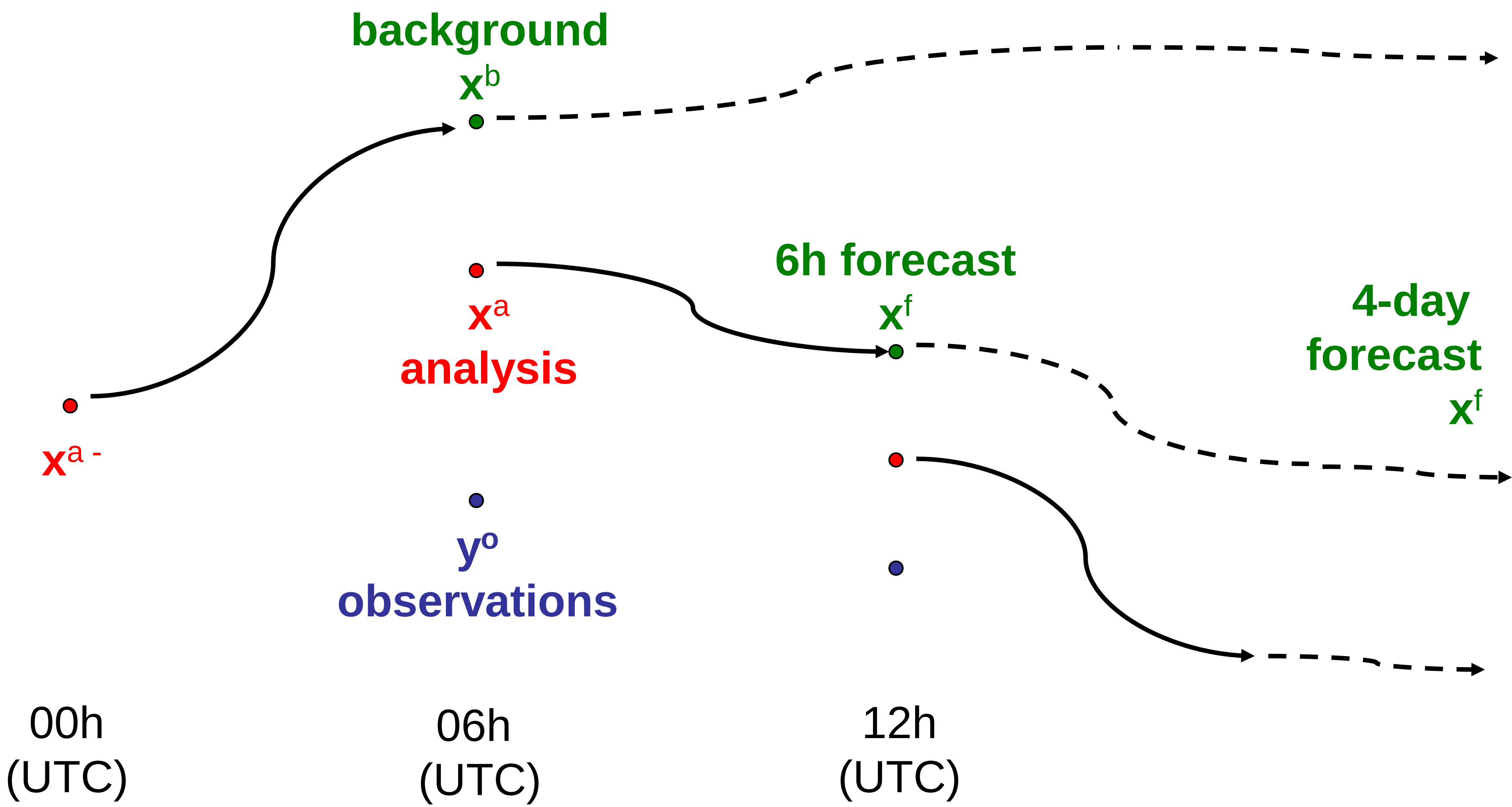
SCATTEROMETER



AIRCRAFT DATA

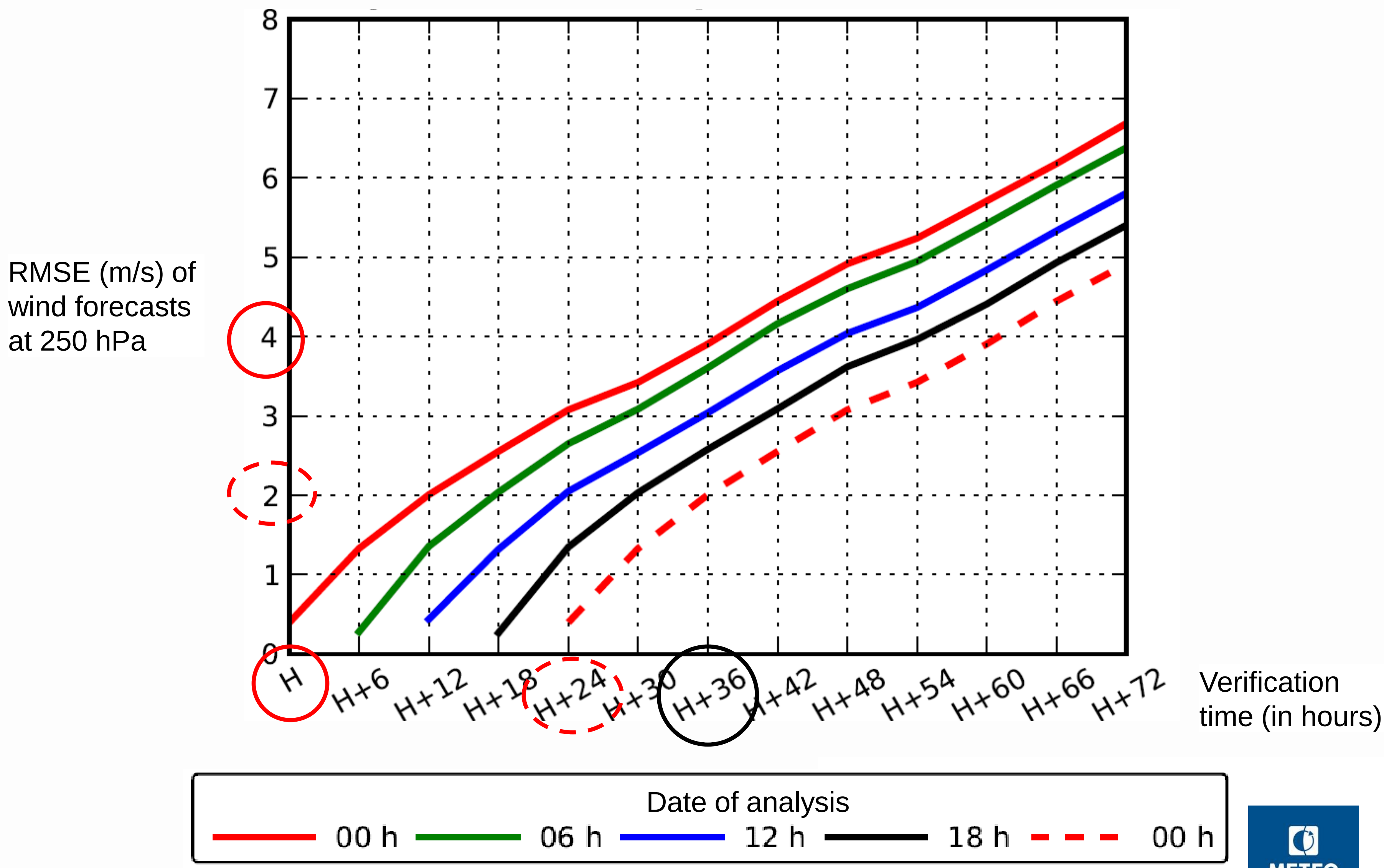


Temporal cycling of data assimilation : succession of analyses and forecasts

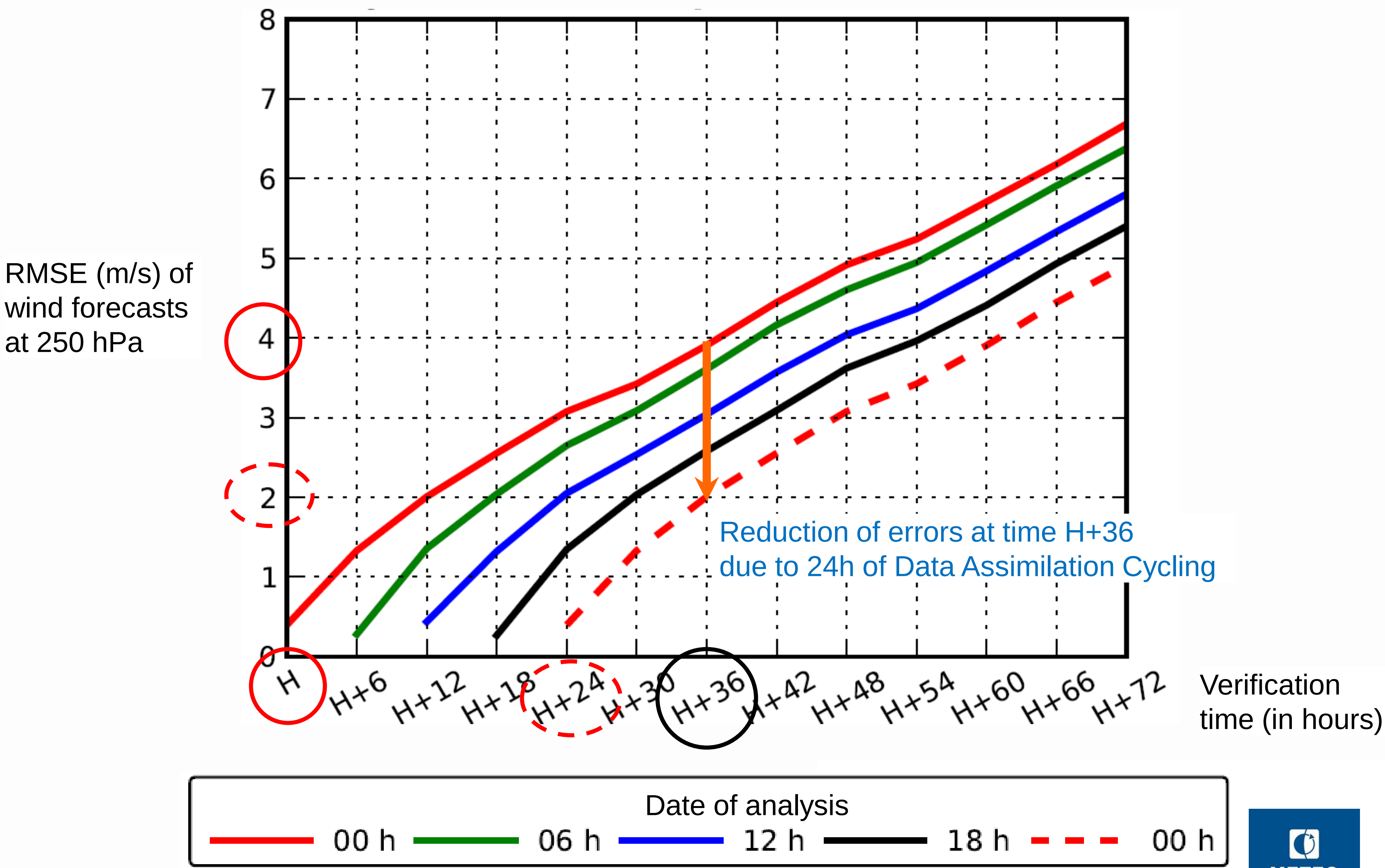


State of DA system is updated ~ continuously

Forecast errors, as a function of verification time and analysis date



Forecast errors, as a function of verification time and analysis date



Quasi-linear analysis (BLUE)

- BLUE analysis equation : $\mathbf{x^a = x^b + K\,(y^o -} \mathit{H}\mathbf{[x^b])}$
- H = non linear observation operator
  = projection from model space to observation space
  (e.g. spatial interpolation, radiative transfer, NWP model).
- **K** = gain matrix :

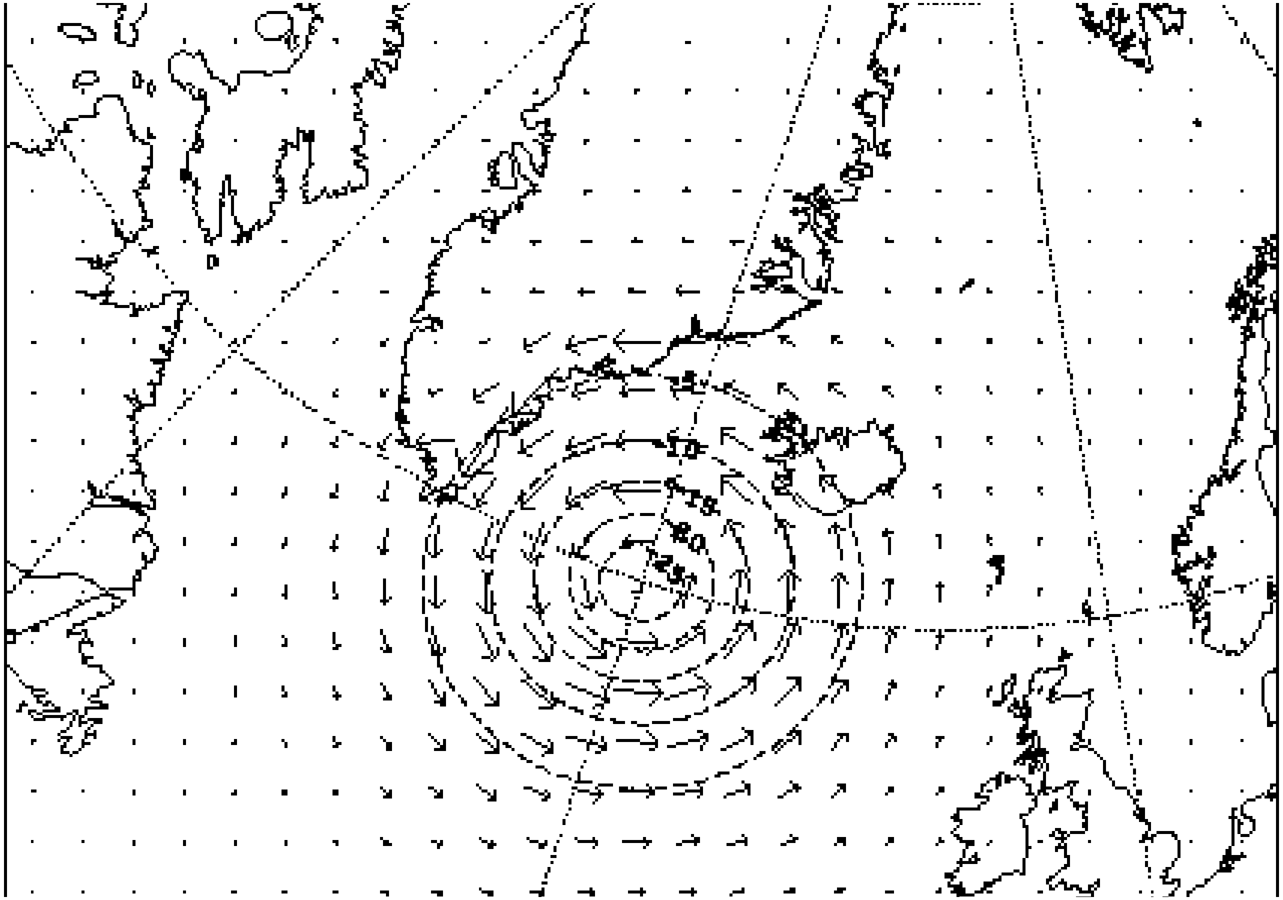
$$\mathbf{K = BH^T\,(\,HBH^T + R\,)^{-1}}$$

with **H** = tangent linear version of H,
B = background error covariance matrix,
R = observation error covariance matrix.

$$\mathbf{H\,K = (\,I + R\,(HBH^T\,)^{-1}\,)^{-1}}$$

$\Rightarrow$ Accounts for relative accuracy of observations,
and for spatial structures of background errors.

Impact of one observation of temperature on the wind analysis (2D)



$\Rightarrow$ Typical spatial scales and multivariate couplings (ex: mass/wind) in **B**.

4D-Var

- Analysis equation (BLUE) :

  $$\mathbf{x}^a = \mathbf{x}^b + \mathbf{K}\,(\,\mathbf{y}^o - H[\mathbf{x}^b]\,) = \mathbf{x}^b + \mathbf{K}\,\delta\mathbf{y}$$

  but **K** is difficult to handle explicitly in a real size system.

- Variational formulation = minimize distance J to all available information :

  cost function : $J(\mathbf{x}) = (\mathbf{x} - \mathbf{x}^b)^T\,\mathbf{B}^{-1}\,(\mathbf{x} - \mathbf{x}^b) + (\mathbf{y}^o - H[\mathbf{x}])^T\,\mathbf{R}^{-1}\,(\mathbf{y}^o - H[\mathbf{x}])$

  minimised when gradient $J'(\mathbf{x})=0$ (equivalent to BLUE).

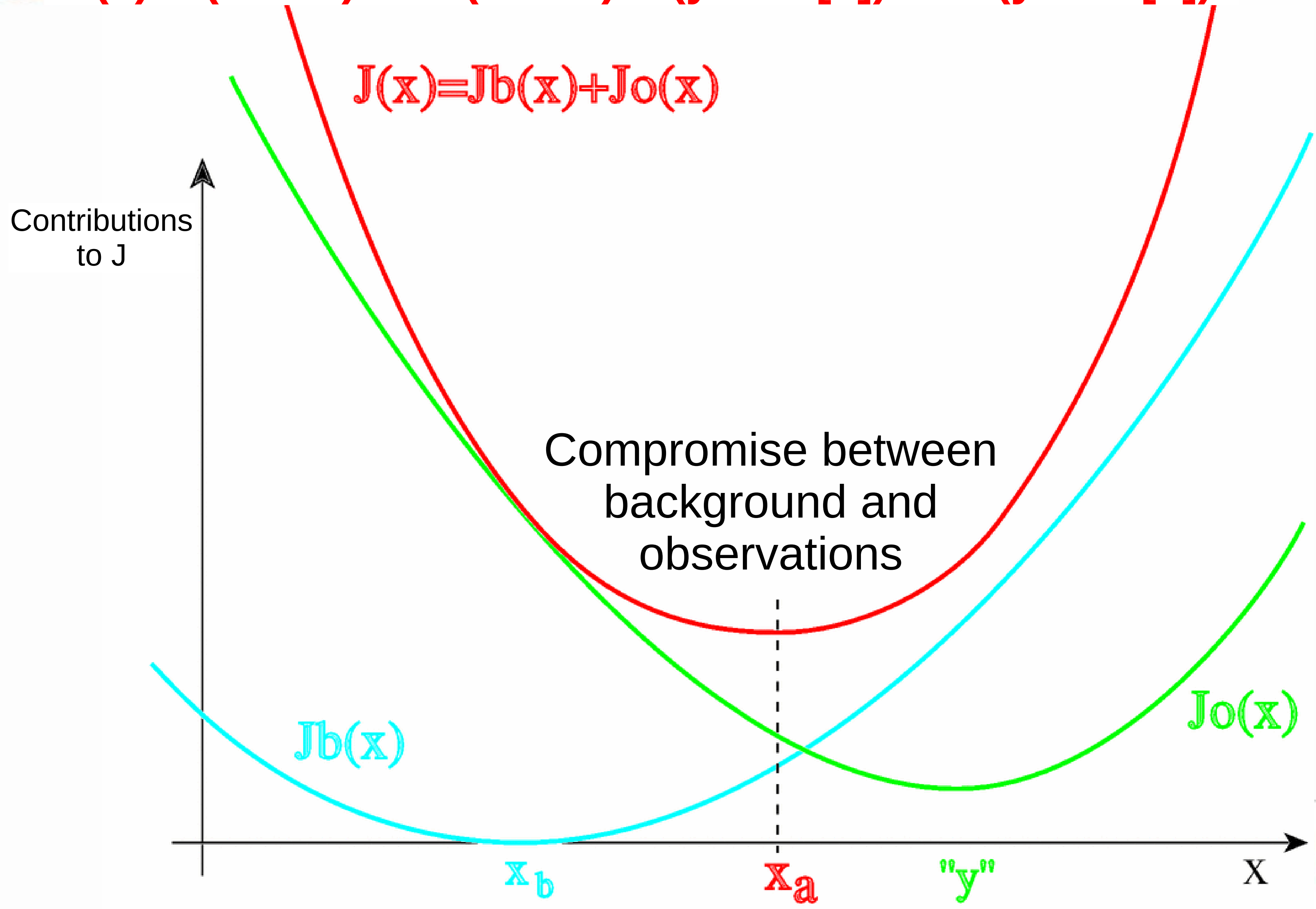
- Computation of J': development and use of adjoint operators (transpose).

