



Data assimilation in meteorology

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Météo-France/CNRM

Plan of the talk

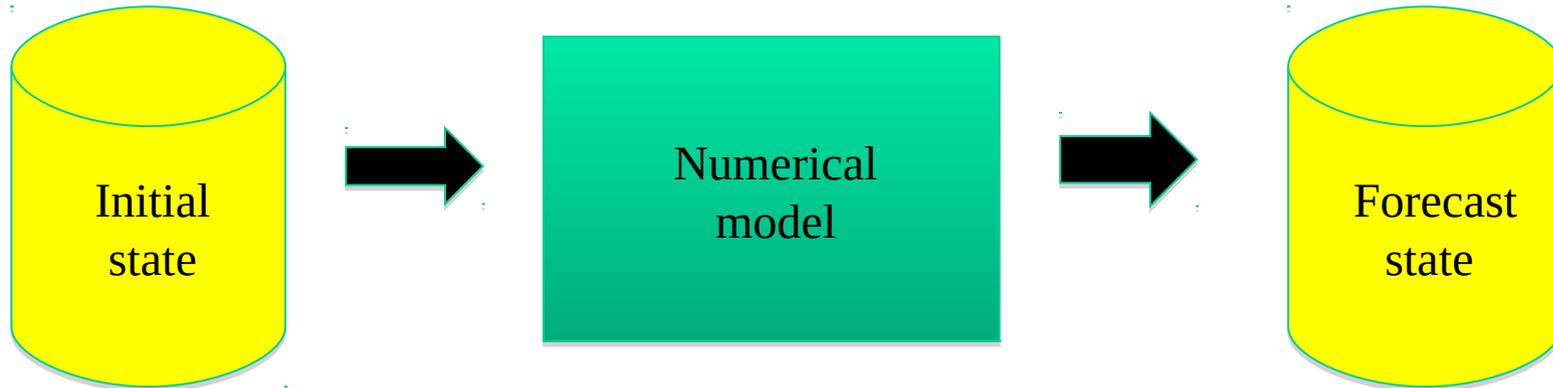
- Numerical Weather Prediction (NWP) and Data Assimilation (DA)
- Observations (in-situ and remote sensing)
- Error covariances : estimation and modelling



1. Numerical Weather Prediction and Data Assimilation

Numerical Weather Prediction

Numerical resolution of fluid mechanics equations (computer code), to **forecast the atmospheric evolution** from an **estimated initial state**.



Vigilance météorologique
La carte est actualisée au moins 2 fois par jour, à 6h et 16h.

- Une vigilance absolue s'impose des situations dangereuses d'intensité exceptionnelle sont prévues...
- Soyez très vigilant, des phénomènes dangereux sont prévus...
- Soyez attentif si vous pratiquez des activités sensibles au risque météorologique...
- Pas de vigilance particulière.

Les Vigilances pluie-inondation et inondation sont élaborées avec le Réseau de Prévision des Crues du Ministère du Développement durable

Diffusion : le lundi 30 janvier 2012 à 22h31
Validité : jusqu'au mardi 31 janvier 2012 à 18h00
Actualité : la carte du lundi 30 janvier 2012 à 18h00

Consultez le **bulletin national**
Épisode neigeux notable en cours des Pays de la Loire au Poitou et au Massif Central, et gagnant mardi en tout début de matinée Rhône-Alpes et PACA.

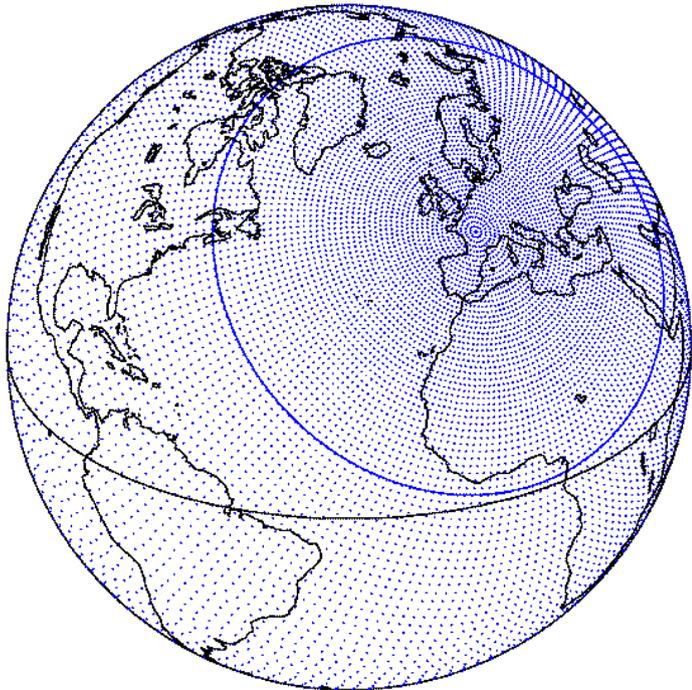
Consultez sur la carte pour lire les **bulletins régionaux**
Conseils des pouvoirs publics : Neige-vergès/orage - Soyez très prudents et vigilants si vous devez absolument vous déplacer. Renseignez-vous sur les conditions de circulation. - Respectez les restrictions de circulation et dérivations. Prenez un équipement minimum en cas d'immobilisation prolongée. - Si vous devez installer un groupe électrogène, placez-le impérativement à l'extérieur des bâtiments. - N'utilisez jamais des chaudières d'appoint à combustion en continu.

31 départements en Orage.

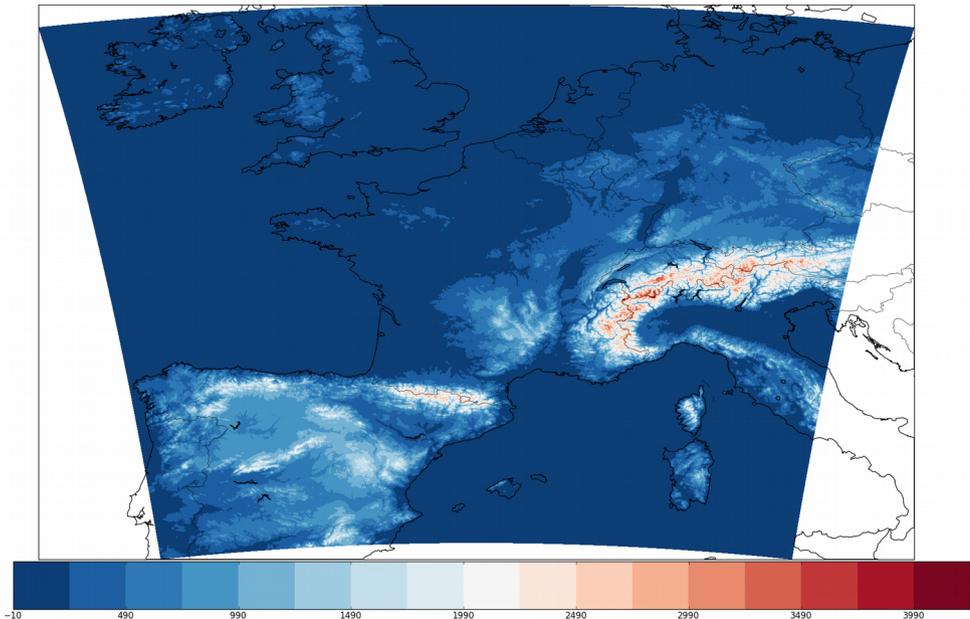
METEO FRANCE
Toujours un temps d'avance

Copyright Météo-France

NWP models at Météo-France (in collaboration with e.g. ECMWF)