- 4D-Var = use generalized observation operator *H* :
  it includes the NWP model *M*
  in order to assimilate observations distributed in time.

- Implicit flow-dependent evolution of **B**, through $\mathbf{HBH}^T$.

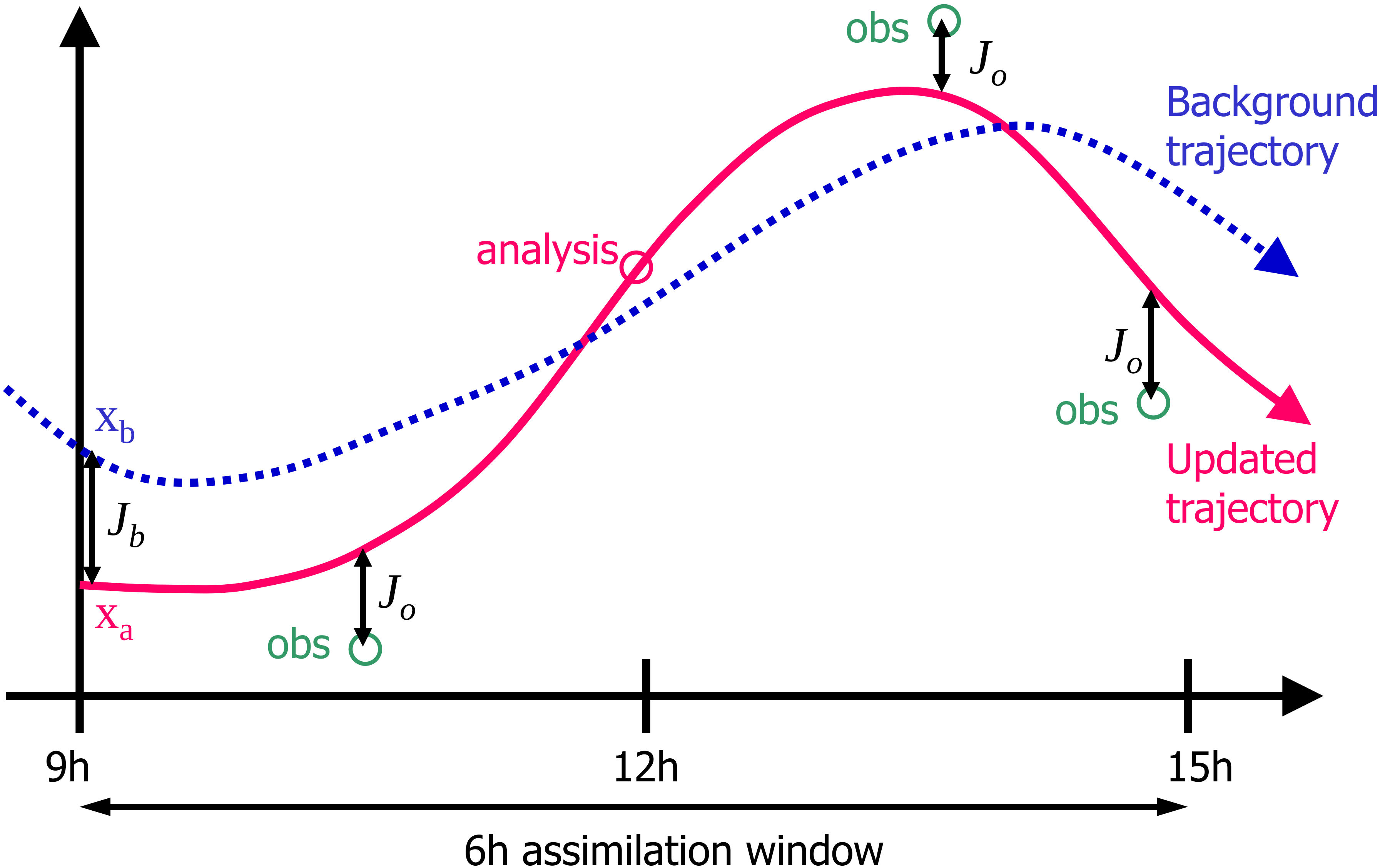
Schematic representation (scalar case) of

$$J(x) = (x - x^b)^T B^{-1} (x - x^b) + (y^o - H[x])^T R^{-1} (y^o - H[x])$$



Principle of 4D-VAR assimilation

(e.g. Talagrand and Courtier 1987, Rabier et al 2000)



Incremental formulation of 4D-Var

- Full variational formulation :

  cost function : $J(\mathbf{x}) = (\mathbf{x} - \mathbf{x}^b)^T \mathbf{B}^{-1} (\mathbf{x} - \mathbf{x}^b) + (\mathbf{y} - H[\mathbf{x}])^T \mathbf{R}^{-1} (\delta\mathbf{y} - H[\mathbf{x}])$

- Incremental formulation : $\delta\mathbf{x} = \mathbf{x} - \mathbf{x}^b$

  $\mathbf{y} - H[\mathbf{x}] = \mathbf{y} - H[\mathbf{x}^b] + H[\mathbf{x}^b] - H[\mathbf{x}] \sim \delta\mathbf{y} - \mathbf{H}\,\delta\mathbf{x}$

  cost function : $J(\delta\mathbf{x}) = \delta\mathbf{x}^T \mathbf{B}^{-1} \delta\mathbf{x} + (\delta\mathbf{y} - \mathbf{H}\,\delta\mathbf{x})^T \mathbf{R}^{-1} (\delta\mathbf{y} - \mathbf{H}\,\delta\mathbf{x})$

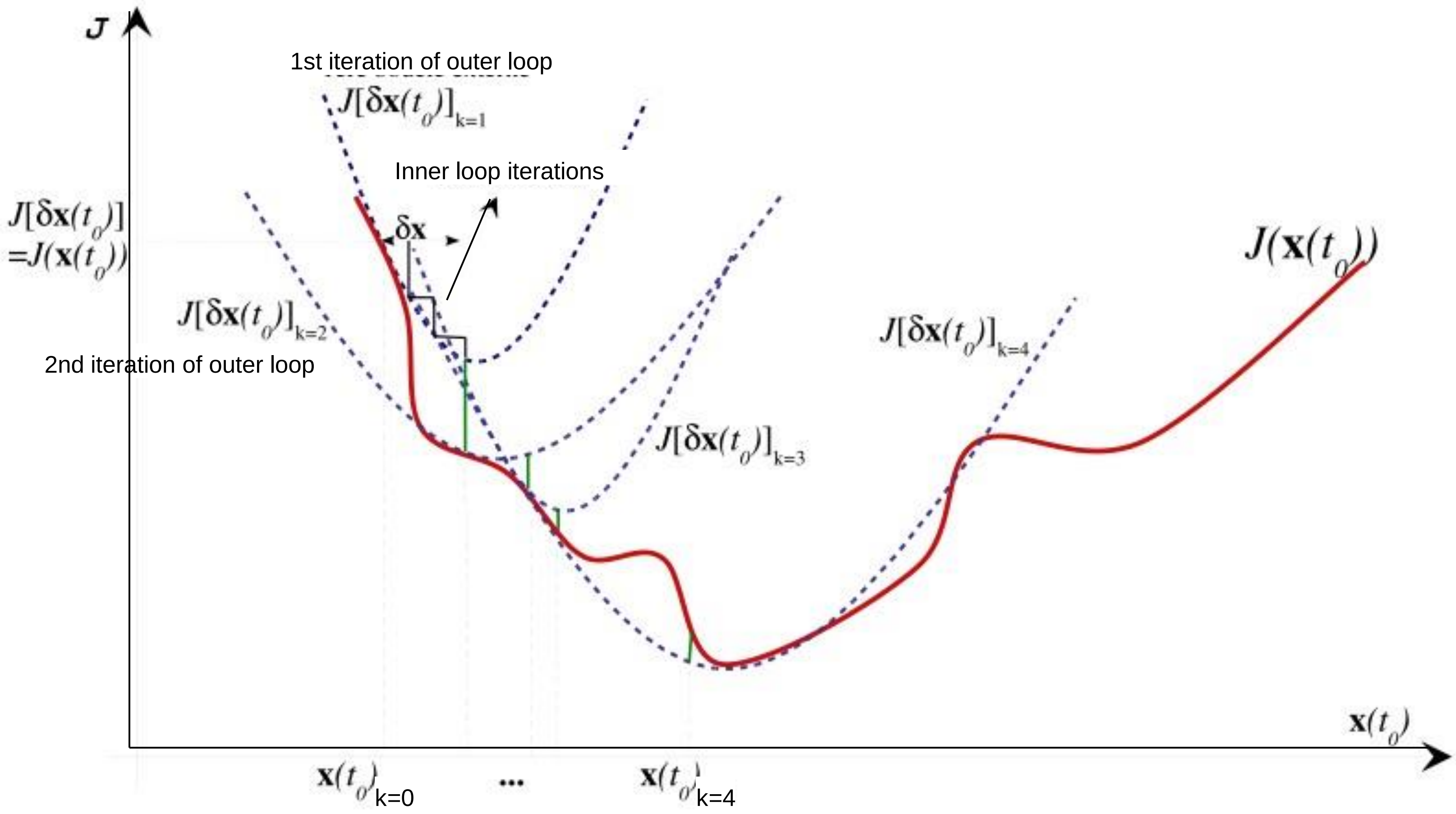
  minimised when gradient $J'(\delta\mathbf{x})=0$ (equivalent to BLUE).

- Cost reduction : analysis increment $\delta\mathbf{x}$ can be computed at low resolution.
  (Courtier, Thépaut and Hollingsworth, 1994)

- Some non linear features accounted / calculation of departures $\mathbf{y}^o$-$H(\mathbf{x}^b)$, and in non linear trajectory updates (outer loop).

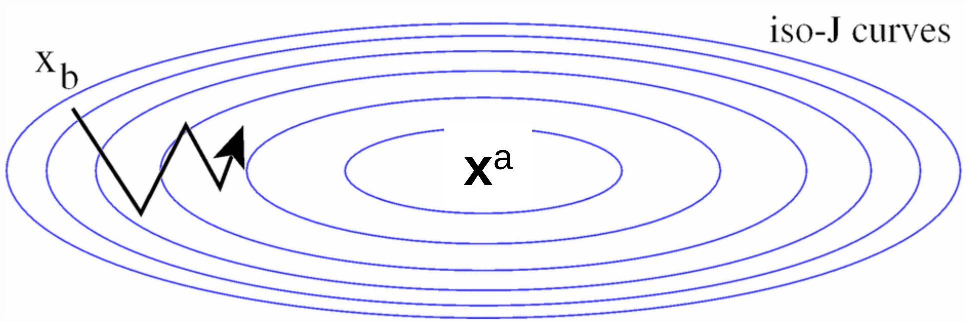
METEO FRANCE

Incremental 4D-Var as a sequence of updated quadratic minimizations

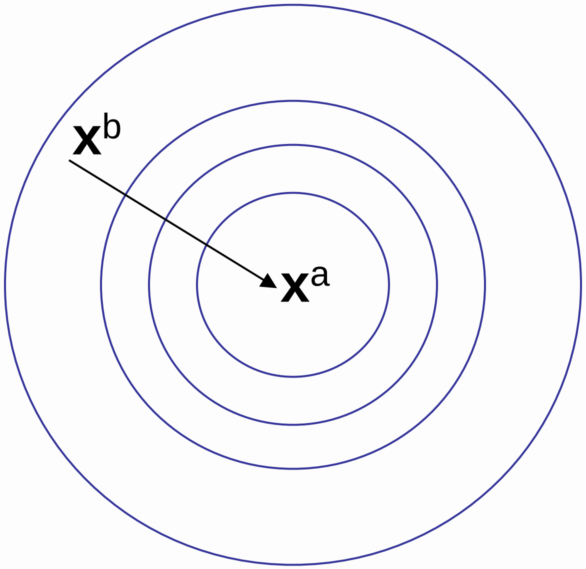


(from A. Vidard ; e.g. Courtier, Thépaut et Hollingsworth, 1994)

Importance of preconditioning of minimization



Bad preconditioning



Good preconditioning

- Slow convergence if iso-J curves are elliptical / quick convergence if circular.

- Shape of J is determined by eigen-structure of its matrix J" of second derivatives :

  $\mathbf{J}'' = \mathbf{B}^{-1} + \mathbf{H}^T \mathbf{R}^{-1} \mathbf{H}$

- Small scale components of **B** = large eigen-values of $\mathbf{B}^{-1}$;
  Large scale components of **B** = small eigen-values of $\mathbf{B}^{-1}$.

=> problem of "narrow valley".

- Use a change of variable ($\chi = \mathbf{B}^{-1/2}\, \delta\mathbf{x}$) such as J becomes nearly "circular".

2. In-situ observations and

remote sensing data

Observation networks in meteorology: in situ measurements

Observation networks in meteorology: satellite data

Constellation of polar orbiting or geostationary satellites

Geostationary satellites

- Advantages

  Very high temporal resolution (15 min)

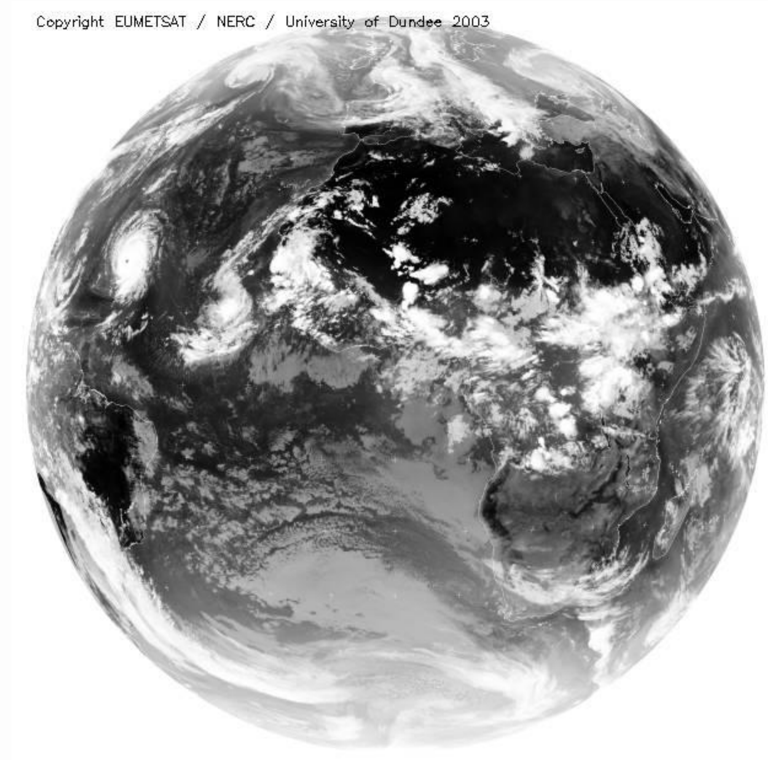
  Useful for nowcasting

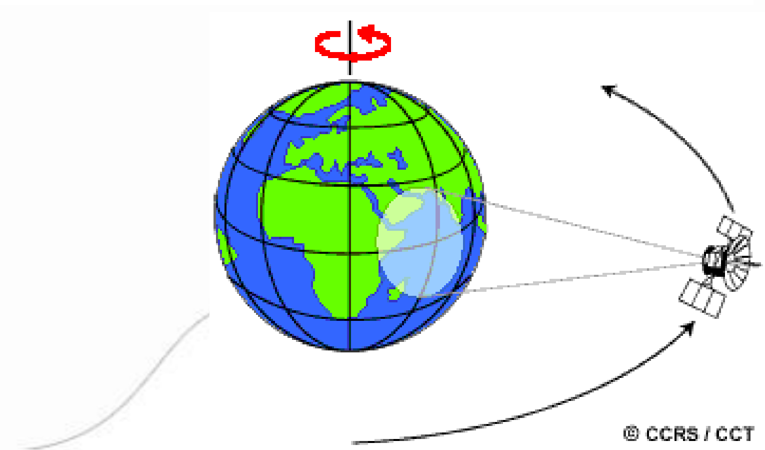
  Dynamics of meteorological structures

- Drawbacks

  Insufficient spatial coverage

  Not adapted to polar regions





Polar orbiting satellites

Low orbit satellites :

- ❑ Advantages

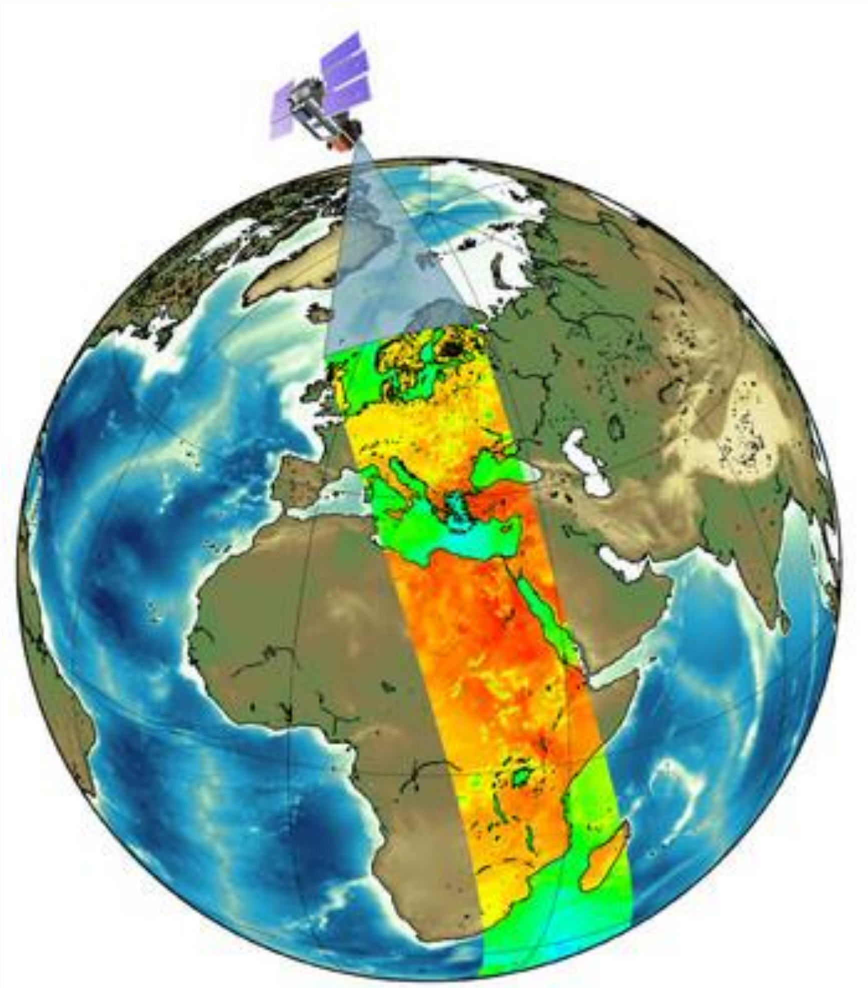
  High spatial resolution

  Global spatial coverage

  Sounding instruments
  (over several vertical layers)

- ❑ Drawbacks

  Insufficient temporal resolution
  (several satellites are needed)

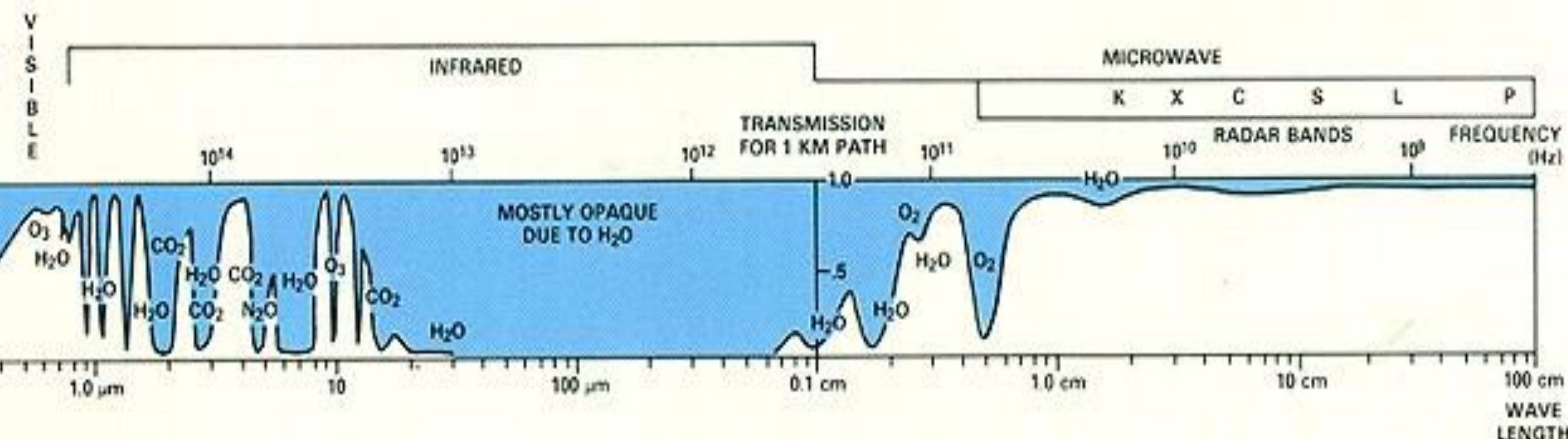


What is measured by satellite sensors ?