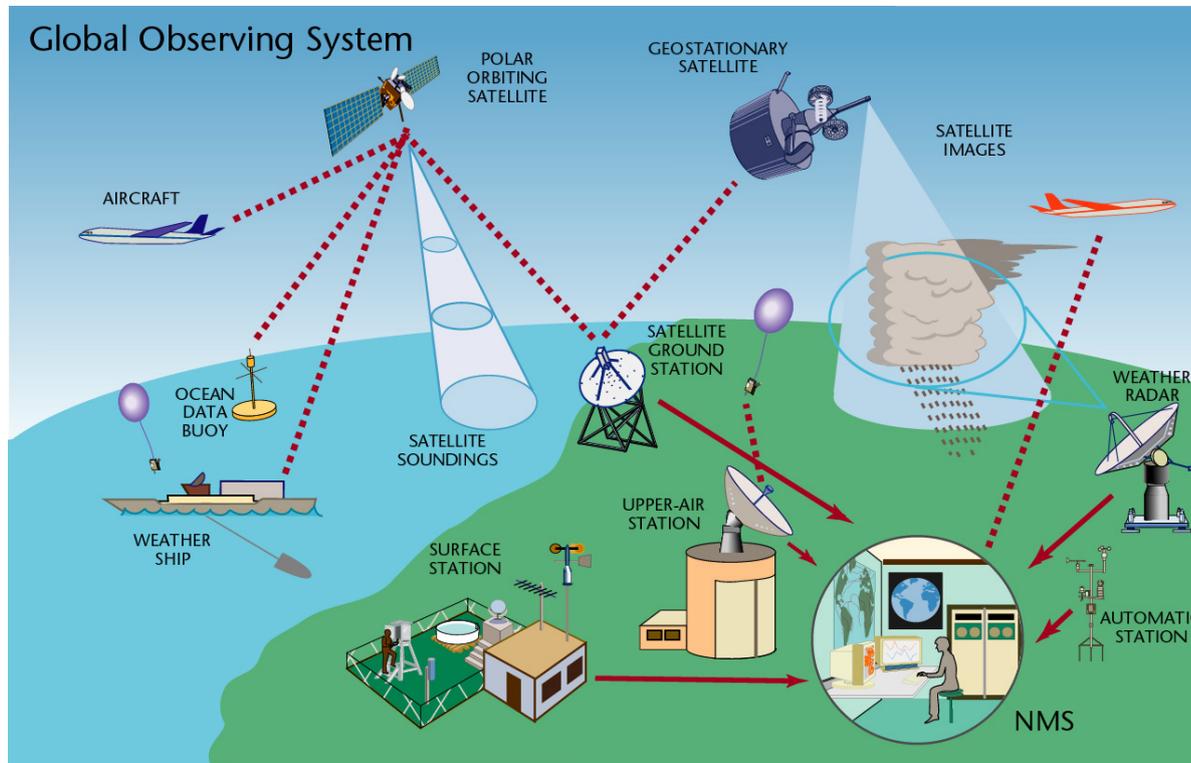
ARPEGE (7 km - 40 km)
 10^9 variables



AROME (1.3 km)
 $1,4 \times 10^9$ variables

Equations of dynamics and physical parametrizations (radiation, convection, ...) to predict the evolution of temperature, wind, humidity, etc.

Data which are assimilated in NWP models



ARPEGE
 10^9 variables

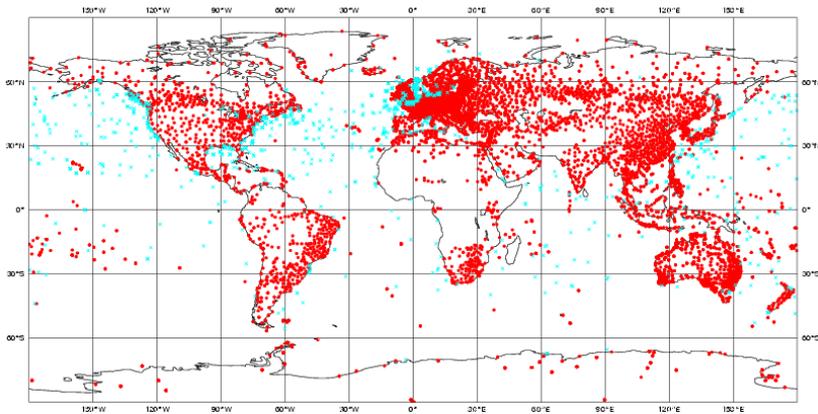
5×10^6 observations / 6h
90 % satellite
4D assimilation : 40 min

AROME
 $1,4 \times 10^9$ variables

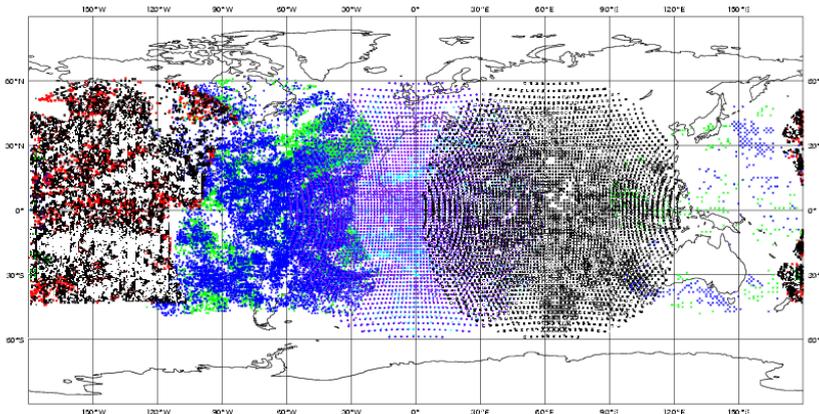
2×10^5 observations / 6h
Up to 70 % radar, 10 % satellite
3D assimilation : 7 min

Spatial coverage and density of observations

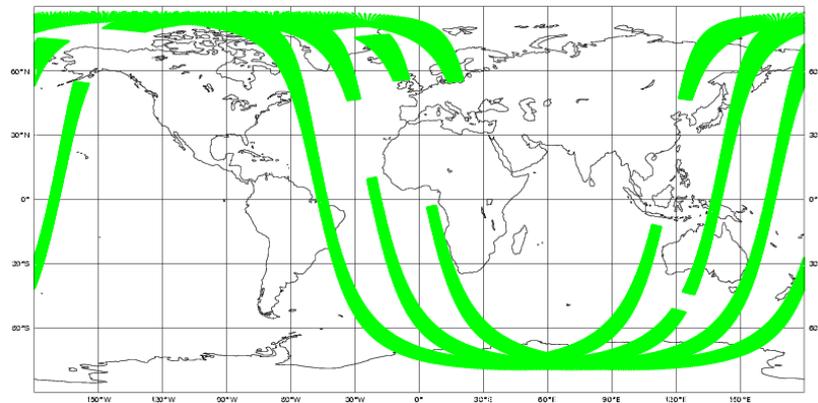
SURFACE DATA



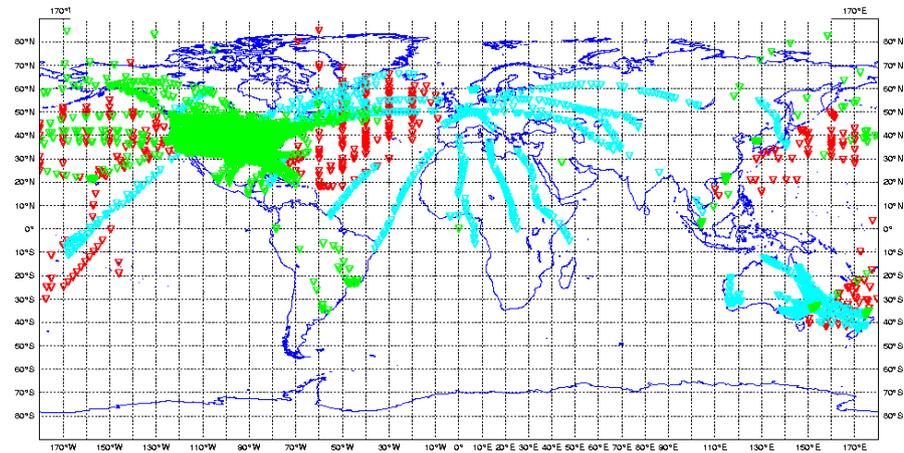
GEOSAT. WINDS



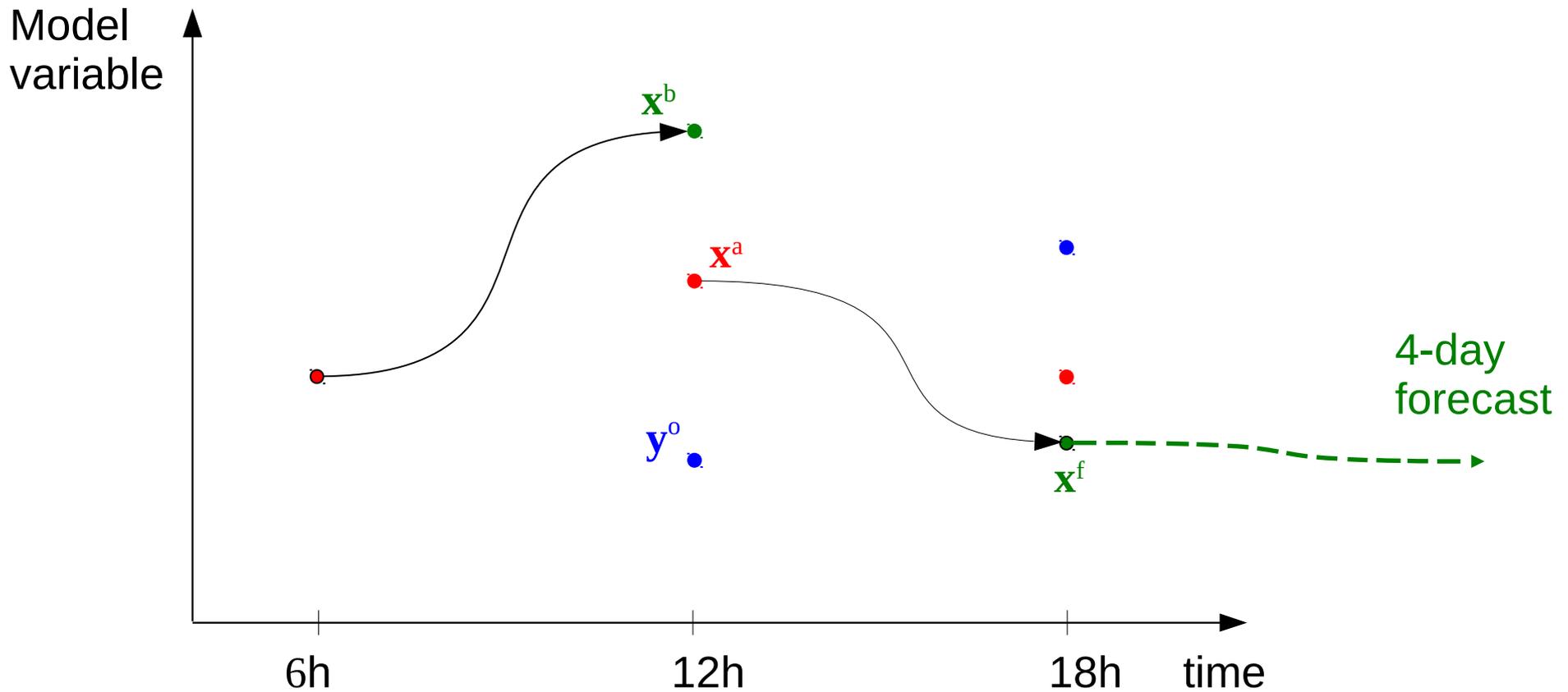
SCATTEROMETER



AIRCRAFT DATA

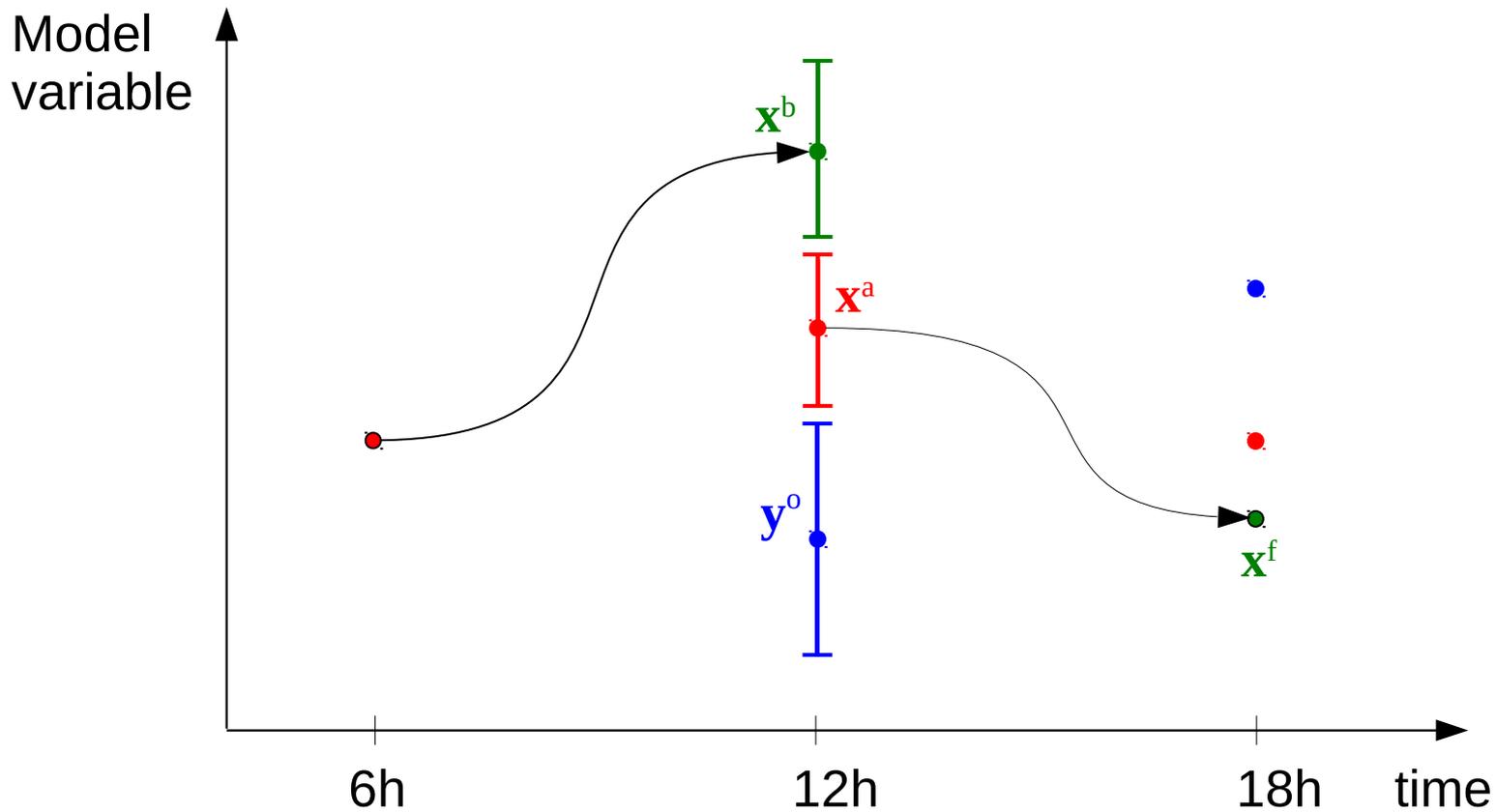


Data assimilation cycling : temporal succession of analysis and forecast steps



=> Memory of DA system is updated ~ continuously in time.

Uncertainties in observations, background, analysis



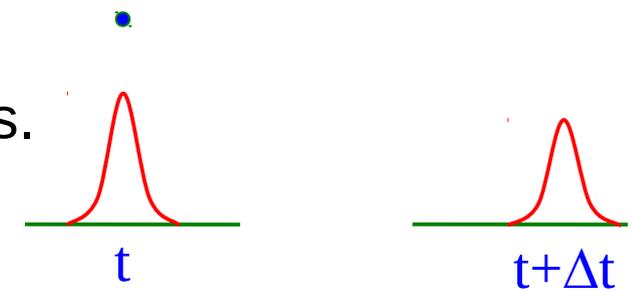
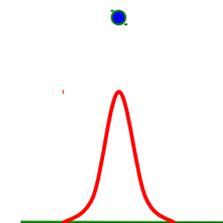
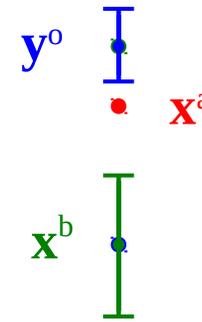
=> Use linear estimation theory to account for errors.