- Sensors do not measure directly atmospheric temperature and humidity, but electromagnetic radiation : brightness temperature or radiance.

- Depending on wave length (or frequency), information on gas concentration or physical properties (temperature or pressure or humidity) of atmosphere.

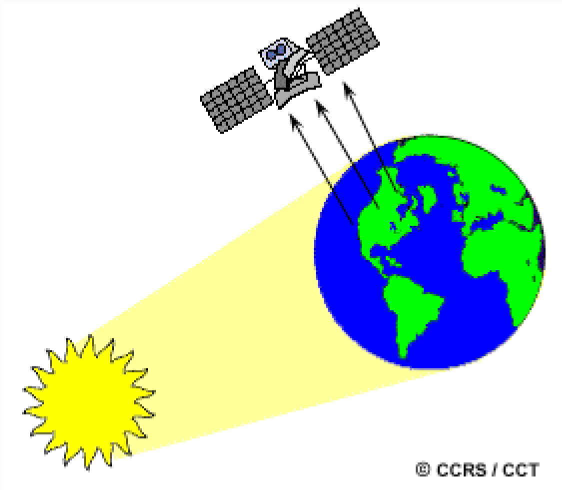
- Observations in atmospheric windows ➔ information on surface.



What is measured by satellite sensors ?

Passive measures

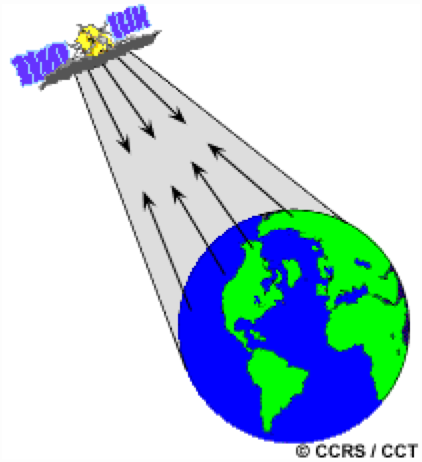
(no energy emitted from instrument)



Measures natural radiation emitted by Earth/Atmosphere from Sun origin

Active measures

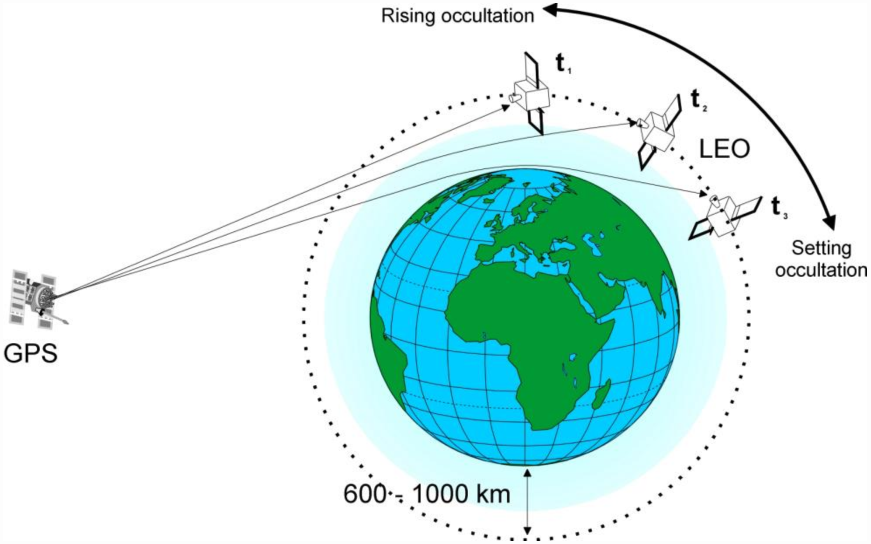
(energy emitted from instrument)



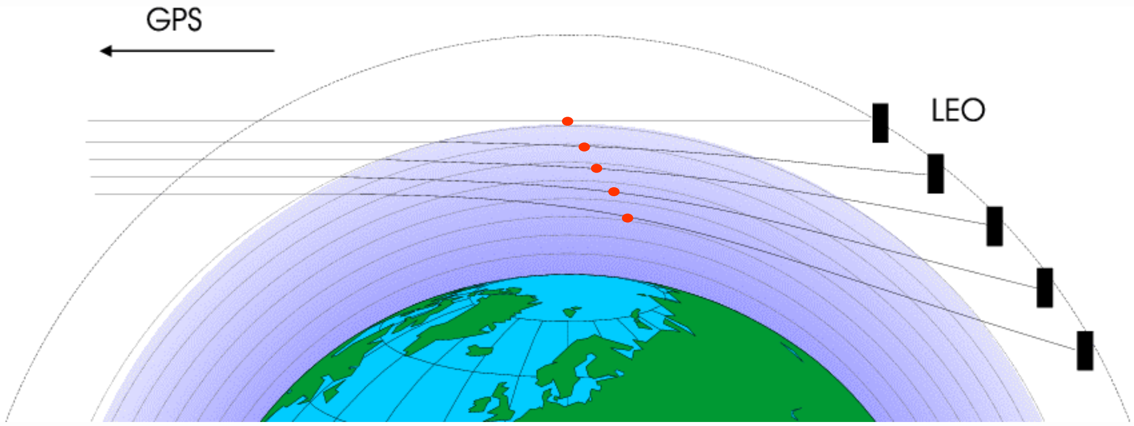
Radiation emitted by satellite and then reflected or diffused by Earth/Atmosphere

Example of active remote sensing

GPS radio occultation:



- Low-Earth Orbit satellites receive a signal from a GPS satellite.
- The signal passes through the atmosphere and gets refracted along the way.
- The magnitude of the refraction depends on temperature, moisture and pressure.
- The relative position of GPS and LEO changes over time => vertical scanning of the atmosphere.

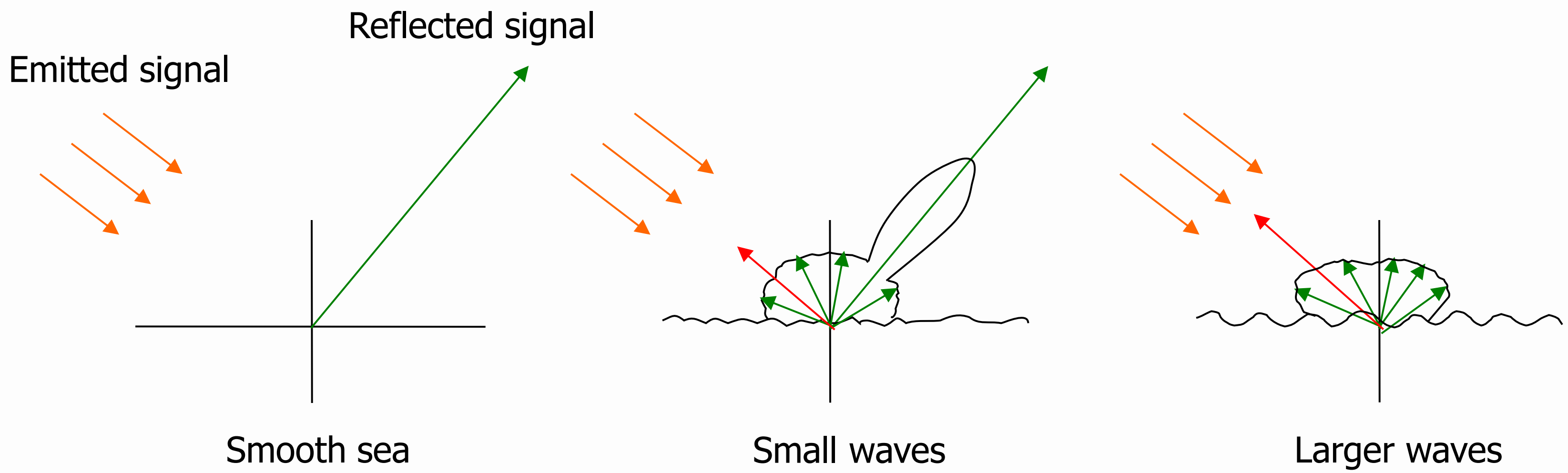


GPS stations of Météo France: Toulouse and Brest

- Propagation of GPS signal is slowed by atmosphere (dry air and water vapour).
- More than 500 GPS stations over Europe provide an estimation of Zenith Total Delay (ZTD) in real time to weather centres.
  - All weather instrument
  - High temporal resolution

Scatterometers

They send out a microwave signal towards a sea target.
The fraction of energy returned to the satellite depends on wind speed and direction.

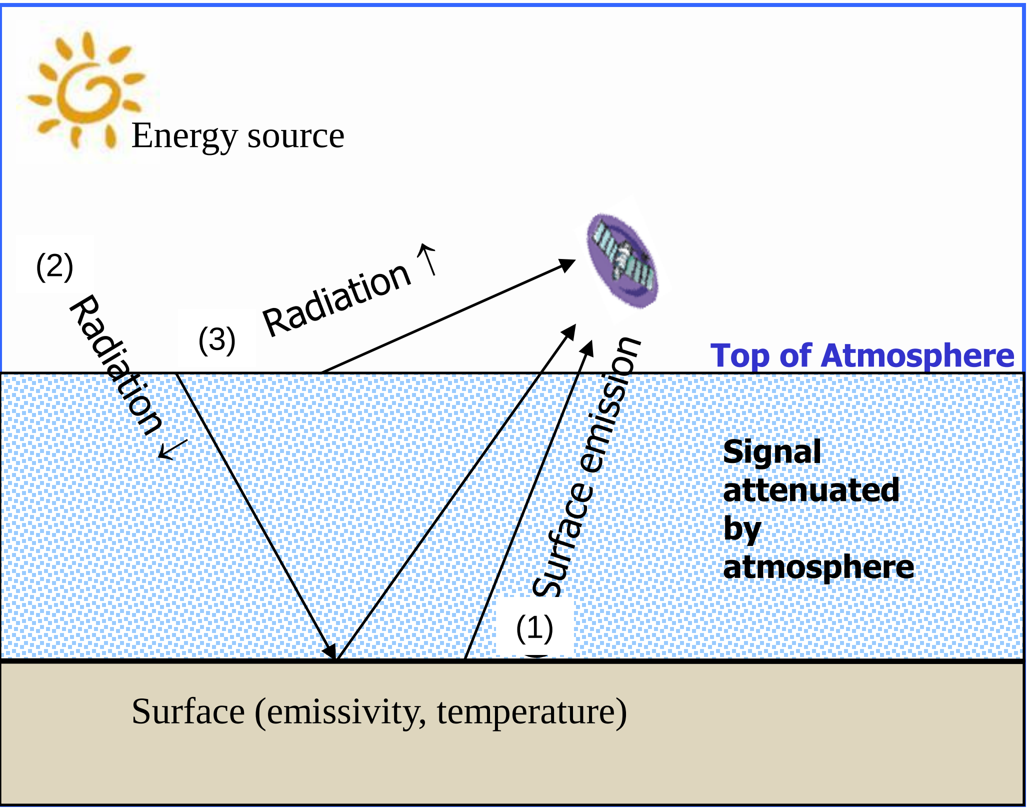


=> Measurements of near surface wind over the ocean,
through backscattering of microwave signal reflected by waves.

Passive remote sensing

Only natural sources of radiation (sun, earth...) are involved, and the sensor is a simple receiver, « passive ».

Atmosphere in Parallel Plan, no diffusion, specular surface



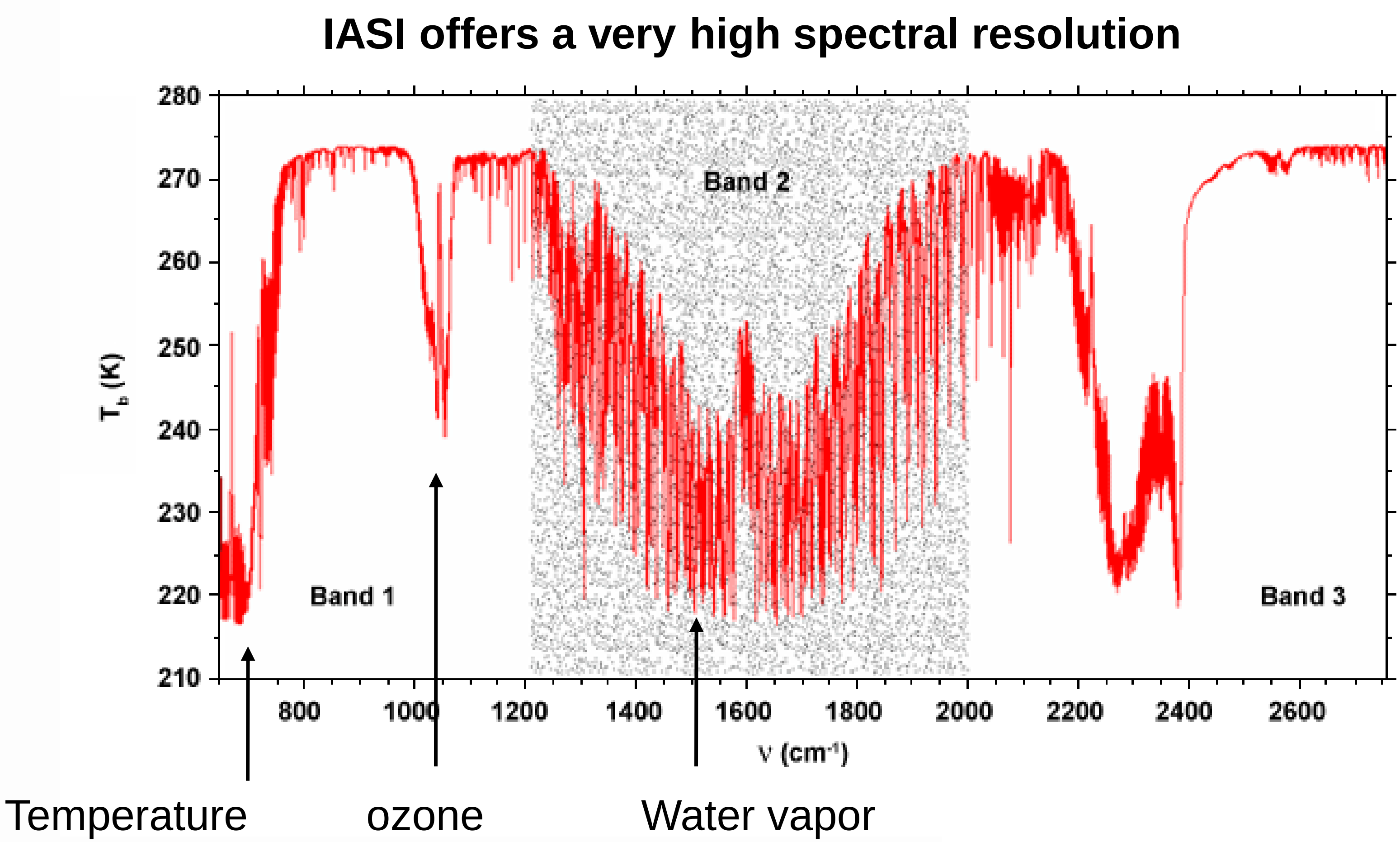
$$T(p,\upsilon) = \varepsilon(p,\upsilon)T_s\,\tau + (1-\varepsilon(p,\upsilon))\tau\, T(\upsilon,\downarrow) + T(\upsilon,\uparrow)$$

Emissivity

Radiative transfer equation, dependent on T,q :

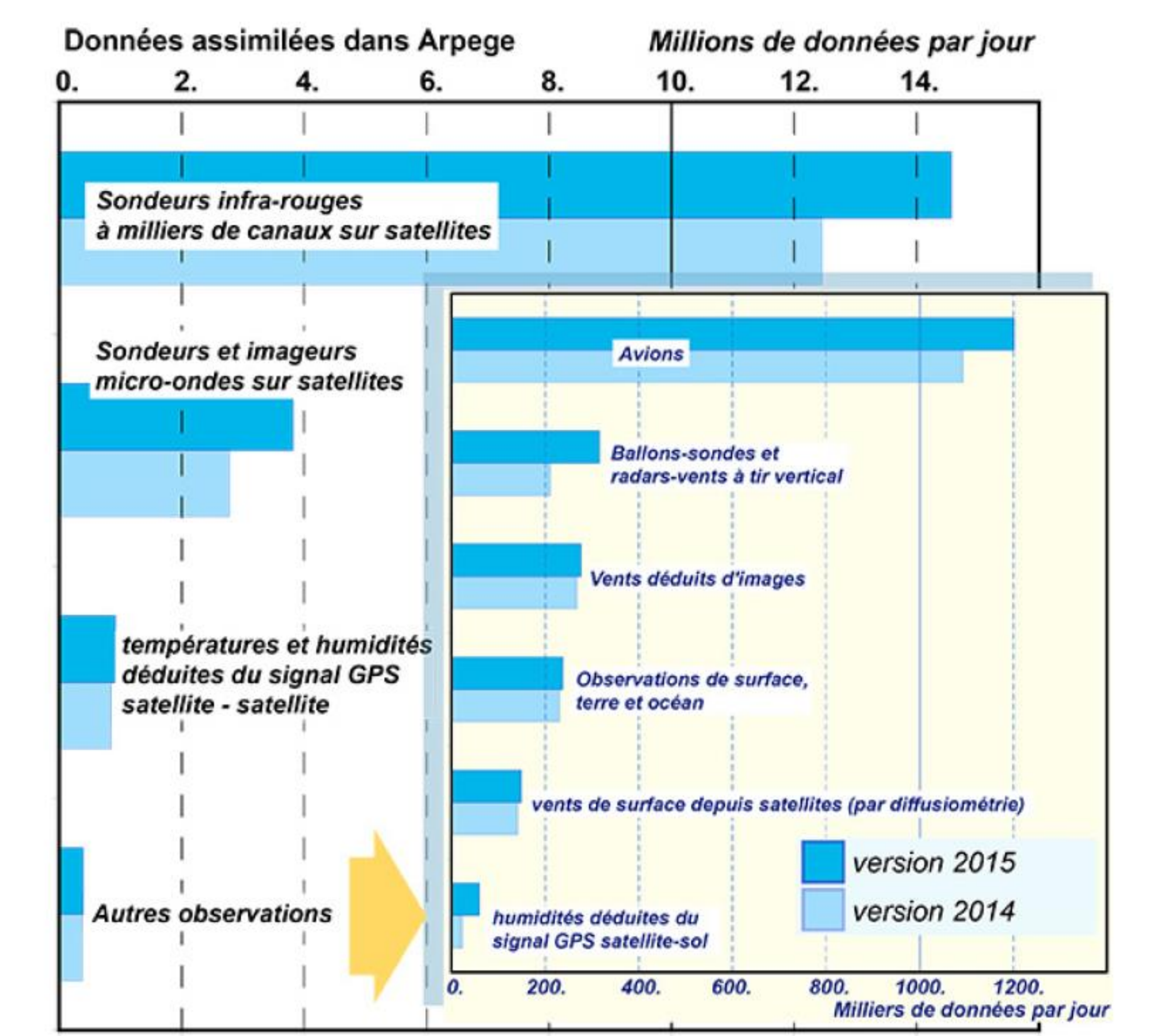
Observation operator for satellite radiances.