Analysis equation

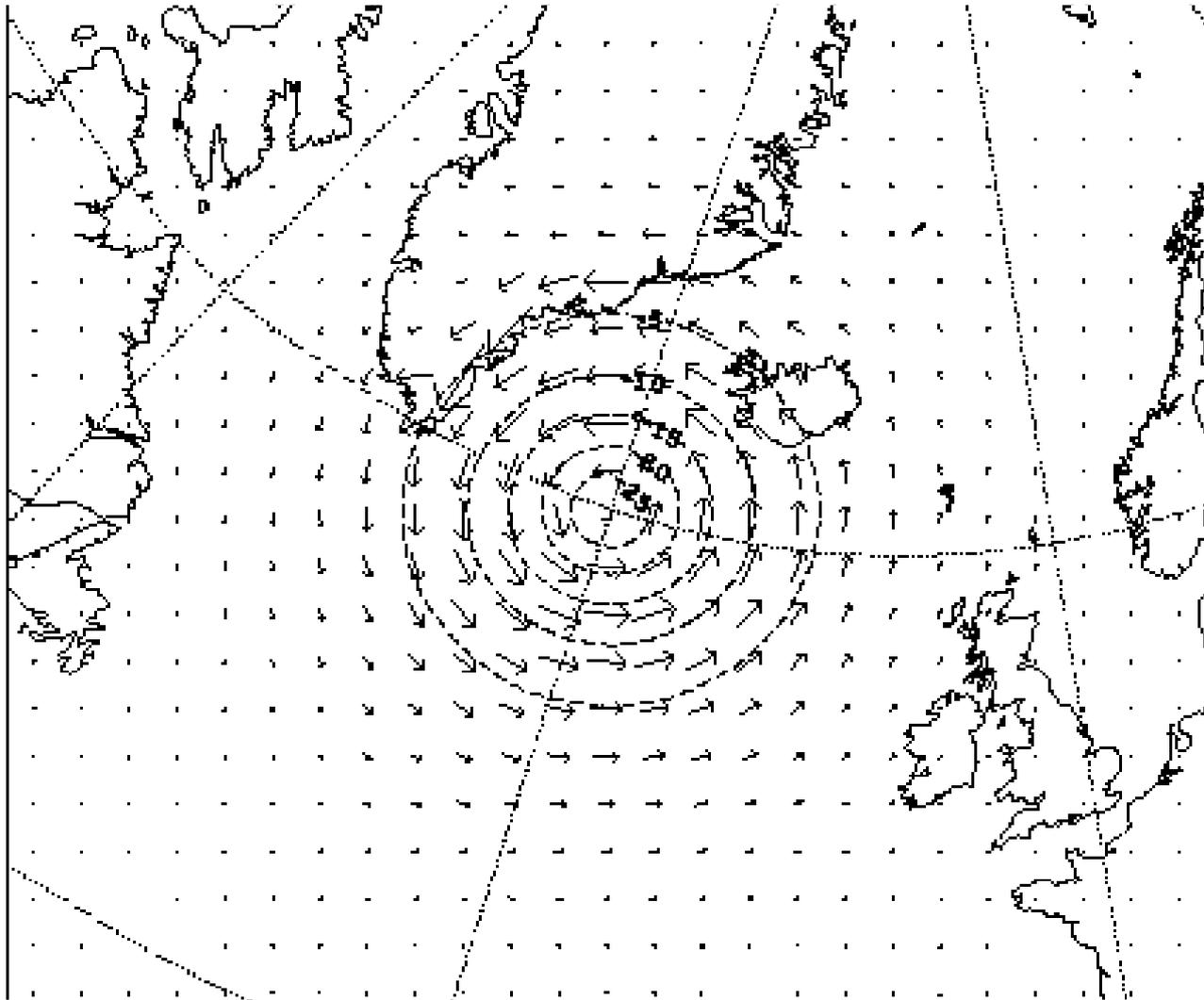
- BLUE formalism : $\mathbf{x}^a = \mathbf{x}^b + \mathbf{K} (\mathbf{y}^o - H(\mathbf{x}^b))$
 - $H =$ non linear observation operator
= projection from model space to observation space : $\mathbf{y} = H(\mathbf{x})$
(e.g. spatial interpolation, radiative transfer, NWP model).
 - $\mathbf{K} =$ observation weights : $\mathbf{K} = \mathbf{B} \mathbf{H}^T (\mathbf{H} \mathbf{B} \mathbf{H}^T + \mathbf{R})^{-1}$
with $\mathbf{H} =$ tangent linear version of H ,
 $\mathbf{B} =$ background error covariance matrix,
 $\mathbf{R} =$ observation error covariance matrix.
- => \mathbf{K} accounts for relative accuracy of observations,
and for spatial structures of background errors.

Background error covariances : filtering and propagation of $y^o - H(x^b)$

- Variances
 - Weighting/filtering of observations.
- 3D spatial correlations
 - Spatial propagation of observations.
 - Spatial coherence of analysis.
- 4D spatio-temporal correlations
 - Spatial and temporal propagation of observations.
 - Spatial and temporal coherence of trajectory.



Impact of one surface pressure observation on the wind analysis (2D)

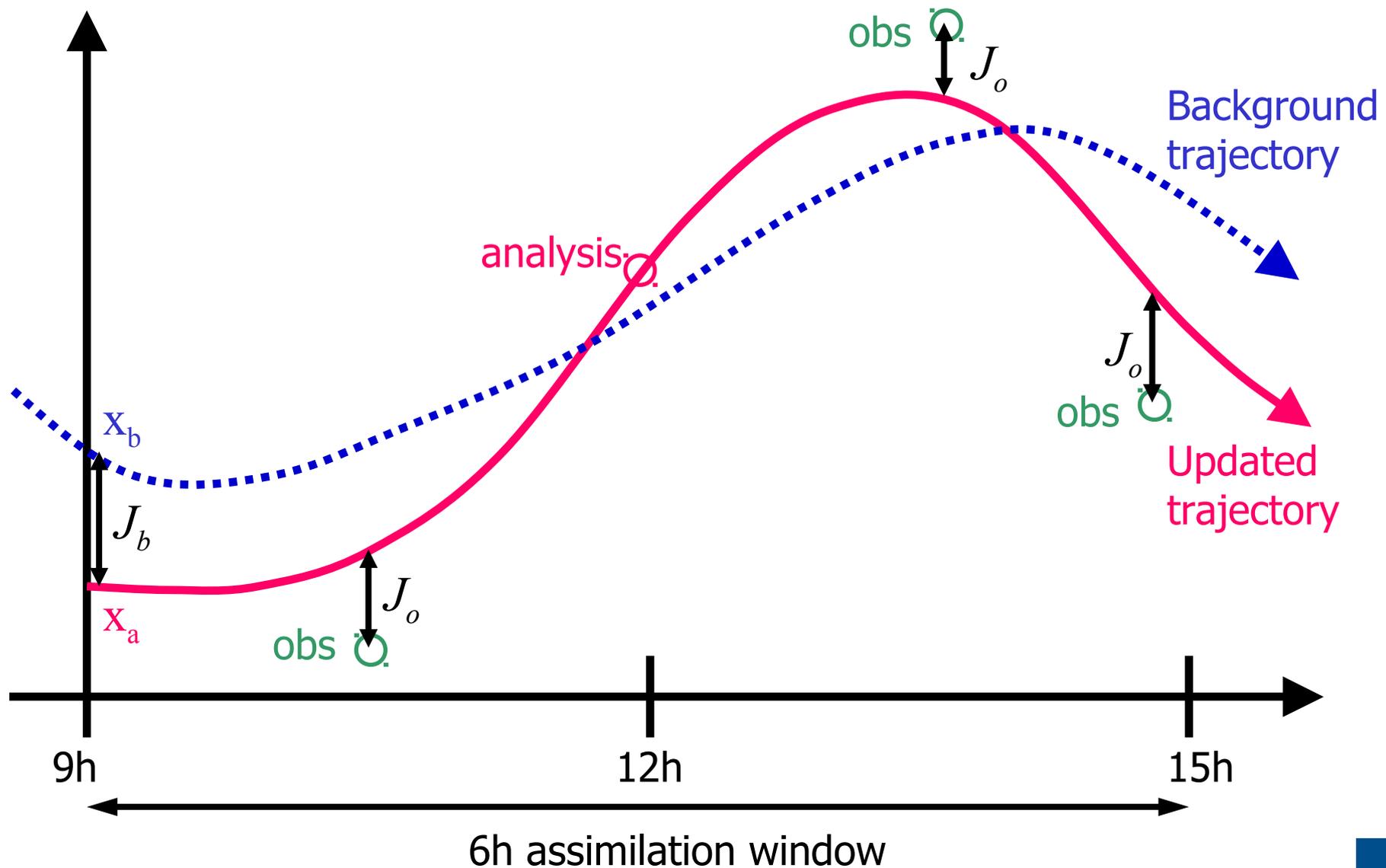


=> Typical scales and multivariate couplings are accounted for, in **B** and thus in the analysis : e.g. geostrophic mass/wind balance.

Variational analysis

- Size of \mathbf{B} is huge : square of model size $\sim (10^9)^2 = 10^{18}$
=> error covariances need to be estimated, simplified and modelled.
- The matrix $(\mathbf{H} \mathbf{B} \mathbf{H}^T + \mathbf{R})$ in $\mathbf{K} = \mathbf{B} \mathbf{H}^T (\mathbf{H} \mathbf{B} \mathbf{H}^T + \mathbf{R})^{-1}$
is too large to be explicitly inverted.
=> minimize distance $J(\mathbf{x}^a)$ to \mathbf{x}^b and \mathbf{y}^o (4D-Var),
without explicit matrix inversions (e.g. Talagrand and Courtier 1987).
- Some (weakly) non linear features are accounted for
in calculation of departures $\mathbf{y}^o - H(\mathbf{x}^b)$,
and by updating the non linear trajectory in 4D-Var.

Principle of 4D-Var assimilation

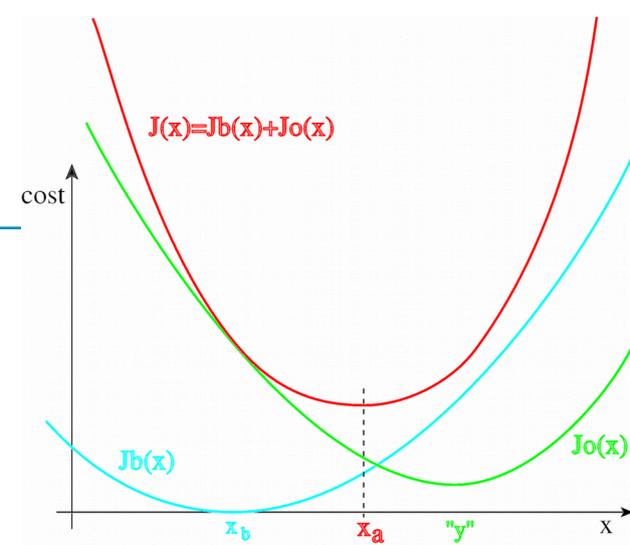


Implementation of 4D-Var

Variational formulation :

$$\text{cost function } J(\mathbf{x}^a) = \|\mathbf{x}^a - \mathbf{x}^b\|_{\mathbf{B}}^2 + \|H(\mathbf{x}^a) - \mathbf{y}^o\|_{\mathbf{R}}^2$$

minimised when gradient $J'(\mathbf{x}^a)=0$ (equivalent to BLUE).



Computation of gradient J' : development and use of adjoint operators (i.e. transpose of tangent-linear operators).

Generalized observation operator H : includes NWP model M , in order to assimilate observations distributed in time over a 6h window.

Reduction of computation cost : analysis increment $\delta\mathbf{x} = \mathbf{x}^a - \mathbf{x}^b$ can be computed at low resolution (Courtier et al 1994).

2. In-situ observations and remote sensing data

Observation networks in meteorology : in situ measurements



- * Direct measurements of T, wind, humidity.
- * Relatively easy to compare with the model, and to assimilate.
- * High quality data, with relatively small biases.
- * Poor horizontal coverage over the globe.

Observation networks in meteorology : satellite data



Constellation of polar orbiting or geostationary satellites

Geostationary satellites

Copyright EUMETSAT / NERC / University of Dundee 2003

Fix position / earth, at 36,000 km height, above equator.

Same area of the globe (disk) is always observed.

□ Advantages

Very high temporal resolution (15 min).

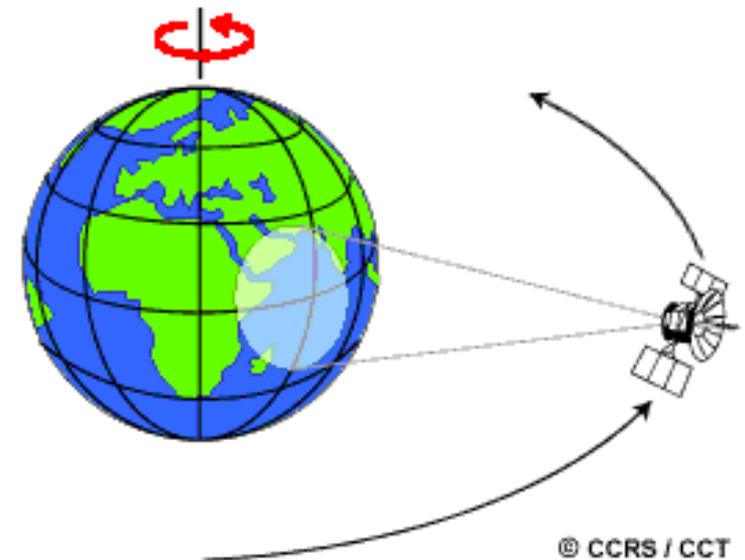
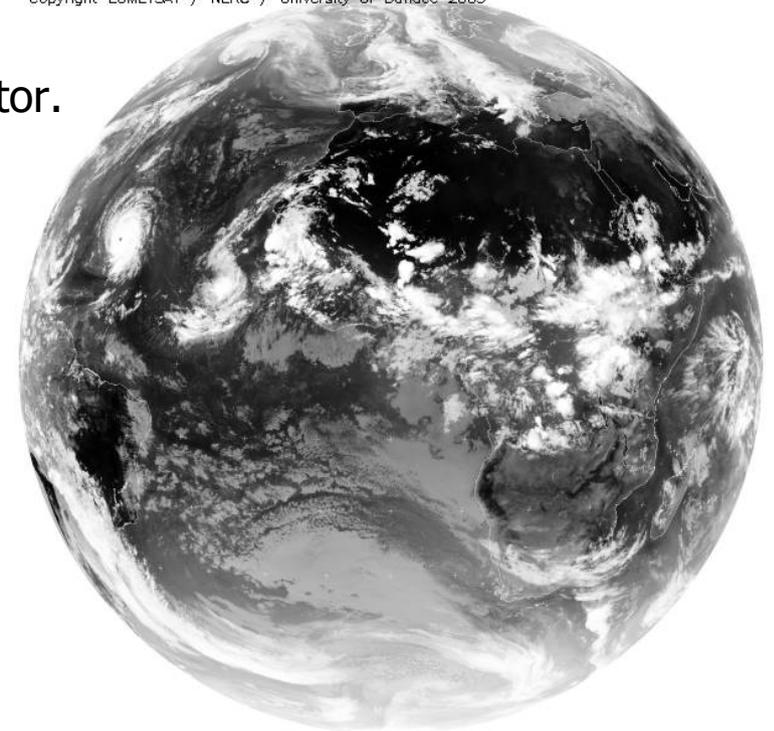
Useful for nowcasting.

Dynamics of meteorological structures.

□ Drawbacks

Insufficient spatial coverage / whole globe.

Not adapted to polar regions, due to position.