IASI, infra-red interferometer developed by CNES and EUMETSAT



Number of observations used in ARPEGE
(global DA at Météo-France)

Total ~ 20 million obs per day



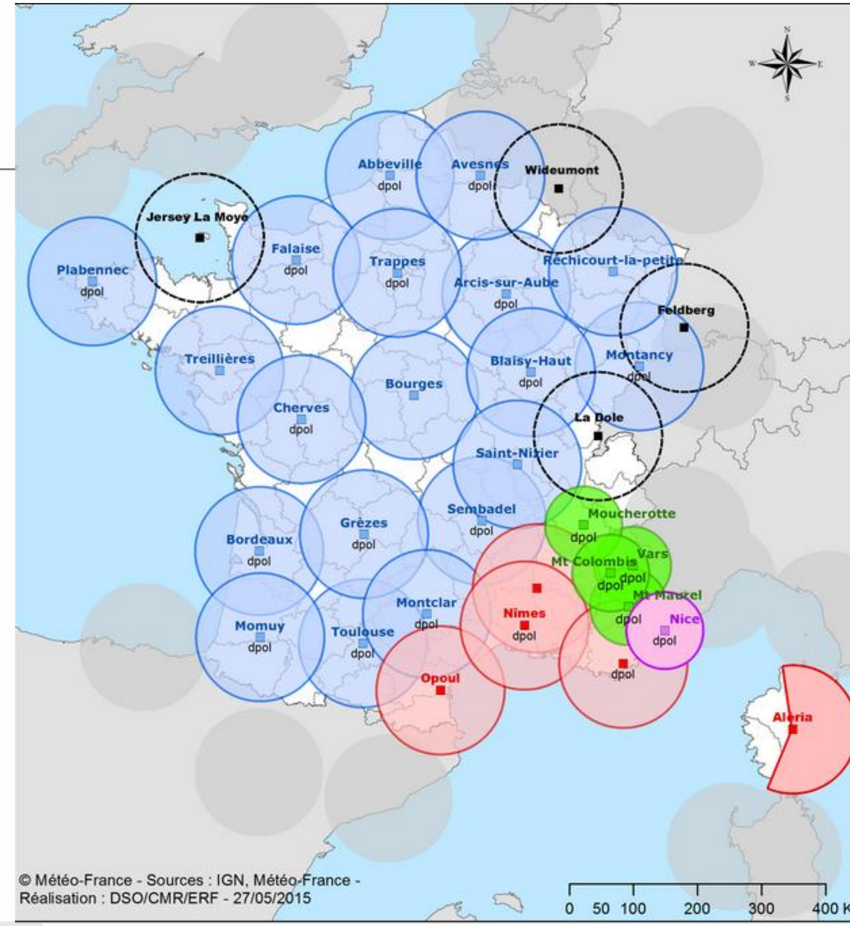
Radar network in France

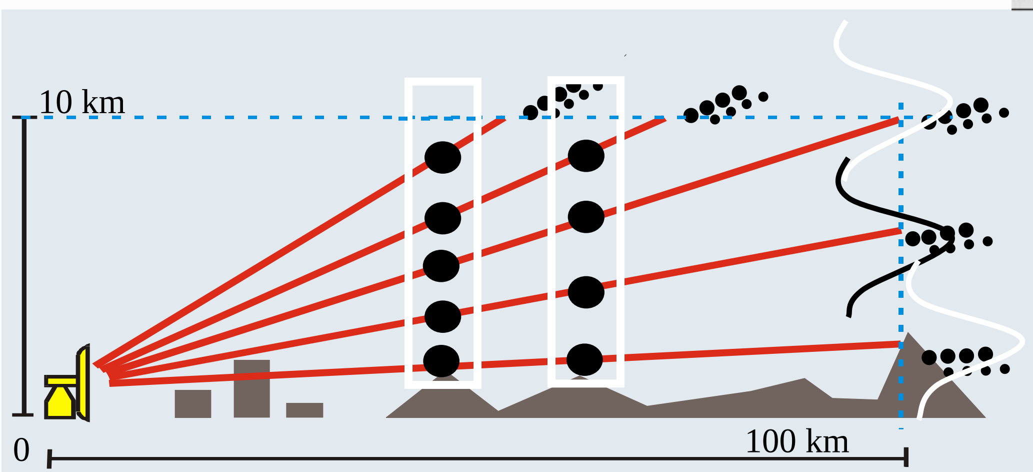
• 29 radars (19 C-band, 5S, 5X
every 15 minutes, at 1 km resolution).

• Observations :

reflectivities Z (related to precipitation),

radial winds Vr (doppler effect : modified frequency of signal, when the target is moving => wind observation).





Observations assimilated as vertical profiles

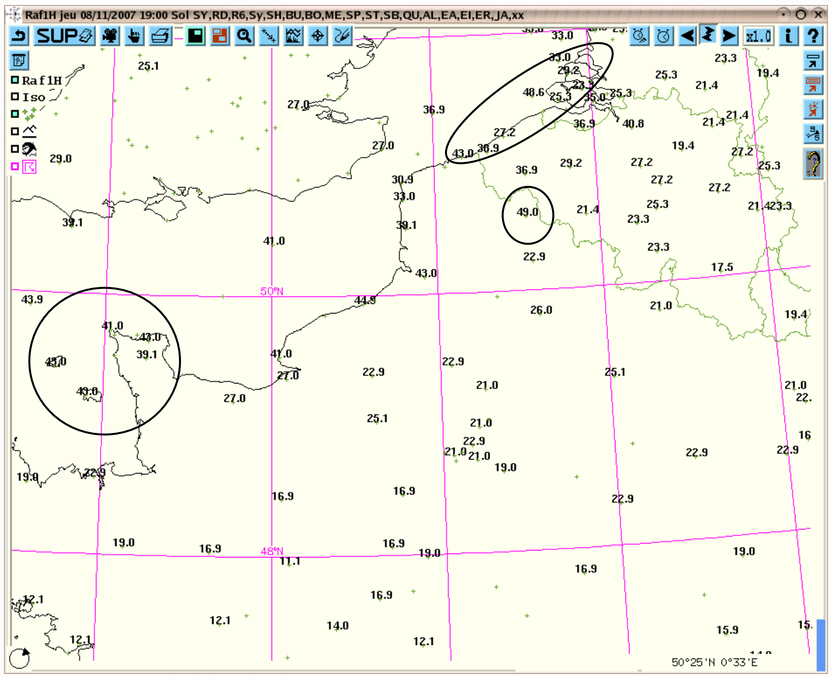
Pixel altitude is computed using a constant refractivity index along the path (effective radius approximation)

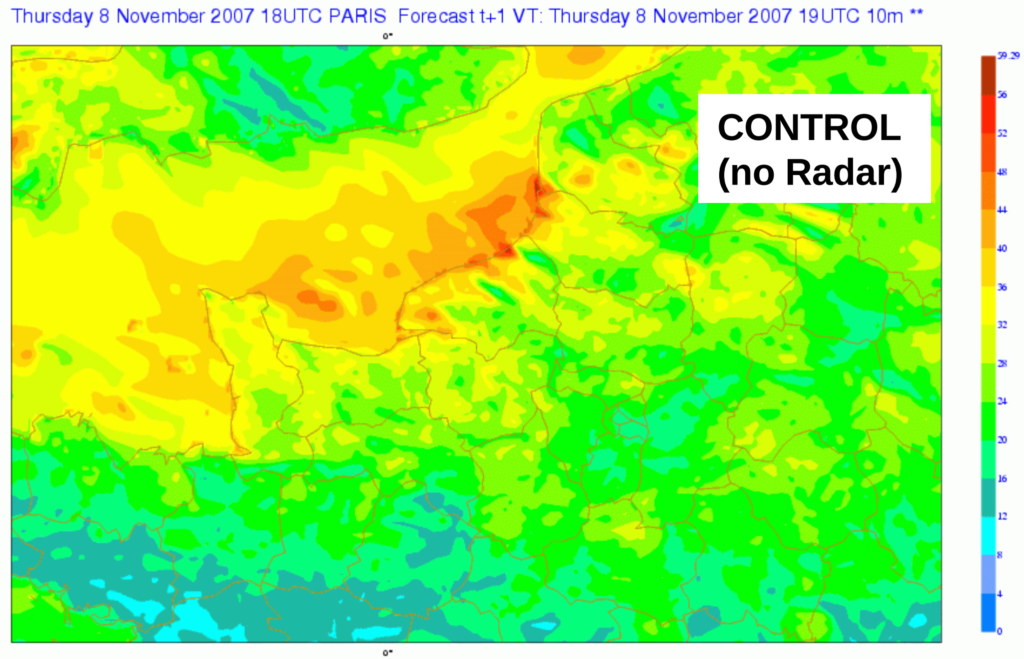
Assimilation of radar radial winds

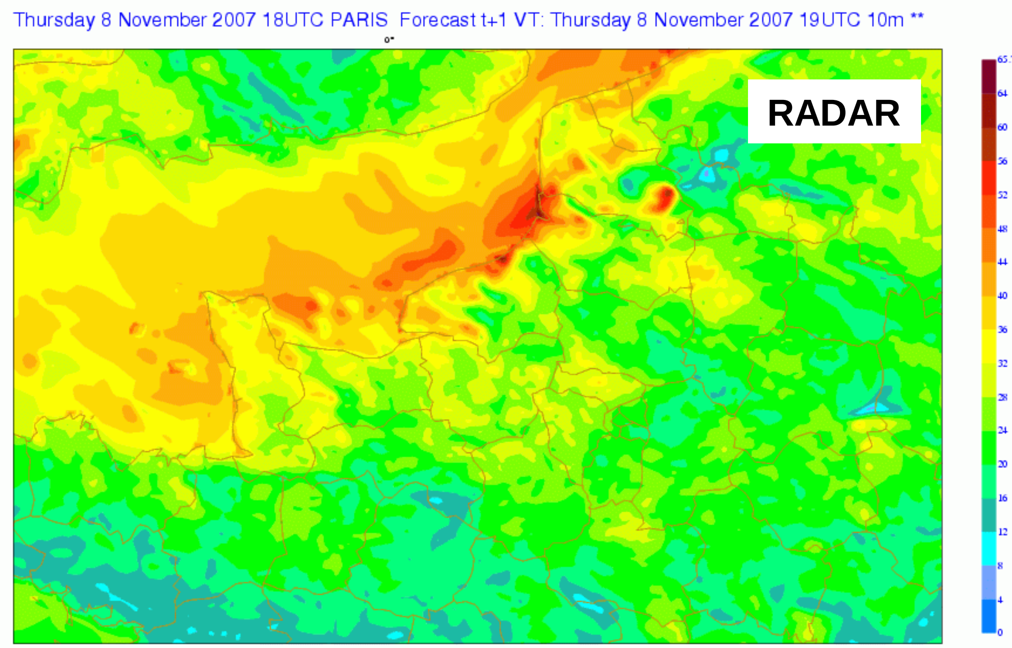
Wind gust at 10 m (kt)

Forecast +1h (19 UTC)

OBS

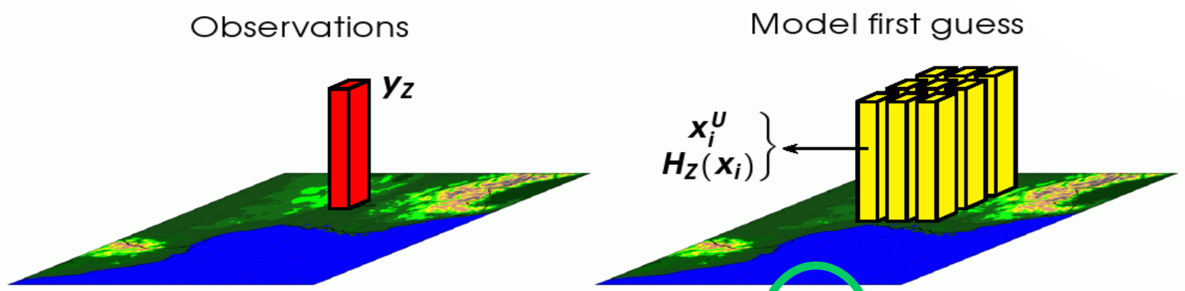






Inversion method of reflectivity profiles

Caumont, 2006: use model profiles in the neighborhood of observations (in 3 steps)



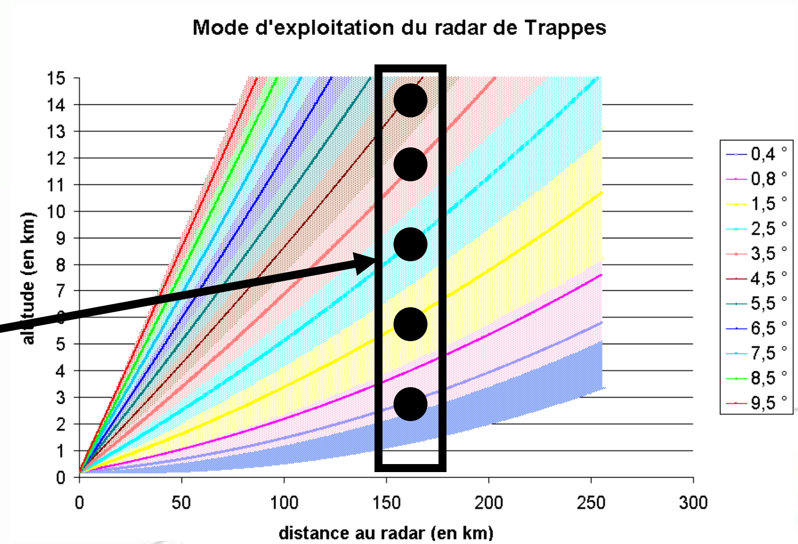
$$y_{po}^U = \sum_{i \in \text{neighbours}} x_i^U \frac{\exp\left(-\frac{1}{2}\|y_Z - H_Z(x_i)\|^2\right)}{\sum_{j \in \text{neighbours}} \exp\left(-\frac{1}{2}\|y_Z - H_Z(x_j)\|^2\right)}$$

Observation operator for reflectivities

y_{po}^U: column of pseudo-observed relative humidity,
y_Z: column of observed reflectivities,
x_i^U: column of relative humidity,
$H_Z(x_i)$: column of simulated reflectivities.

Coherence between the inverted profile and the precipitating cloud that the model is able to create

y_{po}^U



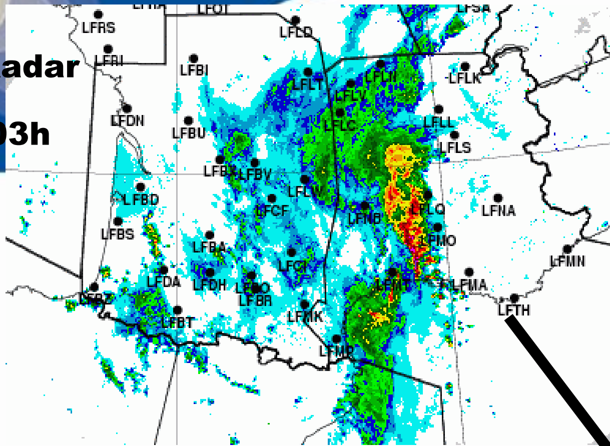
Case 7/8 october: South-East

Comparison of 3h FORECASTS
between REFL runs and CONTROL runs:

line of heavy precipitation is well analysed in REFL.

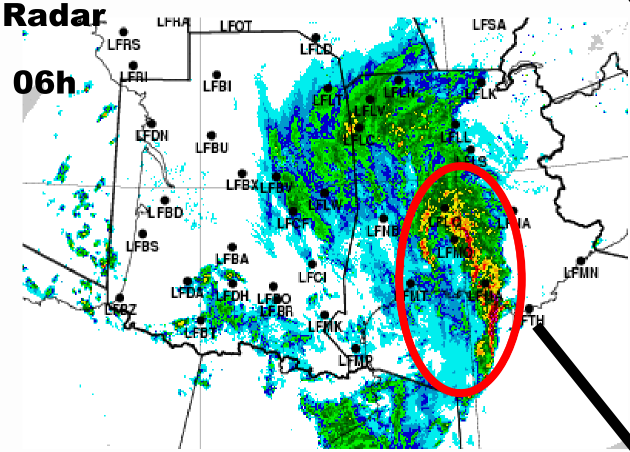
Radar

03h



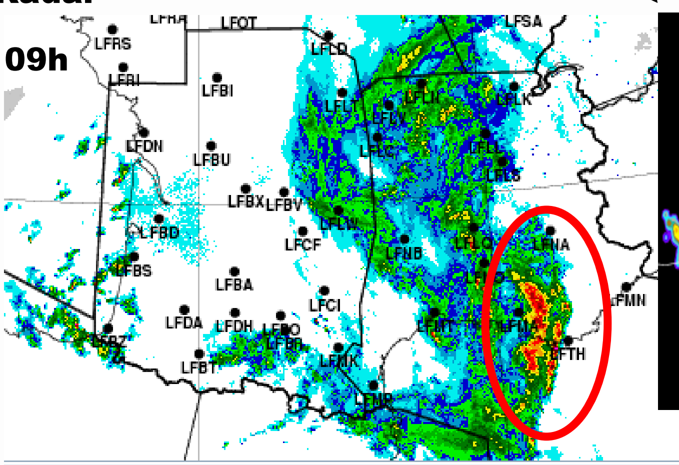
Radar

06h

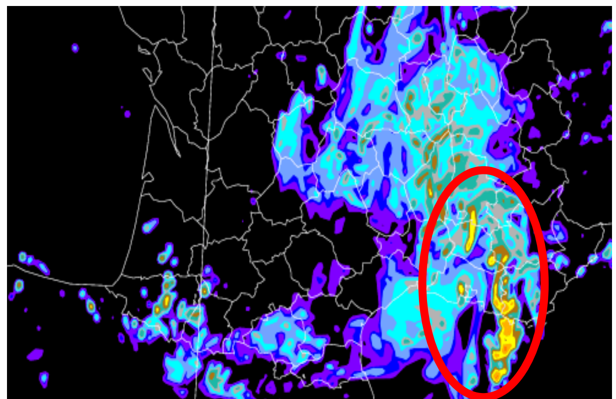


Radar

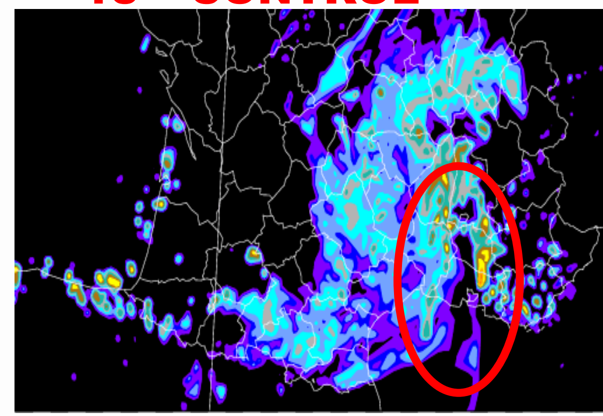
09h



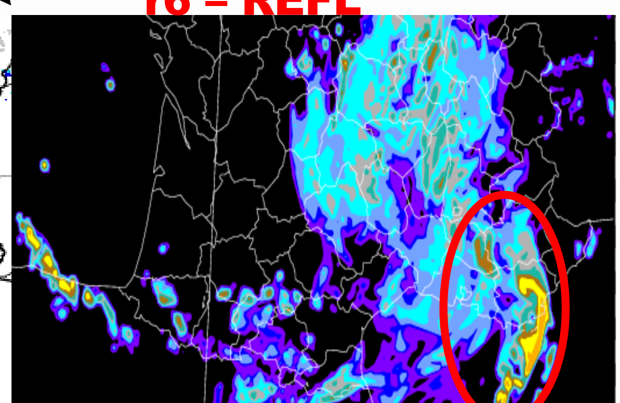
r3 – REFL



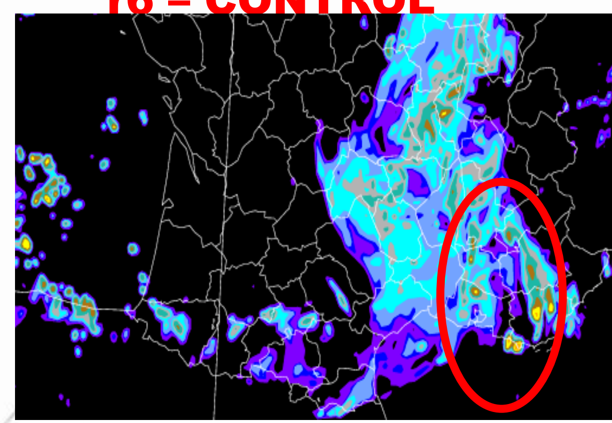
r3 – CONTROL



r6 – REFL



r6 – CONTROL



3. Error covariance estimation and modelling

Estimation and specification of error covariances

- Minimisation of cost function :