Polar orbiting satellites

Low orbit satellites (800 km height) :

❑ Advantages

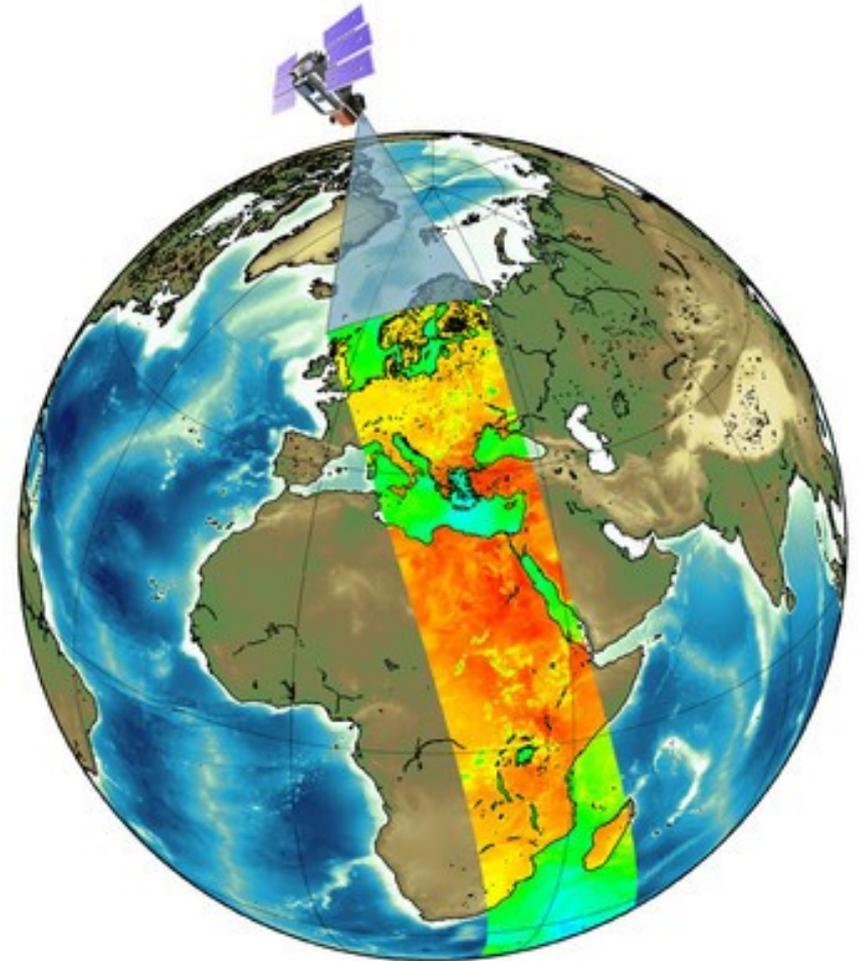
High spatial resolution

Global spatial coverage (twice a day)

Sounding instruments
(over several vertical layers)

❑ Drawbacks

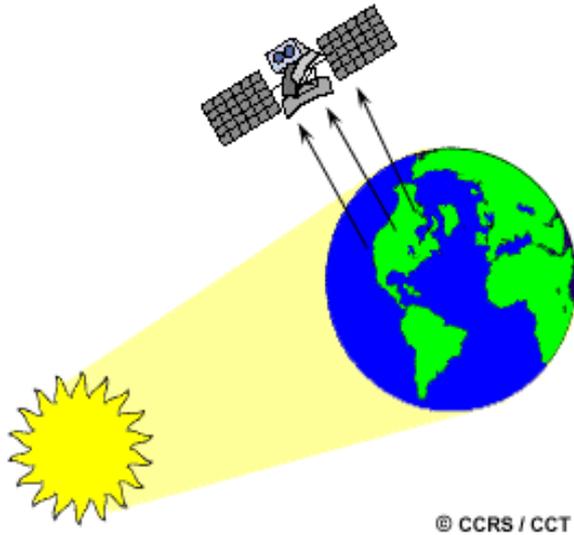
Insufficient temporal resolution :
a given location is only observed every 12h
(several satellites are needed)



Two types of satellite measurements

Passive measures

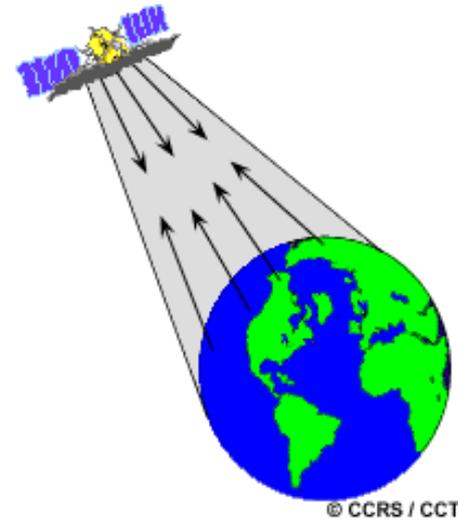
(no energy is emitted from instrument)



Measures natural radiation emitted by Earth or Atmosphere (with Sun origin)

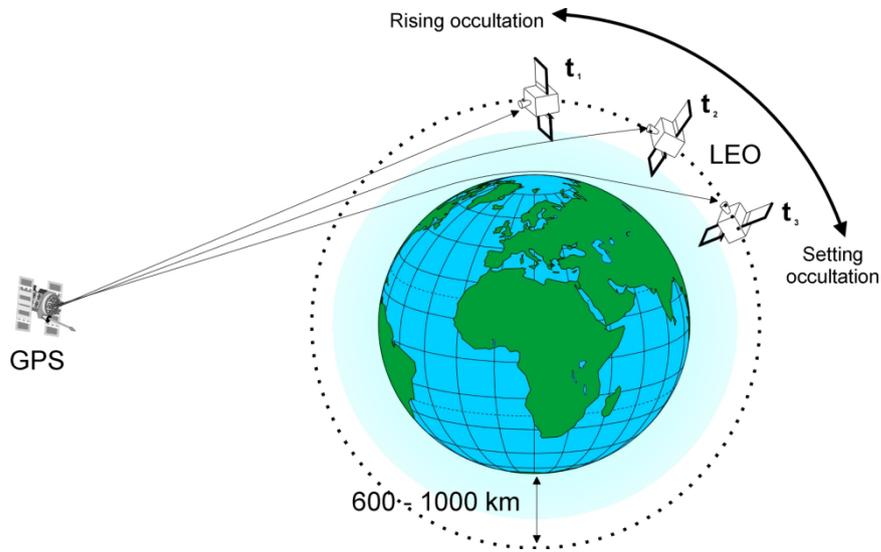
Active measures

(energy is emitted from instrument)

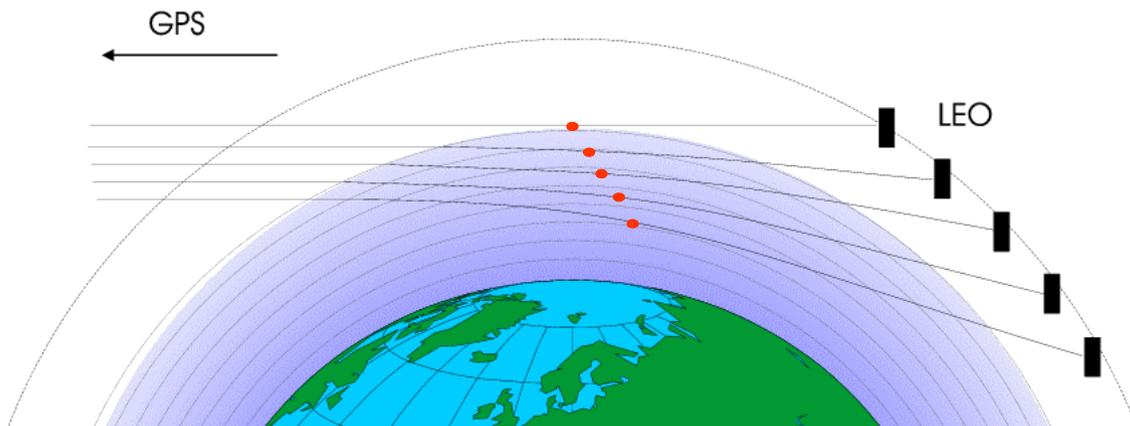


Measures radiation emitted by satellite and then reflected or diffused by Earth or Atmosphere

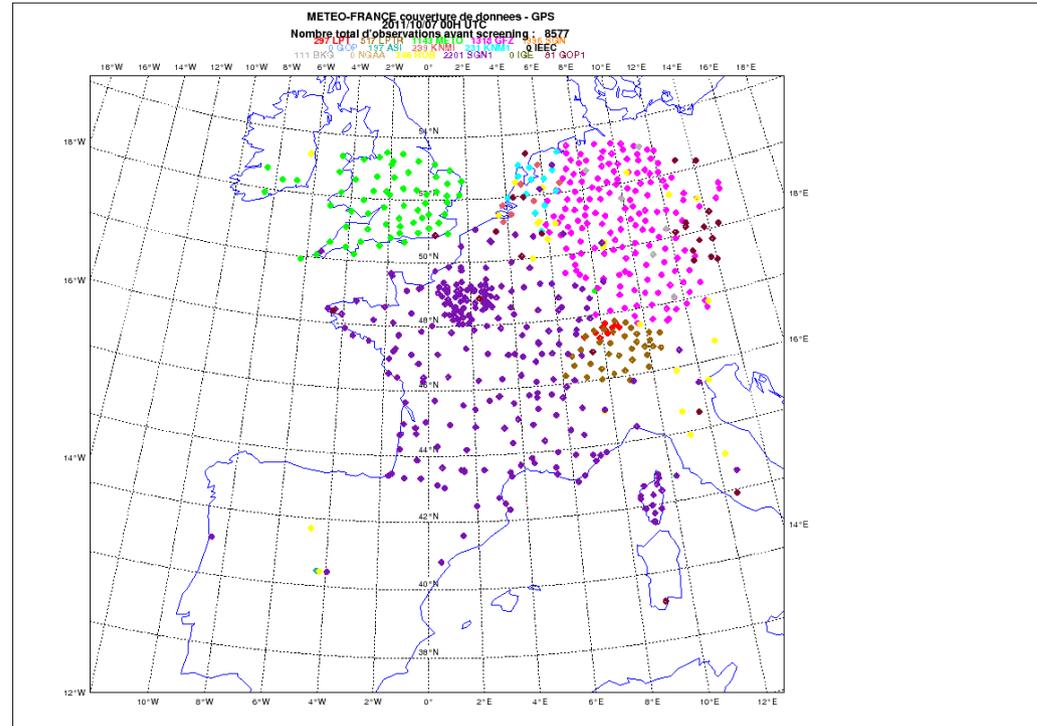
GNSS radio-occultation data (example of active remote sensing)



- Low-Earth Orbit satellites receive a signal from a GNSS satellite (Global Navigation Satellite System).
- The signal passes through the atmosphere and gets refracted along the way.
- The magnitude of the refraction depends on temperature, moisture and pressure.
- The relative position of GNSS and LEO changes over time => vertical scanning of the atmosphere.



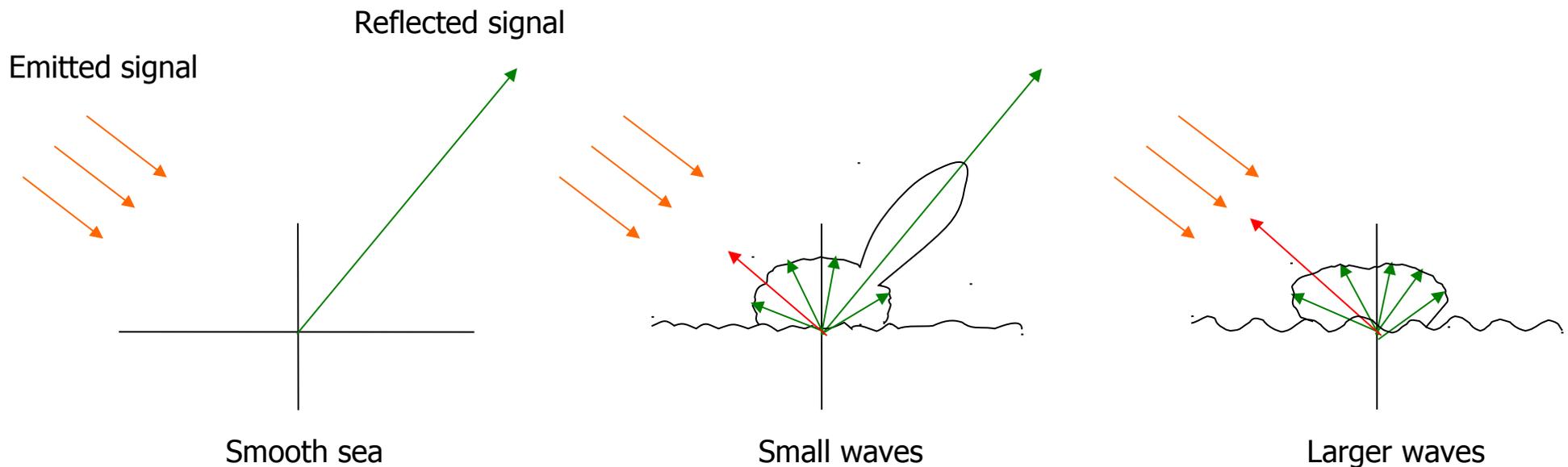
Ground-based data from GNSS (Global navigation Satellite System)



- Propagation of GNSS signal is slowed by atmosphere (dry air and water vapour) : information about humidity in particular.
- More than 900 GNSS stations over Europe provide an estimation of Zenith Total Delay (ZTD) in real time to weather centres.
 - All weather instrument
 - High temporal resolution

Scatterometers

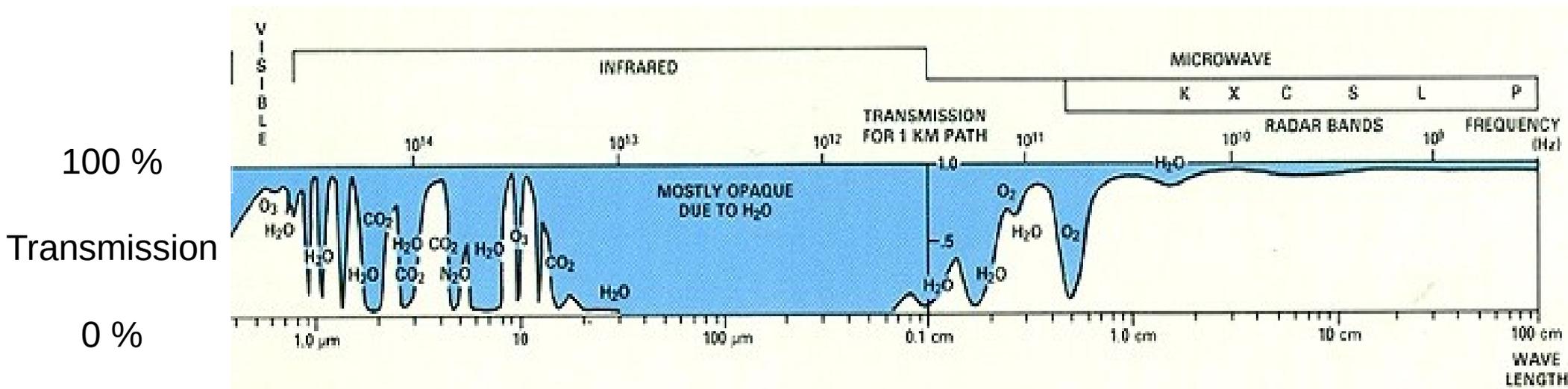
They send out a microwave signal towards a sea target.
The fraction of energy returned to the satellite depends on wind speed and direction.



=> Measurements of near surface wind over the ocean,
through backscattering of microwave signal reflected by waves.

Passive remote sensing : what is measured by satellite sensors ?

- ❑ Sensors do not measure directly atmospheric temperature and humidity, but electromagnetic radiation : brightness temperature or radiance.
- ❑ Depending on wave length, indirect information on gas concentration (e.g. humidity) or on physical properties of atmosphere (temperature or pressure).
- ❑ Observations are often made in atmospheric windows (in white, below : frequencies with low transmission, e.g. in microwave and some infrared).