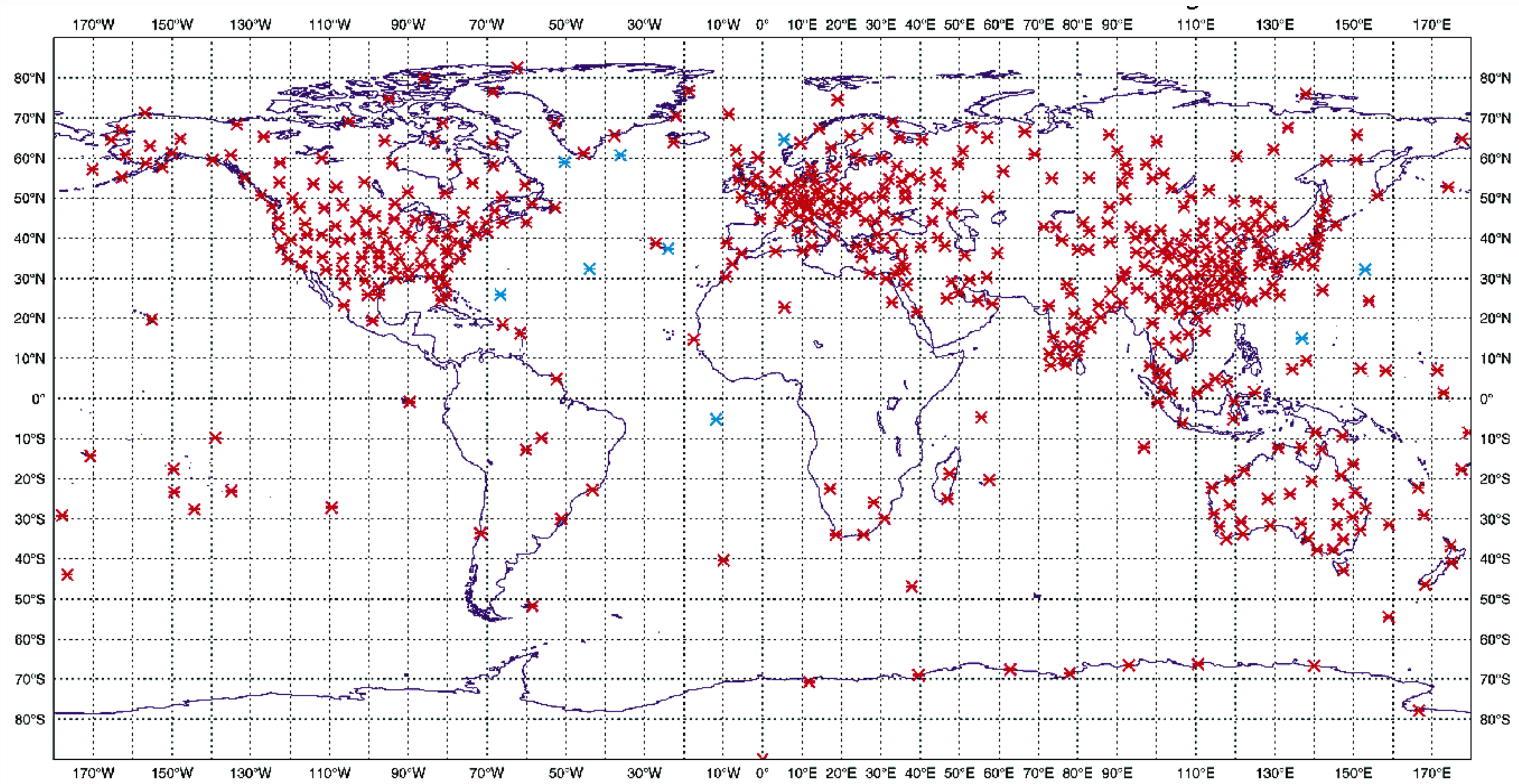
$$J(\delta \mathbf{x}) = \delta \mathbf{x}^T \mathbf{B}^{-1} \delta \mathbf{x} + (\delta \mathbf{y} - \mathbf{H}\, \delta \mathbf{x})^T \mathbf{R}^{-1} (\delta \mathbf{y} - \mathbf{H}\, \delta \mathbf{x})$$

$\Rightarrow$ Need to estimate **B** and **R**, in order to define J.

How can we estimate error covariances ?

- The true atmospheric state is never known.
- Use observation-minus-background departures
to estimate some space/time-averaged features of **R** and **B**,
using assumptions on spatial structures of errors.
- Use ensemble to simulate the error evolution and
to estimate flow-dependent background error structures.
- Use covariance modelling to filter sampling noise
and other uncertainties in the ensemble.

RADIOSONDE OBSERVATIONS



Covariances of innovations

- Innovation = observation-minus-background :

$$\mathbf{y_o} - H(\mathbf{x_b}) = \mathbf{y_o} - H(\mathbf{x_t}) + H(\mathbf{x_t}) - H(\mathbf{x_b})$$

$$\sim \mathbf{e_o} - \mathbf{H}\,\mathbf{e_b}$$

- Innovation covariances :

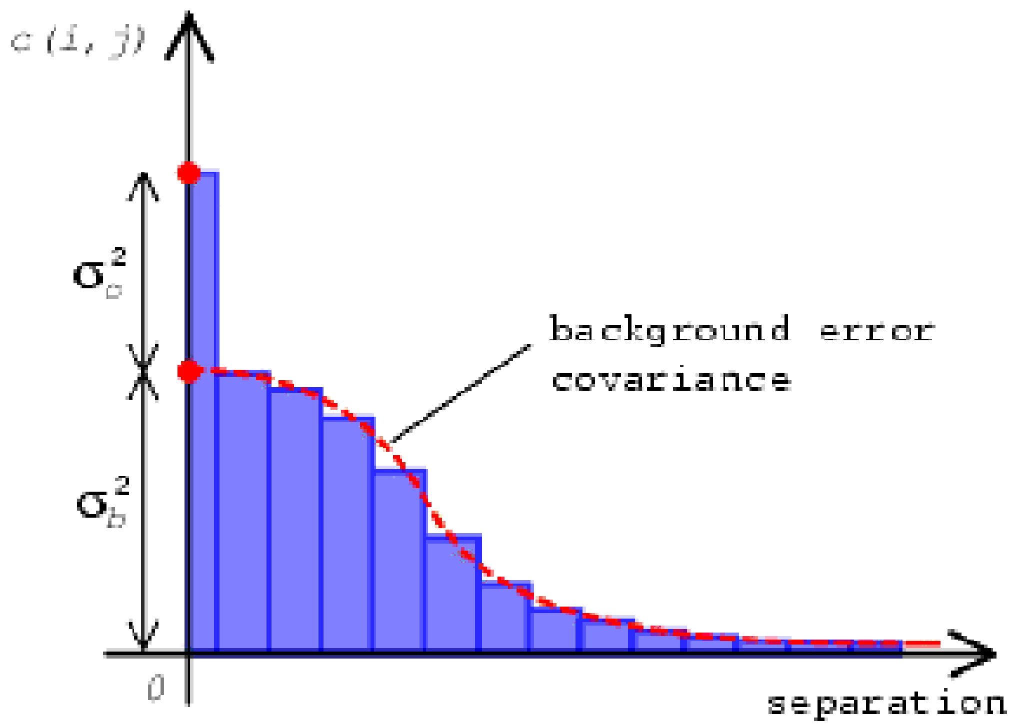
$$\mathbf{E}[(\mathbf{y_o}-H(\mathbf{x_b}))(\mathbf{y_o}-H(\mathbf{x_b}))^T] \sim \mathbf{E}[(\mathbf{e_o}-\mathbf{He_b})(\mathbf{e_o}-\mathbf{He_b})^T]$$

$$\sim \mathbf{R} + \mathbf{HBH}^T$$

assuming that $\mathbf{E}[\mathbf{e_o}(\mathbf{He_b})^T] = 0$.

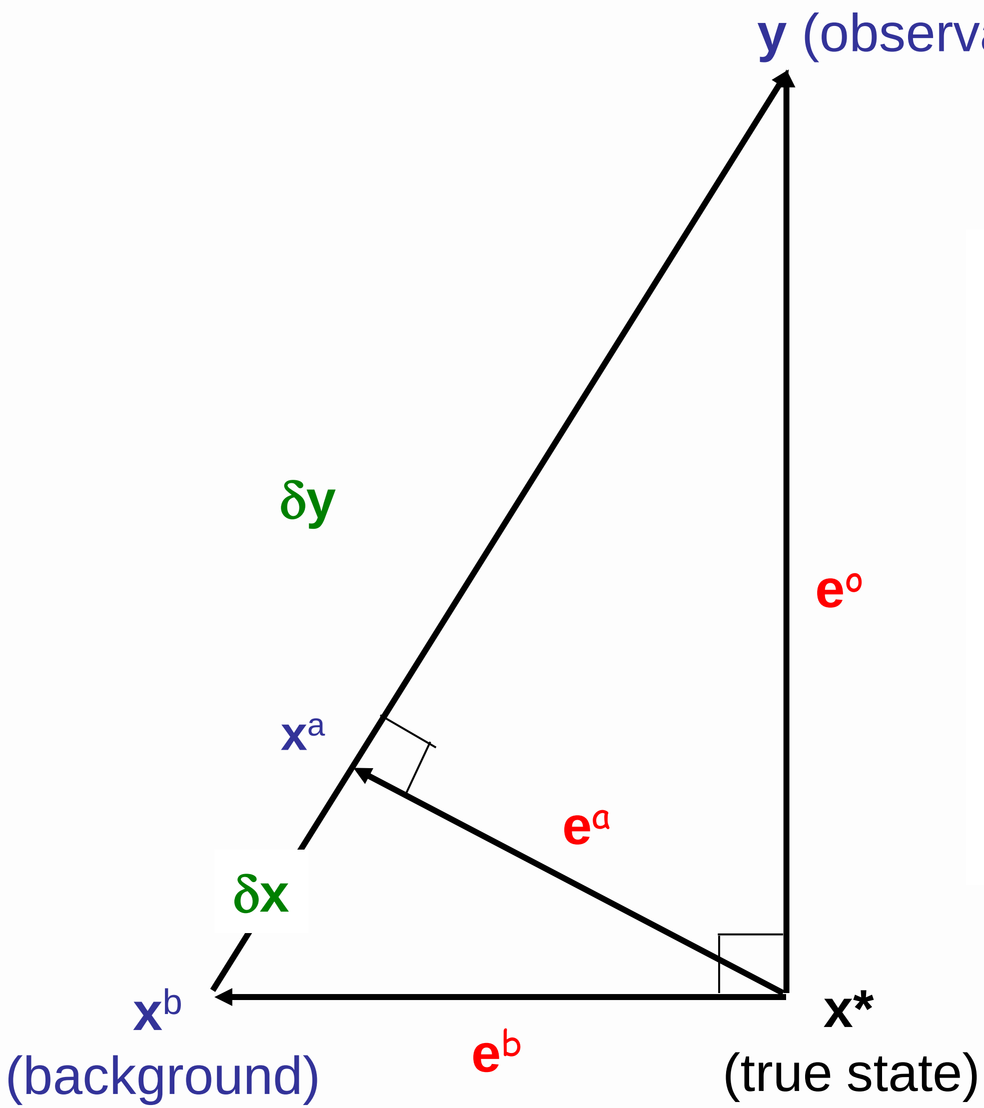
(e.g. Hollingsworth and Lönnberg 1986)

Contributions to innovation covariances



$$E[(y_o - H(x_b))(y_o - H(x_b))^T] = R + HBH^T$$

Covariances of analysis residuals



$\delta \mathbf{y} = \mathbf{y} - H(\mathbf{x}^b)$ (innovation)

$\mathbf{H}\,\delta \mathbf{x} = H(\mathbf{x}^b) - H(\mathbf{x}^a)$ (increment)

$E[\mathbf{H}\,\delta \mathbf{x}\,\delta \mathbf{y}^T] = \mathbf{HBH}^T$

$E[(\mathbf{y} - H(\mathbf{x}^a))\,\delta \mathbf{y}^T] = \mathbf{R}$

(Desroziers et al 2005)

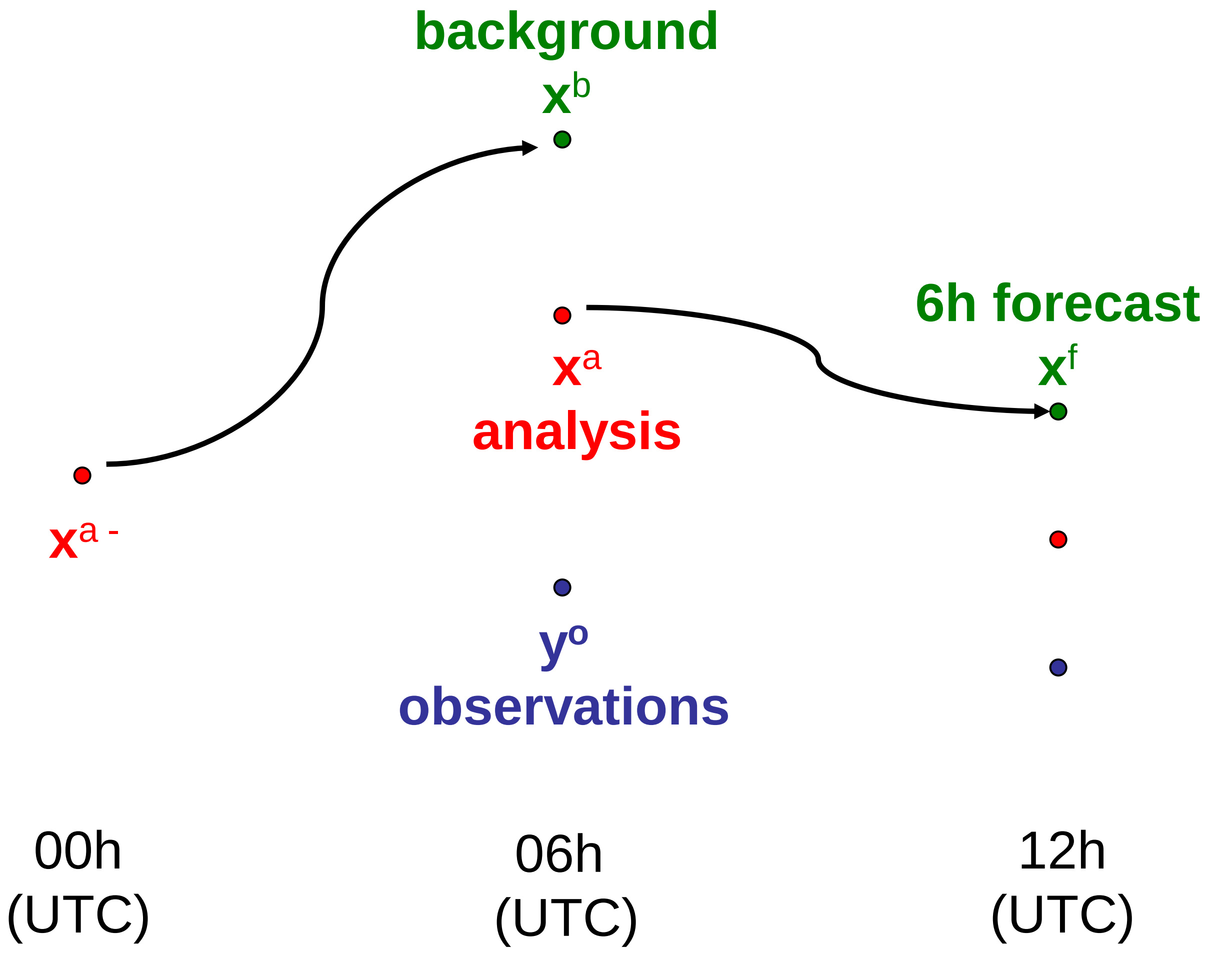
Innovation method : properties

- Provides estimates in observation space only.
- A good quality data dense network is needed.
- Assumption that observation errors are spatially uncorrelated (white noise).
- An objective source of information on **B** and **R**.
- At a given location and time, only 1 innovation value :
  a single realization of errors is available.

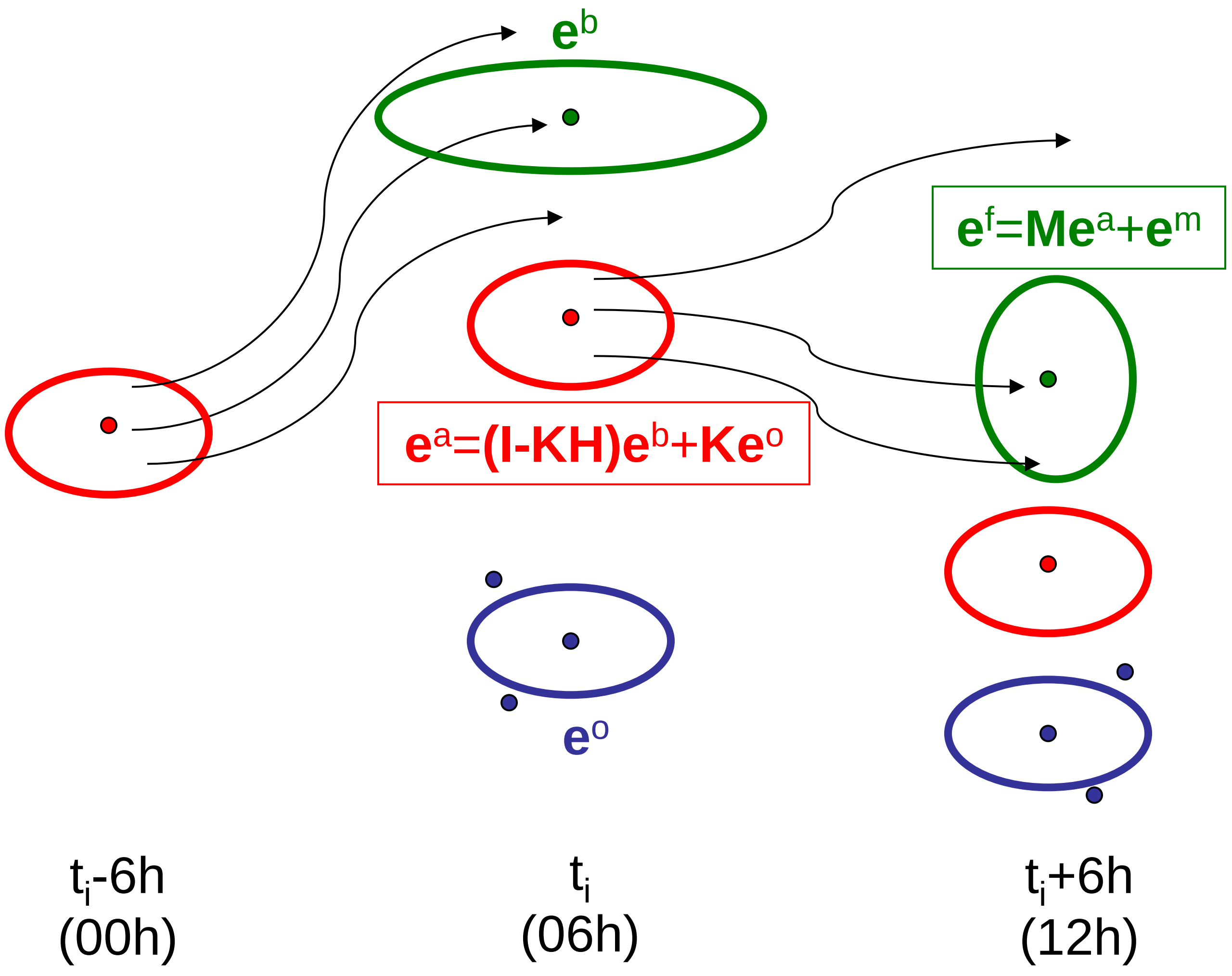
$\Rightarrow$ Statistical averages (expectations) are replaced by
space and time averages (ergodicity assumption).

$\Rightarrow$ Estimation of space/time-averages of **B** and **R**.

Data Assimilation cycling



Simulation of error cycling using an Ensemble of Data Assimilations (EDA)

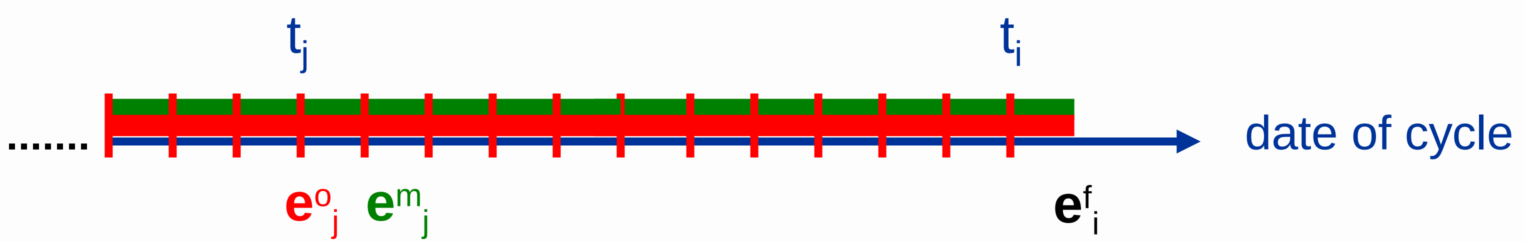


Contributions to error cycling

Quasi-linear expansion of forecast errors / cycling of observation and model errors :

$$\mathbf{e}^f_i = \sum_{j \le i} \mathbf{T}_{i-j}\, \mathbf{M}_j \mathbf{K}_j\, \mathbf{e}^o_j + \sum_{j \le i} \mathbf{T}_{i-j}\, \mathbf{e}^m_j$$

where $\mathbf{T}_{i-j} = \prod_k \mathbf{M}_k\, (\mathbf{I}-\mathbf{K}_k\mathbf{H}_k)$ = cycling operator (over k successive analysis/forecast steps from t_j to t_i).



Estimation and simulation of observation errors

- Observation perturbations / random draws of **R** :

$$\boldsymbol{\varepsilon}^o = \mathbf{R}^{1/2}\, \boldsymbol{\eta}^o \qquad \text{with } \boldsymbol{\eta}^o \sim \mathcal{N}(\mathbf{0},\mathbf{I})$$

- Estimation of **R** / spatio-temporal averages

  via covariances of innovations ($\delta\mathbf{y} = \mathbf{y}^o - \mathbf{H}\mathbf{x}^b = \boldsymbol{\varepsilon}^o - \mathbf{H}\boldsymbol{\varepsilon}^b$, $E[\delta\mathbf{y}\, \delta\mathbf{y}^T] = \mathbf{R} + \mathbf{HBH}^T$) :

$$\mathbf{R} \simeq E[\delta\mathbf{y}\, \delta\mathbf{y}^T] - \mathbf{HBH}^T$$

  after subtracting estimated contribution of background errors.

- Filtering and propagation of observation perturbations /

  contribution to forecast errors : $\boldsymbol{\varepsilon}^{f,o}_i = \sum_{j \leq i} \mathbf{T}_{i-j}\, \mathbf{M}_j \mathbf{K}_j\, \boldsymbol{\varepsilon}^o_j$

Estimation and simulation of model errors

- Forecast errors = cycling of observation and model errors :

$$\mathbf{e}^{f}_{i} = \mathbf{e}^{f,o}_{i} + \mathbf{e}^{f,m}_{i} = \sum_{j \le i} \mathbf{T}_{i-j}\, \mathbf{M}_{j}\mathbf{K}_{j}\, \mathbf{e}^{o}_{j} + \sum_{j \le i} \mathbf{T}_{i-j}\, \mathbf{e}^{m}_{j}$$

- Model perturbations / random draws of **Q** :

$$\boldsymbol{\varepsilon}^{m} = \mathbf{Q}^{1/2}\, \boldsymbol{\eta}^{m} \qquad \text{with } \boldsymbol{\eta}^{m} \sim \mathcal{N}(\mathbf{0},\mathbf{I})$$

- Estimation of $\mathbf{HQH}^{T}$ / spatio-temporal averages via covariances of innovations ; ex : accumulated contribution of $\mathbf{e}^{f,m}$:

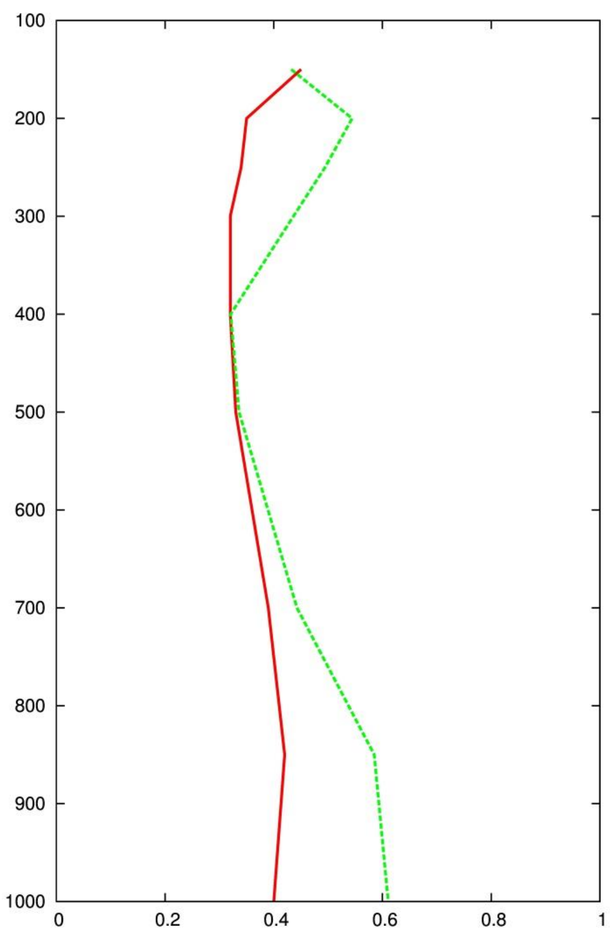
$$\mathbf{HQ}^{f,m}\mathbf{H}^{T} \simeq \mathbf{HB}^{f}\mathbf{H}^{T} - E[\mathbf{H}\boldsymbol{\varepsilon}^{f,o}(\mathbf{H}\boldsymbol{\varepsilon}^{f,o})^{T}]$$

after subtracting estimated contribution of cycled observation errors.

- Different variants of representation (*multiplicative inflation (oper.)* or additive, SPPT, SKEB, etc) (e.g. Houtekamer et al 2009).

Model error accumulated during cycling

Pressure (hPa)



$$\sigma_{\delta y/ens}\left(e^{f,m}_i = \sum_j T_{i-j}\, e^{m}_j \right)$$

$$\sigma_{ens}\left(e^{f,o}_i = \sum_j T_{i-j}\, M_j K_j e^{o}_j \right)$$

(Raynaud et al 2012)

Standard deviations of contributions to forecast errors

(aircraft observations of temperature (K))

Modelling and filtering covariances

Huge size of **B** : $(10^9)^2 = 10^{18}$ elements.
=> modelling with operators that are sparse and/or of small size.

Sampling noise, and other uncertainties.
=> spatio-temporal filtering.

Factorisation :

$$\mathbf{B} = \mathbf{B}^{1/2}\,\mathbf{B}^{T/2}$$

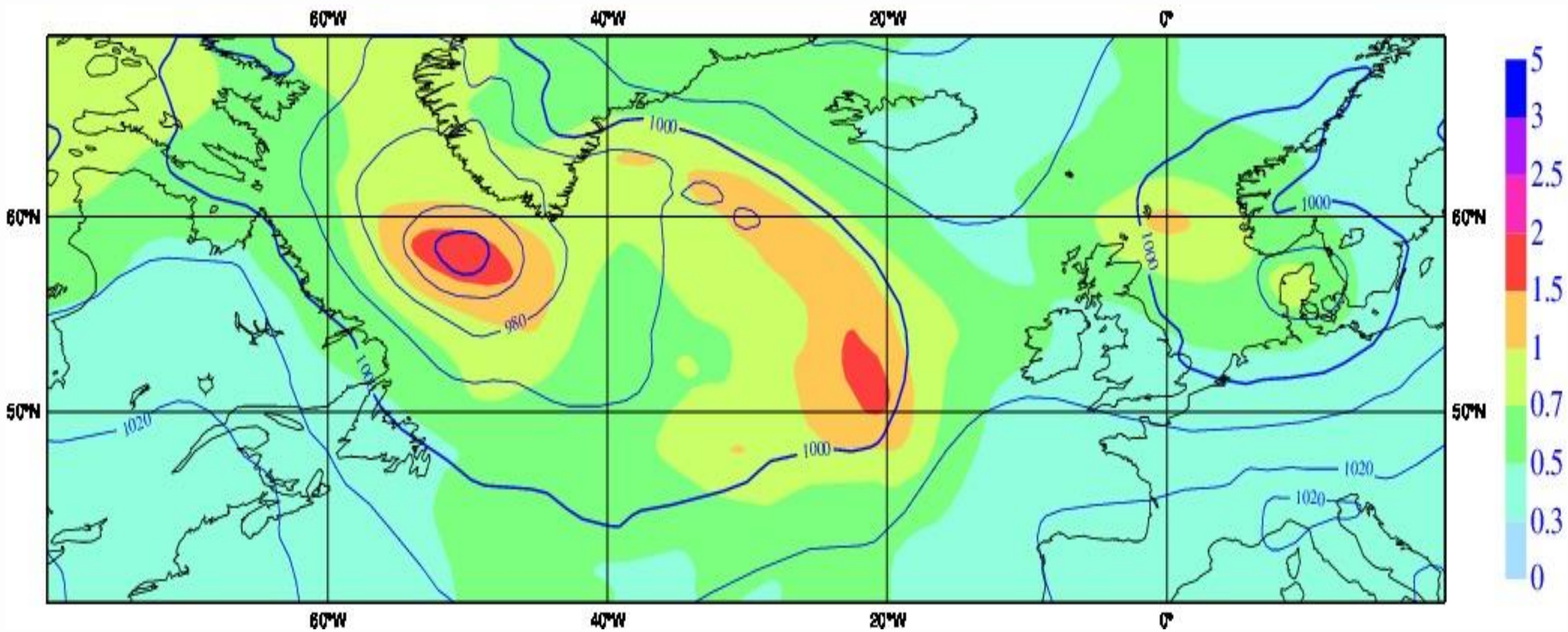
$$\mathbf{B}^{1/2} = \mathbf{L}\;\mathbf{S}\;\mathbf{C}^{1/2}$$

L ~ mass/wind cross-covariances (related to geostrophy), including flow-dependence (non linear balances).

S flow-dependent standard deviations (~ expected error amplitudes), filtered spatially.

C matrix of 3D spatial correlations (~ spatial structures of errors), filtered in wavelet space (block-diagonal model).

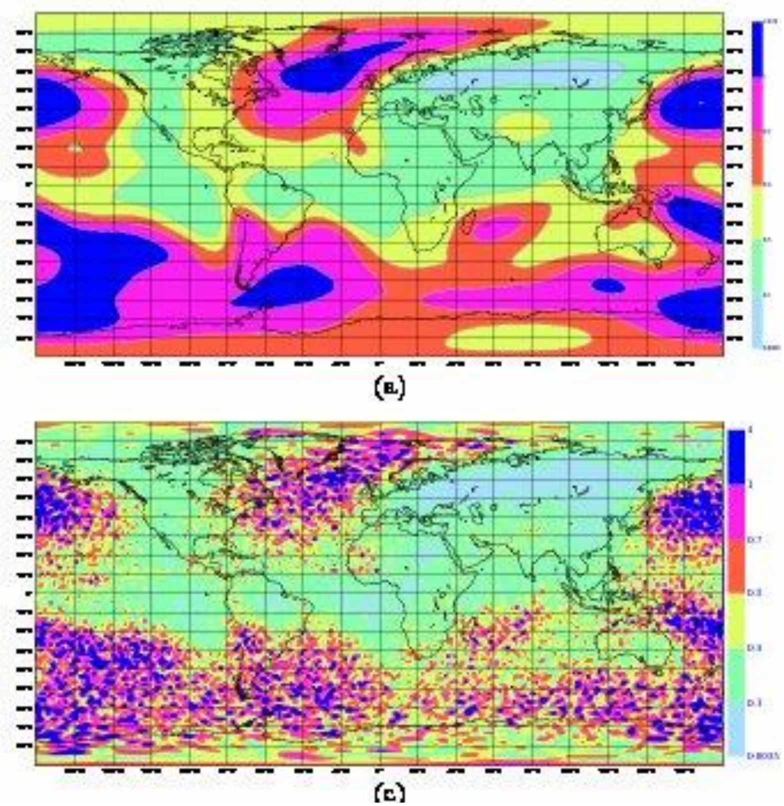
Dynamics of background error variances



Standard deviations of surface pressure (hPa) (2/2/2010), superimposed with mean sea level pressure analysis (hPa)

Spatial filtering of variance field

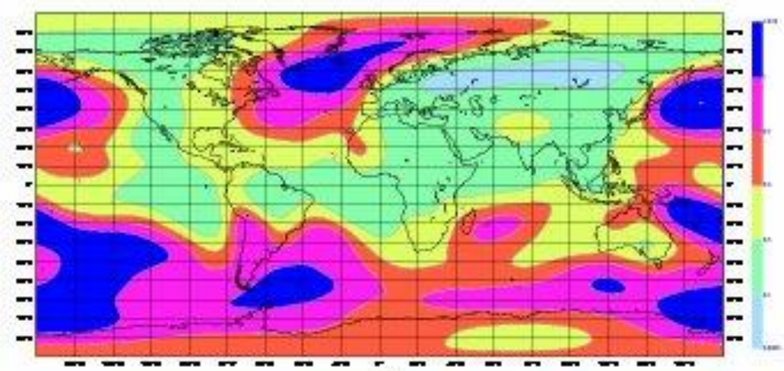
« true » variances



raw variances $\mathbf{v}_b$ (N = 6)

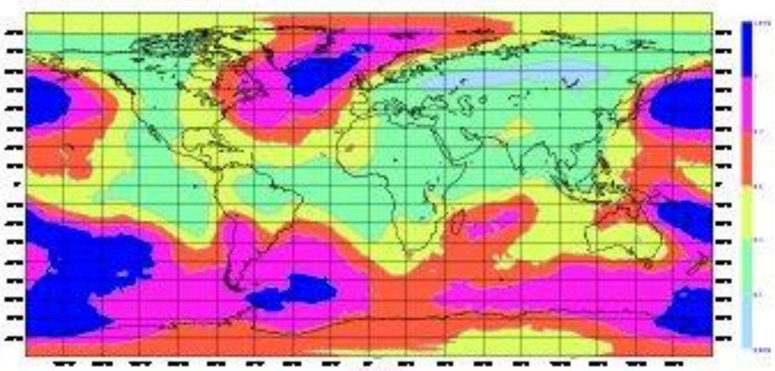
Spatial filtering of variance field

« true » variances

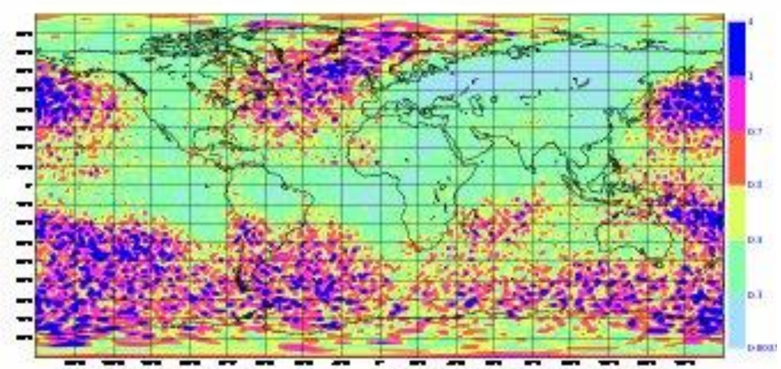


(a)

filtered variances $\mathbf{v'}_b$ (N = 6)



(b)



(c)

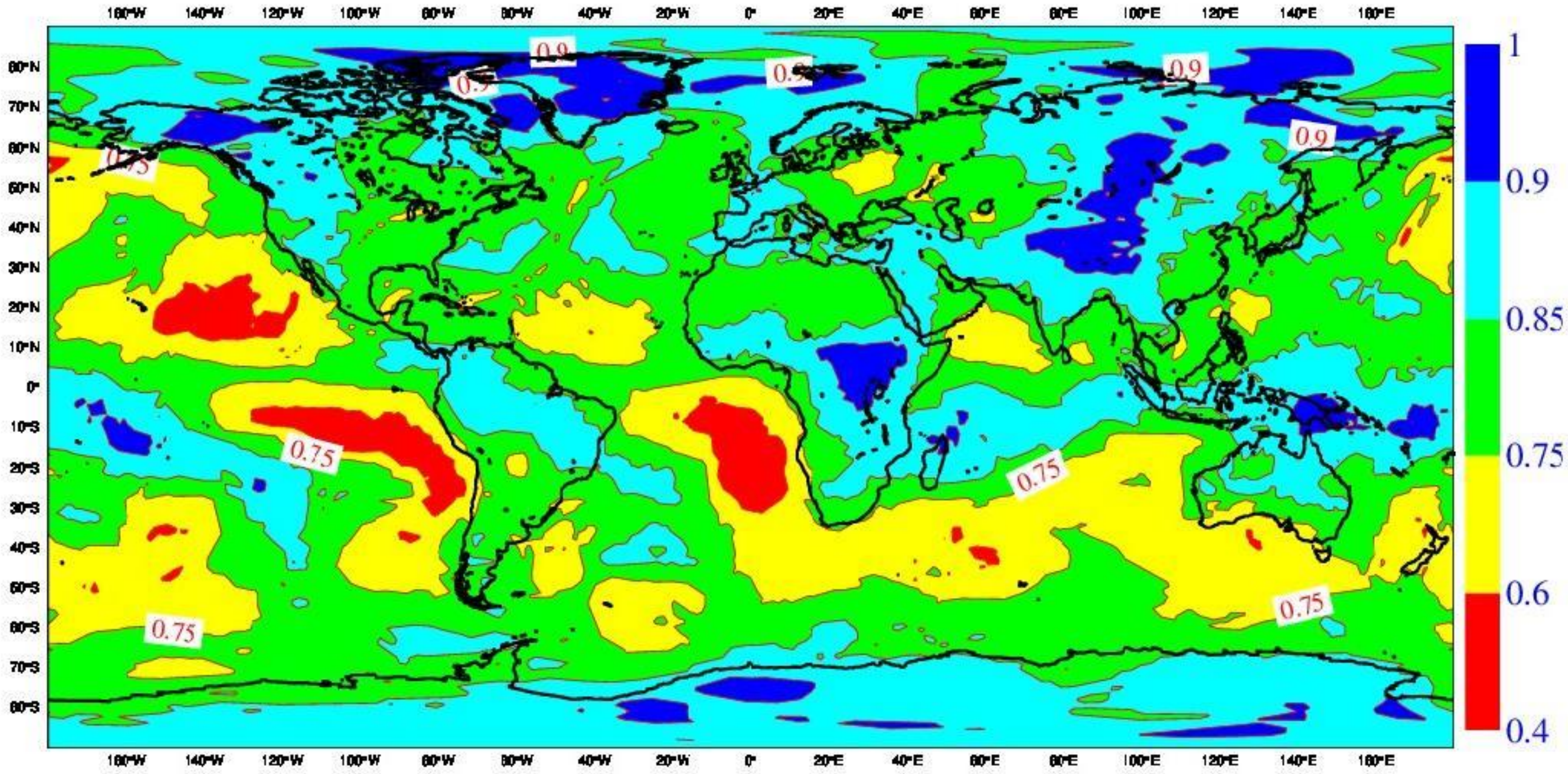
raw variances $\mathbf{v}_b$ (N = 6)

$$\mathbf{v'}_b = \mathbf{F}\ \mathbf{v}_b$$

$$F = 1 / (1 + E[noise^2]/signal^2)$$

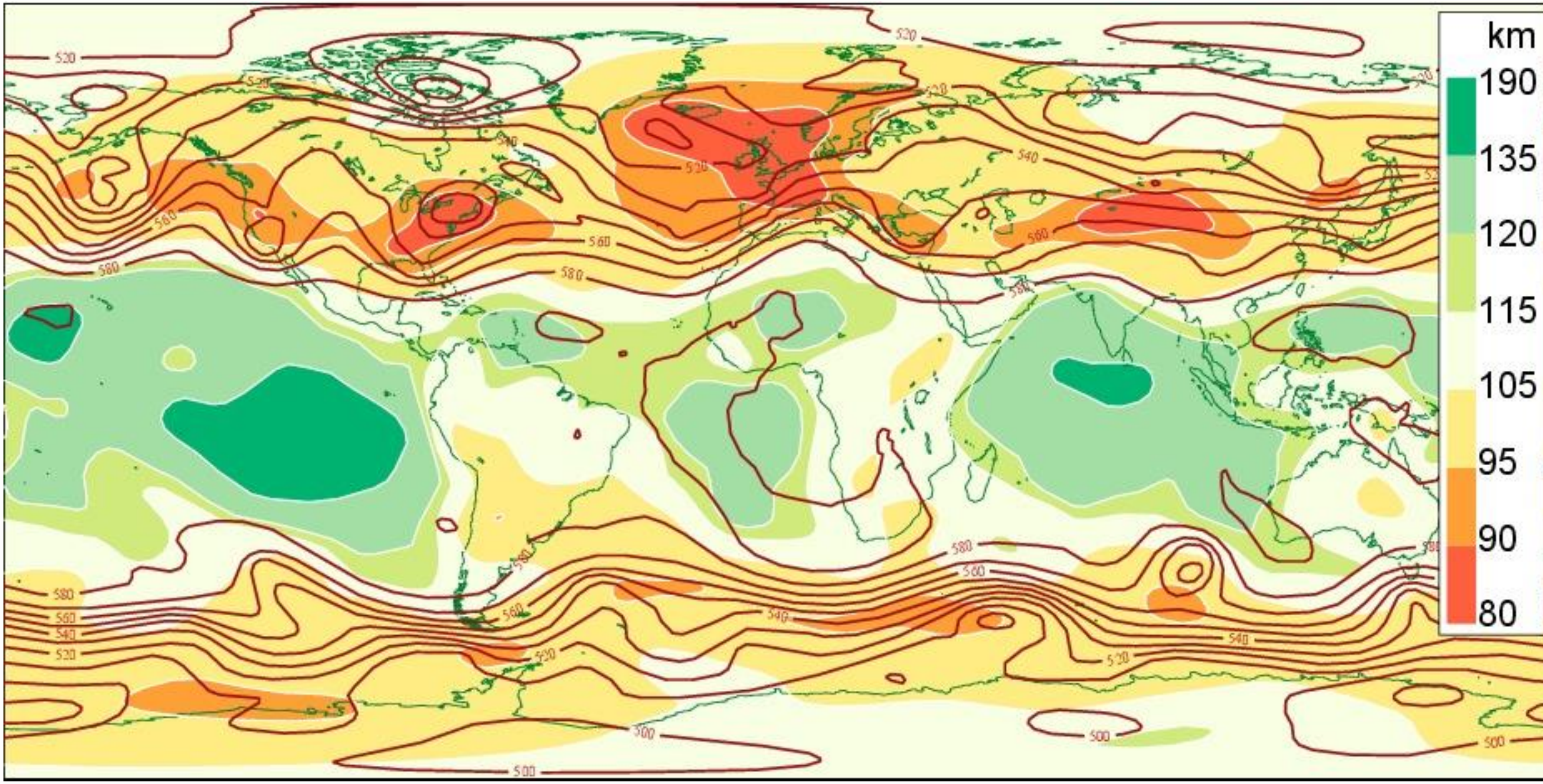
with $E[v_b^2] = signal^2 + E[noise^2]$.

Dynamics of vertical correlations (ensemble + wavelet filtering)



Vertical correlations of temperature between 850 & 870 hPa (28/2/2010)

Dynamics of horizontal correlations (ensemble + wavelet filtering)



Length-scales (in km) of wind near 500 hPa, superimposed to geopotential.

Modelling of covariances in ensemble space

$$\mathbf{B}_{\mathbf{raw}} = \mathbf{X'}_{b}\,\mathbf{X'}_{b}^{T} / (N-1)$$

$$\mathbf{B}_{\mathbf{raw}}^{1/2} = \mathbf{X'}_{b} / \sqrt{N-1}$$

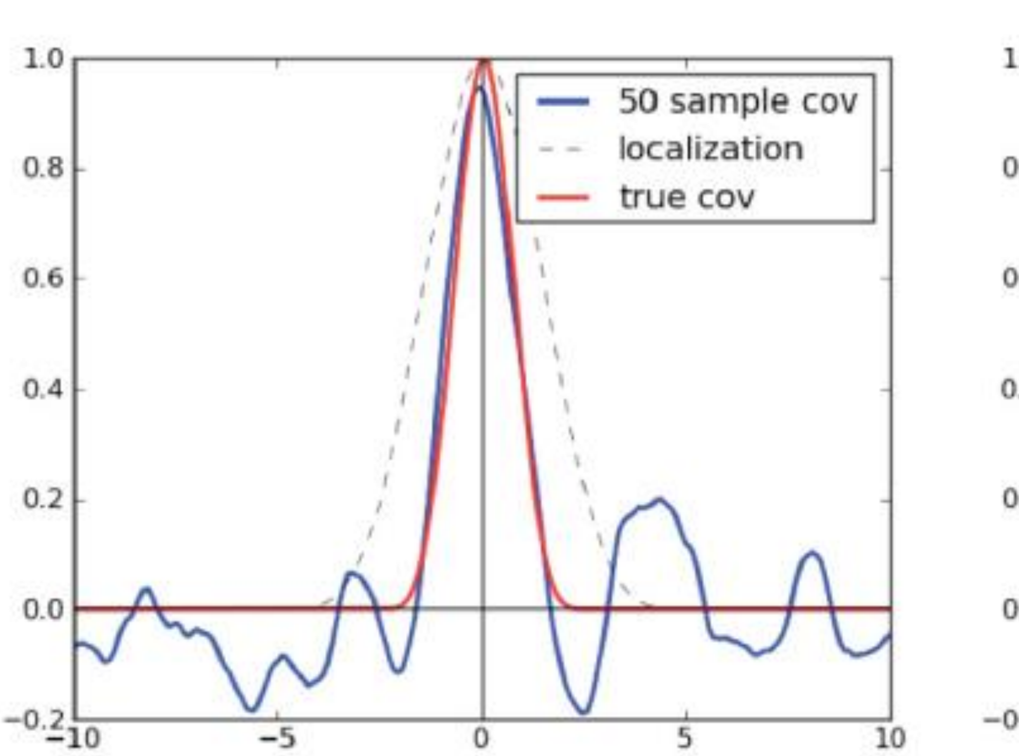
where the perturbation matrix $\mathbf{X'}_{b}$ contains the perturbation field of each member as a column (Lorenc 2003).

Schur filtering for long-distance correlations :

$$\mathbf{B} = \mathbf{B}_{\mathbf{raw}} \circ \mathbf{C}_{\mathbf{L}}$$

where $\mathbf{C}_{\mathbf{L}}$ is a localisation matrix (~ correlation model).

Schur filtering of covariances



(Figure Whitaker (2011), N=50)

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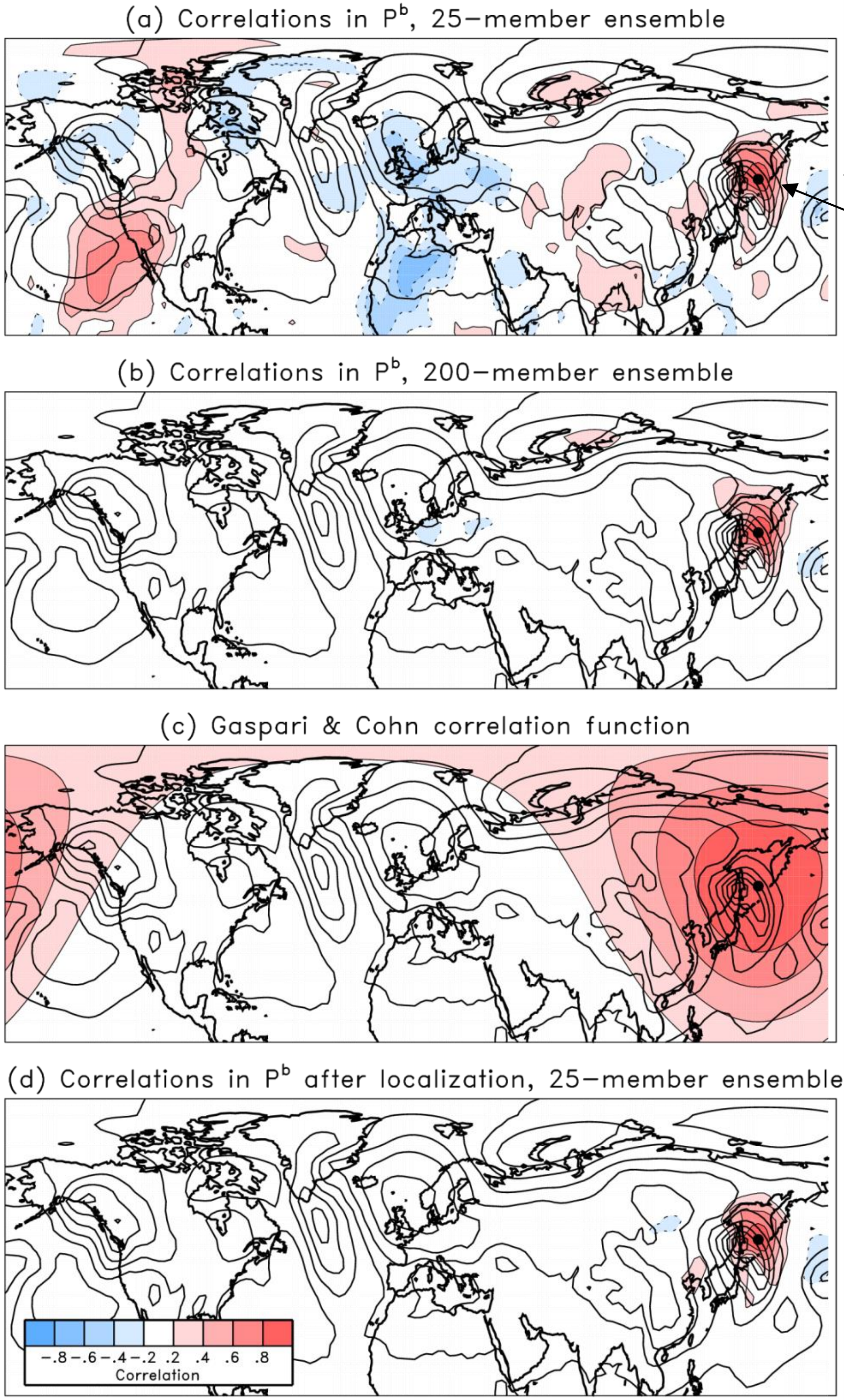

Observation location

Schur filtering of long-distance correlations

from Hamill, Chapter 6 of "*Predictability of Weather and Climate*"

Conclusions

- Data Assimilation (DA) is vital for weather forecasting (NWP).
- Observations are very diverse in type, density and quality.
- 4D-Var for temporal and non linear aspects.
- Innovation departures for space-averaged estimation of error covariances.
- Ensemble DA methods for error simulation and flow-dependent estimation.
- Covariance modelling/filtering for sampling noise and other uncertainties.

Thank you

for your attention

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Some references

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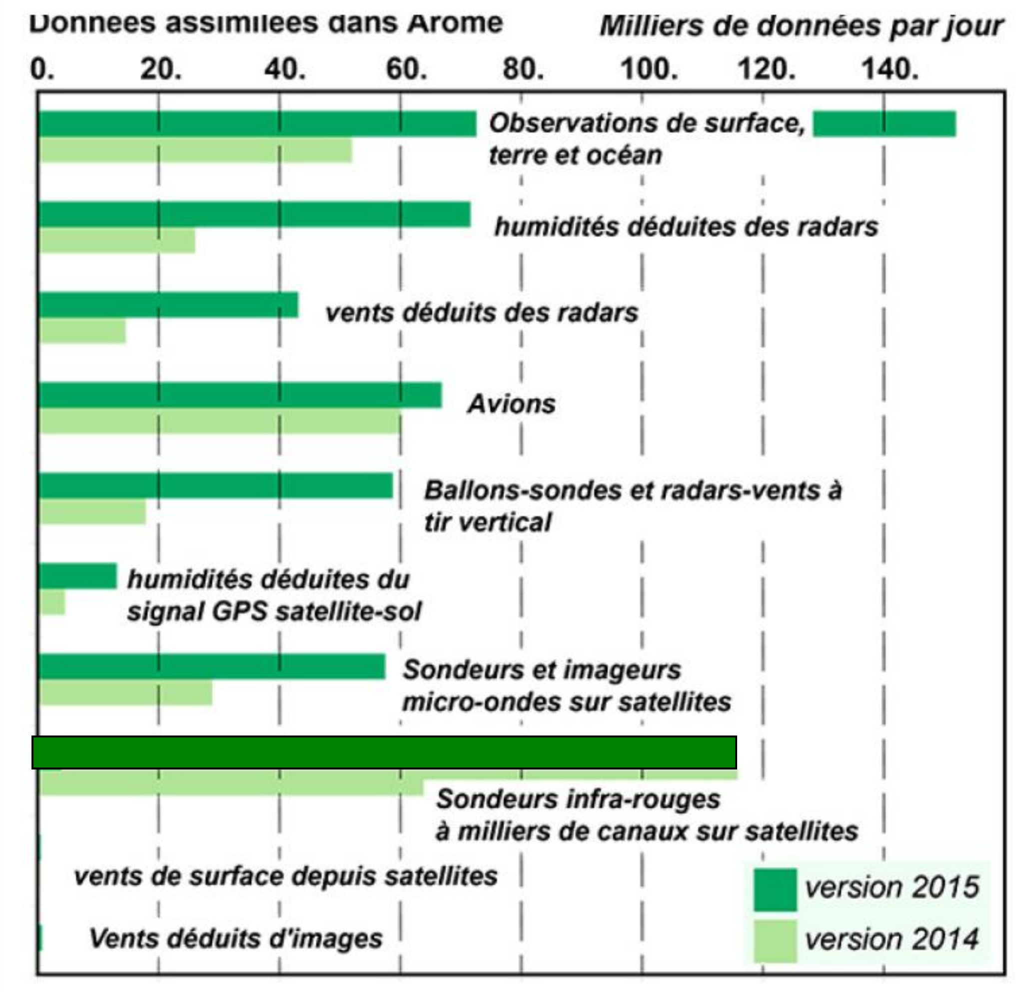
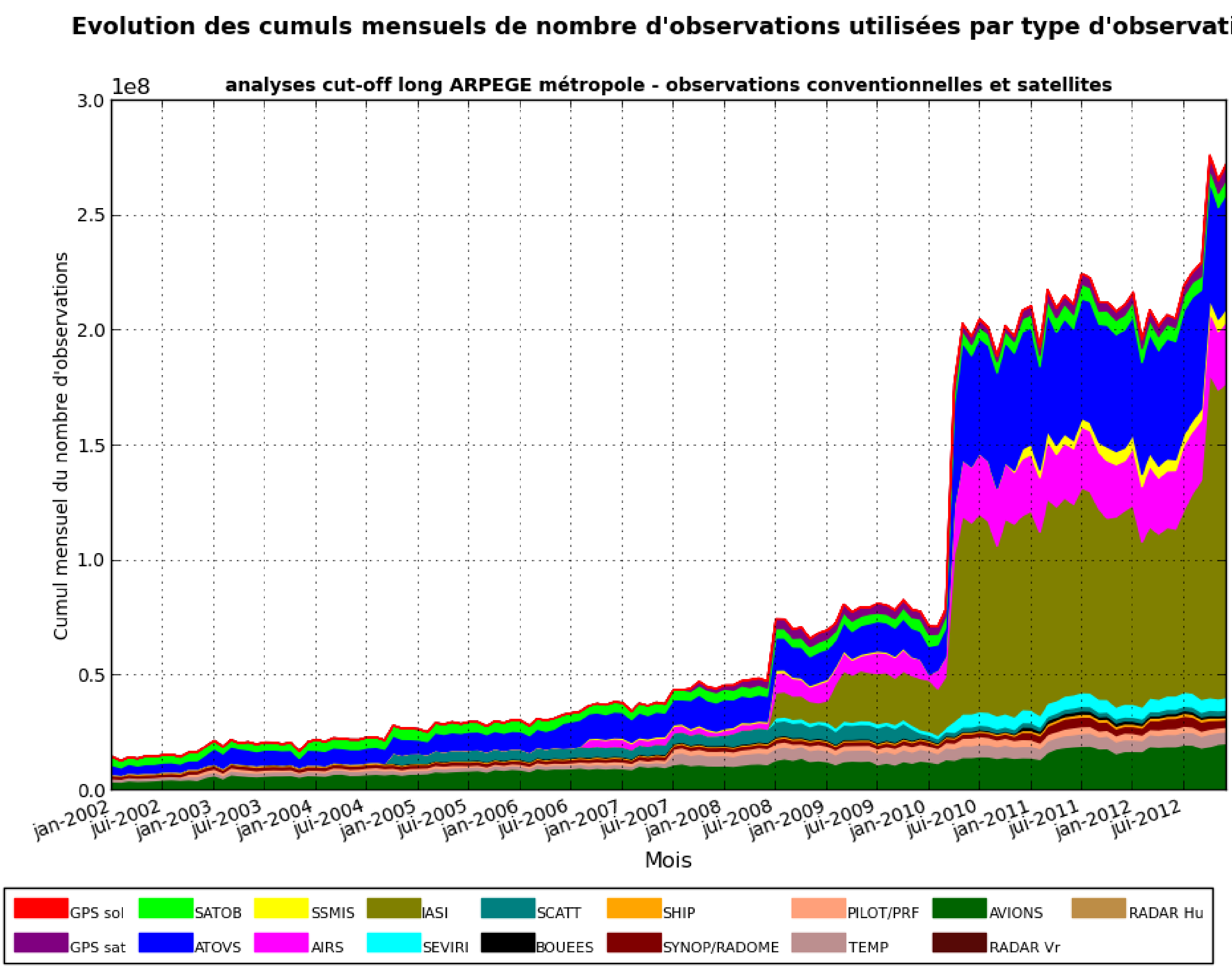
Number of observations used in AROME (regional DA at Météo-France)



Total ~ 800,000 obs per day

Number of observations used in ARPEGE (global DA at Météo-France)

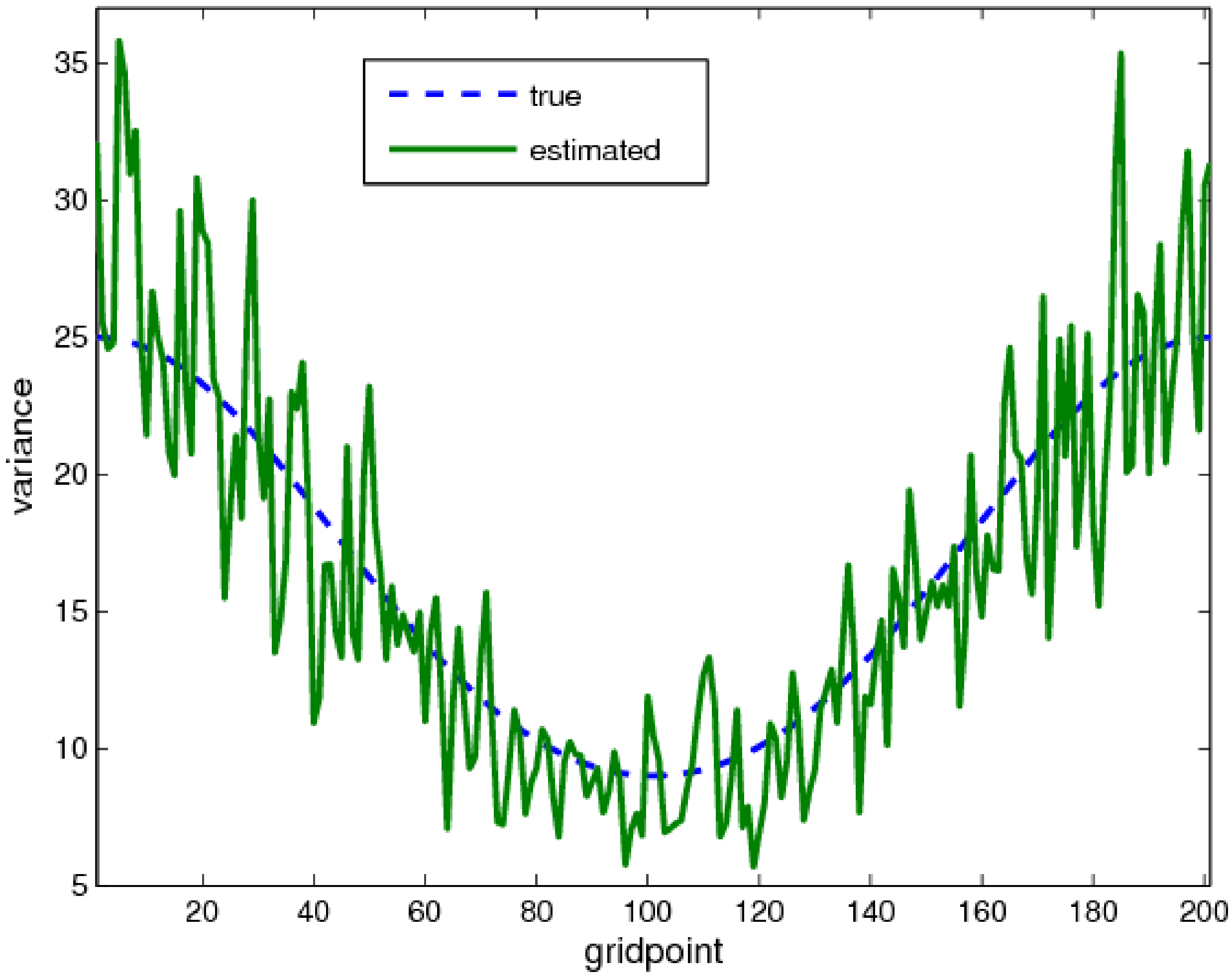


~100 million obs per day

Estimation of background error variances from ensemble spread

$$Var(e_b) = 1/(N-1) \sum_n [\, x'_b(n) - \overline{x'_b} \,]^2$$

Spatial structure of sampling noise for variances



$$\varepsilon_b = \mathbf{B}^{1/2}\,\eta$$

$$\eta \sim \mathcal{N}(0,\mathbf{I})$$

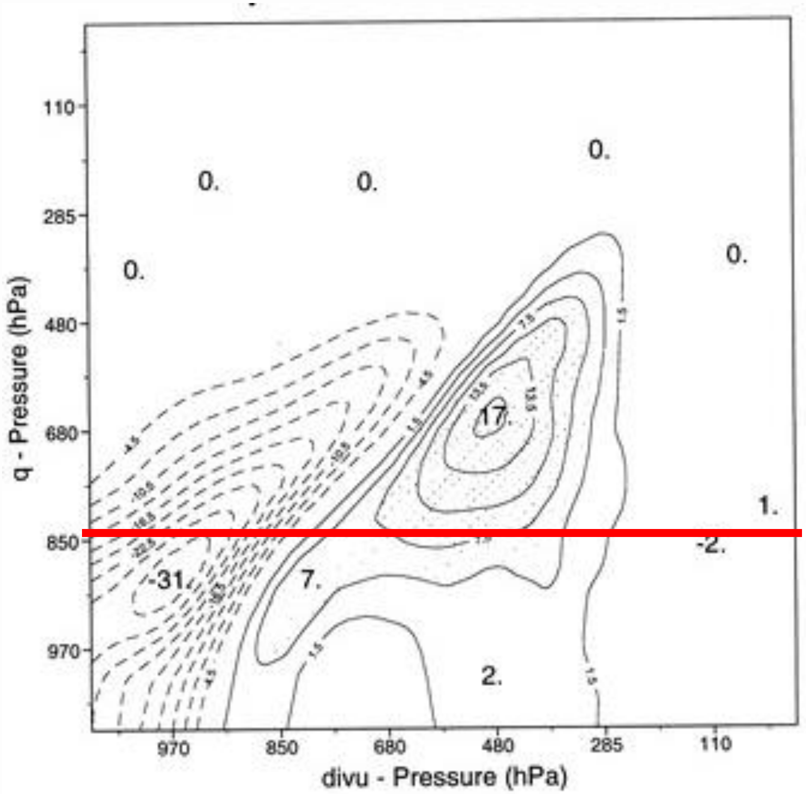
N = 50 members

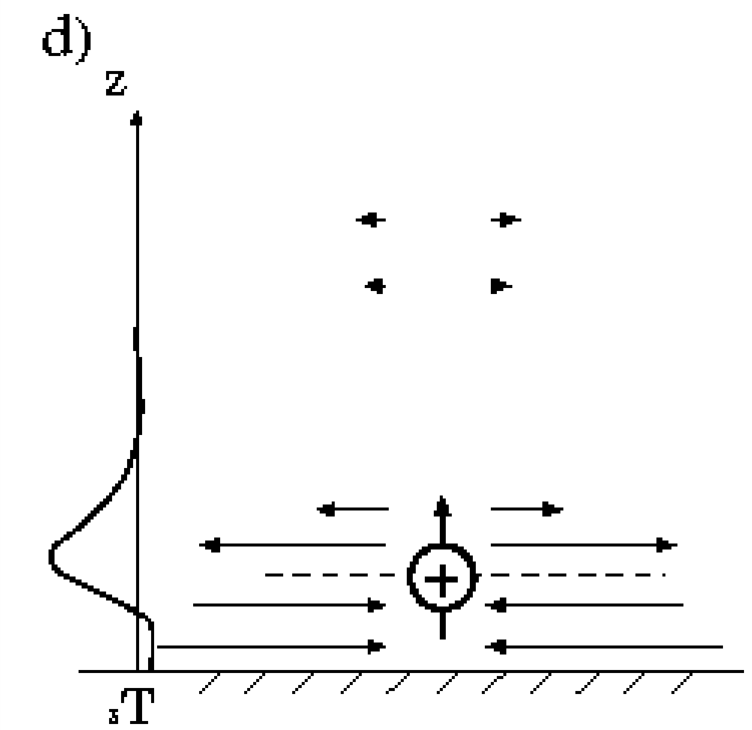
$L(\varepsilon_b) = 200$ km

$$\overline{V^e\,(V^e)^T} = 2/(N-1)\ \mathbf{B}^* \circ \mathbf{B}^*$$

$\Rightarrow$ Spatial filtering in order to extract large scale signal, and remove small scale sampling noise. (e.g. Raynaud et al 2009)

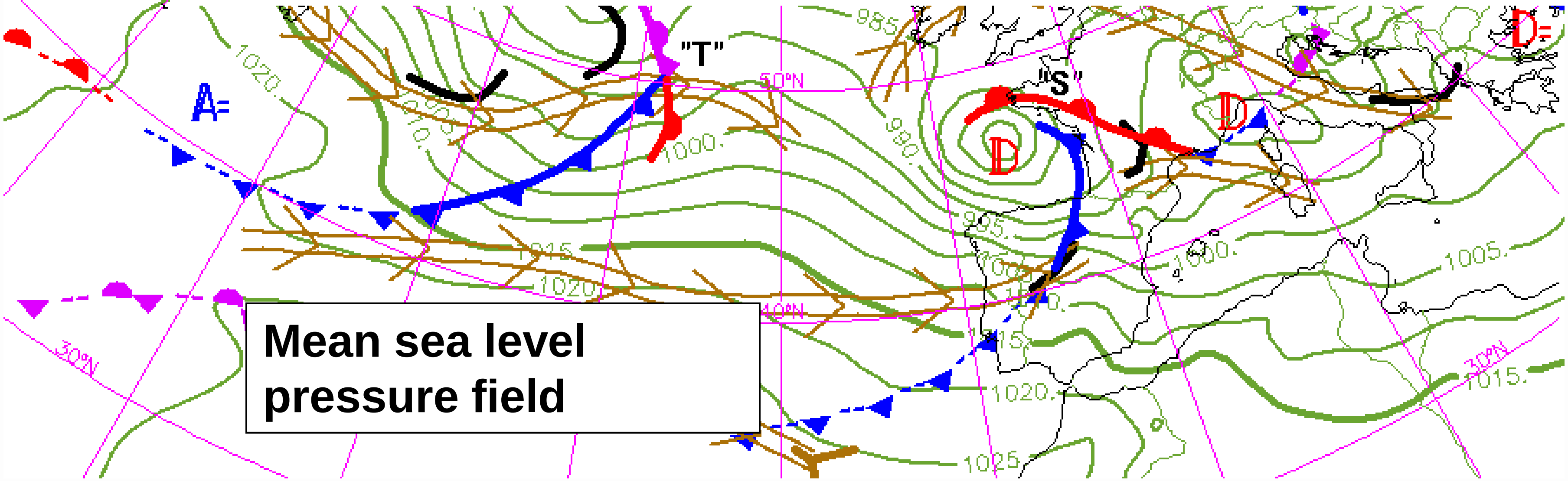
Divergence/humidity couplings



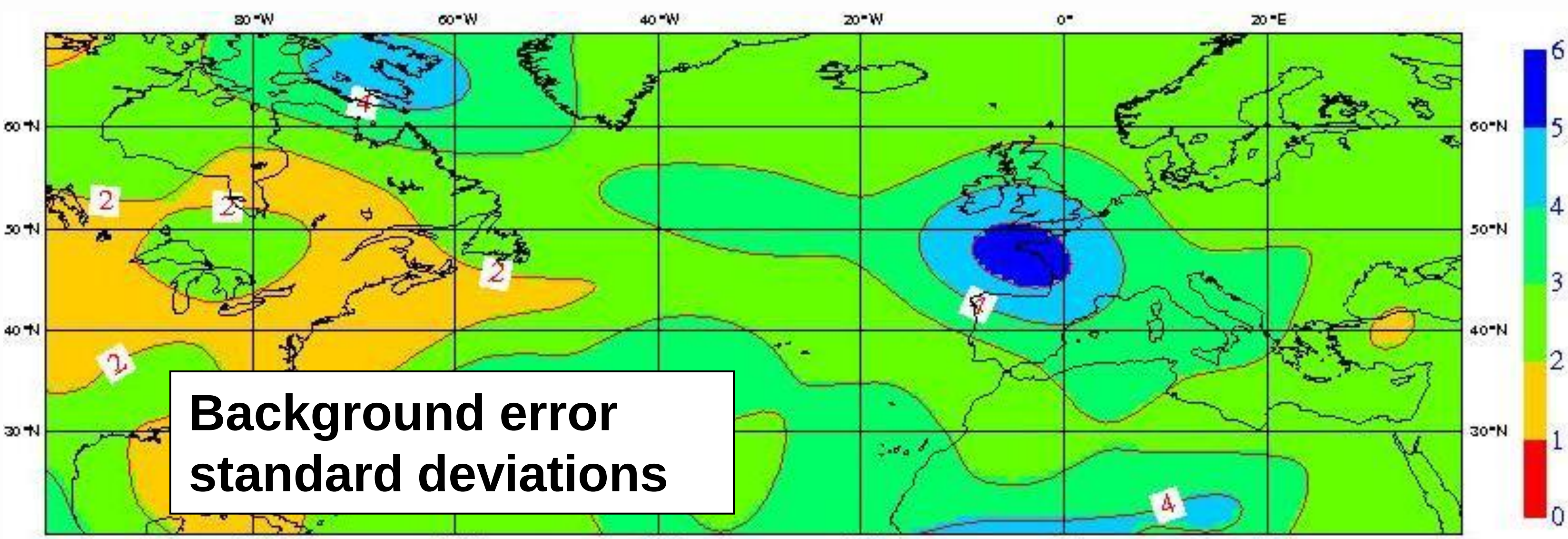


(Berre 2000, Montmerle et al 2006)

Connexion between large errors and intense weather
(Klaus storm, 24/01/2009, 00/03 UTC)

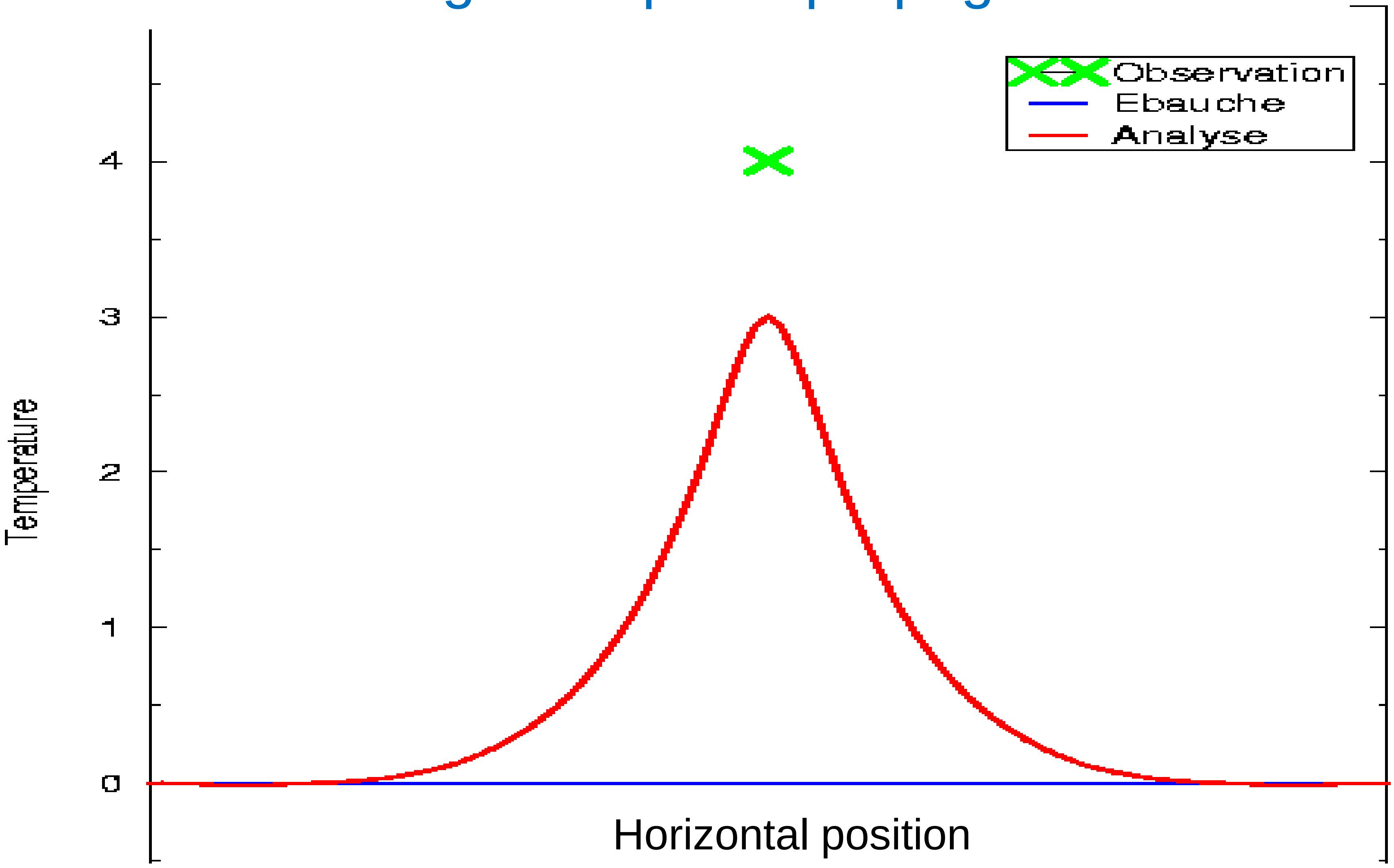


Mean sea level pressure field



Background error standard deviations

Impact of one observation (1D) : filtering and spatial propagation



$\Rightarrow$ relative accuracies of observations and background, and characteristic spatial scales of background errors are accounted for.

Analysis error equation

- Analysis state (BLUE, K = 4D-Var gain matrix) :

$$\mathbf{x_a = (I\text{-}KH)\ x_b + K\ y_o}$$

- True state :

$$\mathbf{x_t = (I\text{-}KH)\ x_t + K\ Hx_t}$$

- Analysis error :

$$\mathbf{e_a = x_a - x_t}$$

i.e.

$$\mathbf{e_a = (I\text{-}KH)\ e_b + K\ e_o}$$

Analysis perturbation equation

- Perturbed analysis :

$$x'_a = (I\text{-}KH)\ x'_b + K\ y'_o$$

- Unperturbed analysis :

$$x_a = (I\text{-}KH)\ x_b + K\ y_o$$

- Analysis perturbation :

$$e_a = x'_a - x_a$$

i.e. $$e_a = (I\text{-}KH)\ e_b + K\ e_o$$

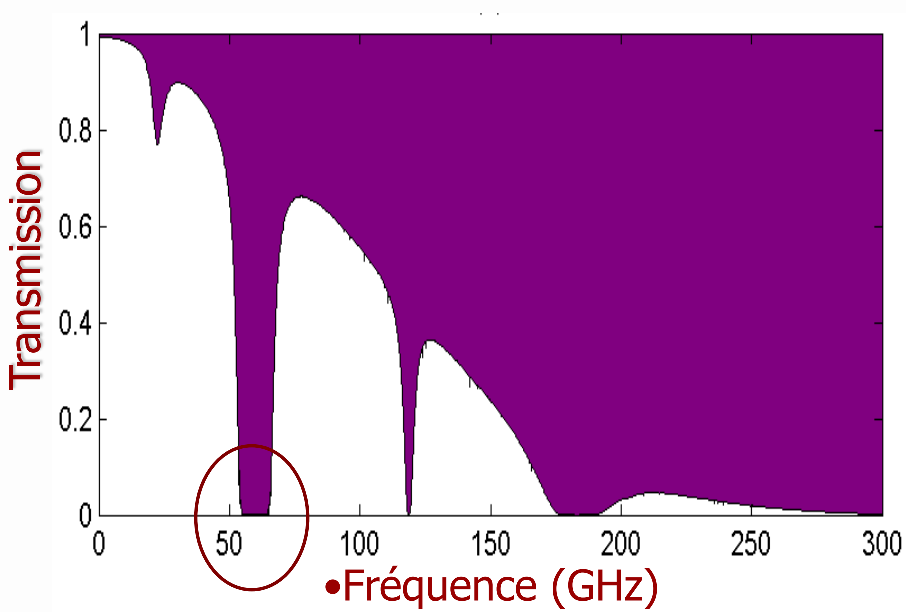
=> Estimate 4D-Var errors by using perturbed inputs.

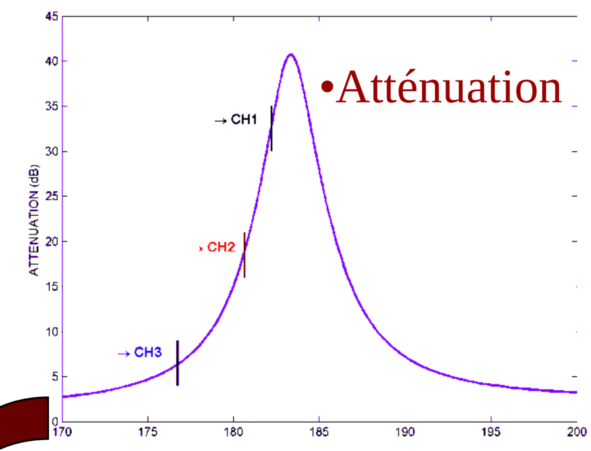
What is measured by satellite sensors ?

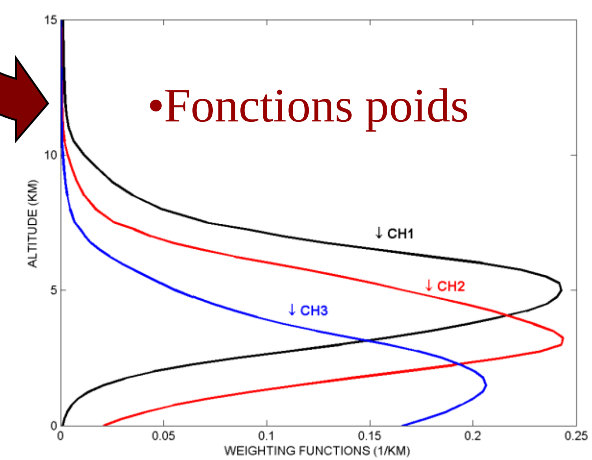
Soundings of atmosphere ?

- In micro-waves: absorption par by water vapor, oxygen
- Largeur des bandes d'absorption: Pression (altitude) (< 60km): les bandes d'absorption plus larges quand la pression augmente

<u>Les mesures loin (proches) d'une bande d'absorption: information sur les basses (hautes) couches atmosphériques</u>







Covariances of residuals

- Analysis increment : $H\,\delta x = HK\,(y_o - Hx_b)$

  with $HK = HBH^T\,(HBH^T + R)^{-1}$

- Covariances between $H\delta x$ and omb :

$$E[(H\,\delta x)(y_o - Hx_b)^T] = HK\,E[(y_o - Hx_b)(y_o - Hx_b)^T]$$
$$\sim HK\,(HB_tH^T + R_t)$$
$$\sim HBH^T\,(HBH^T + R)^{-1}\,(HB_tH^T + R_t)$$
$$\sim HB_tH^T$$

either assuming K ~ optimal,

or, for averaged σ_b, assuming that structures

in B,R are much different. (Desroziers et al 2005)