Passive remote sensing : radiative transfer equation

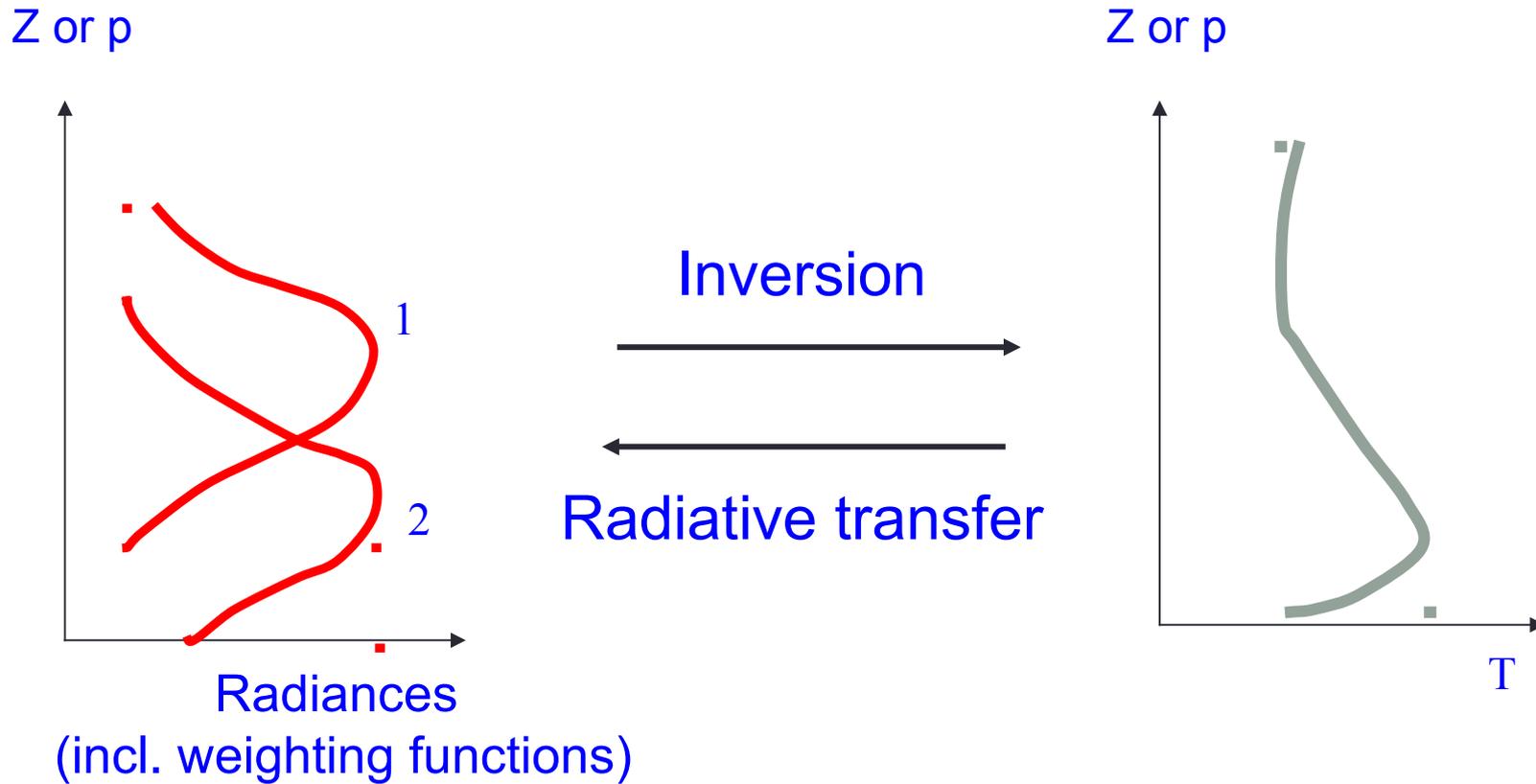
- What is observed is a **radiance** = quantity of energy per time unit, going through a surface, in a solid angle and for a wave number interval.
Unit [W/m²Sr.cm⁻¹]
- Planck function:
 $B_\nu(T)$ = radiance emitted by a black body at temperature T, for a wave number ν

- Intensity of the radiation, emitted by the atmosphere at wave number ν :

$$R_\nu = (I_0)_\nu \tau_\nu(z_0) + \int_{z_0} B_\nu(T(z)) (d\tau_\nu(z)/dz) dz$$

- $(I_0)_\nu$ is the surface emission at altitude z_0
- $\tau_\nu(z)$ is the *transmittance* from z to the top of the atmosphere, accounts for absorption.
- $K_\nu(z) = d\tau_\nu(z)/dz$ is called *weighting function*. It weights the Planck function and determines the region of the atmosphere that is sounded at this frequency.

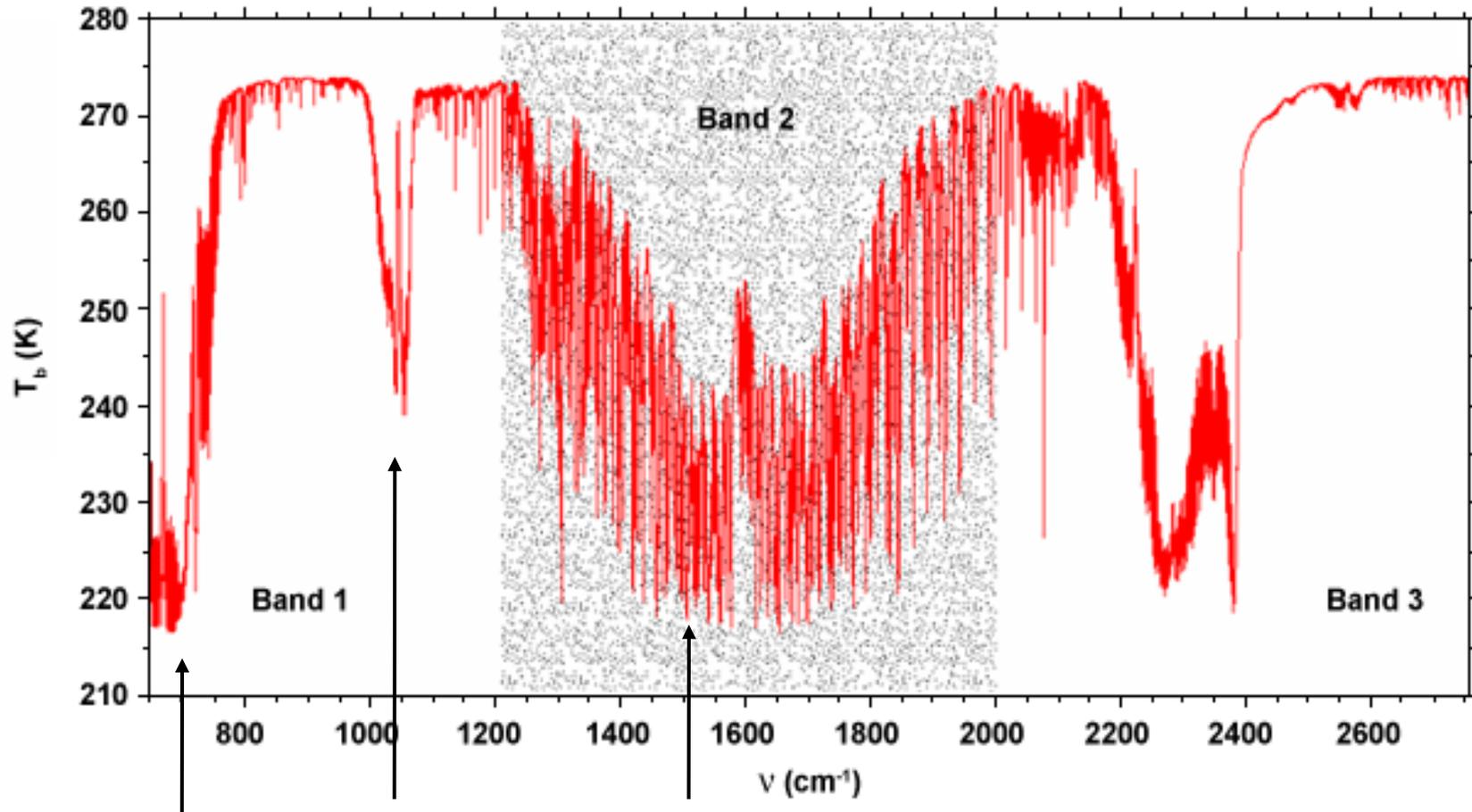
Retrieval of temperature vertical profiles



$$R_i = B_i[T(p_0)] \cdot \tau_i(p_0) + \int_p B_i[T(p)] \cdot d\tau_i(p)$$

IASI : infra-red interferometer developed by CNES and EUMETSAT

IASI offers a very high spectral resolution



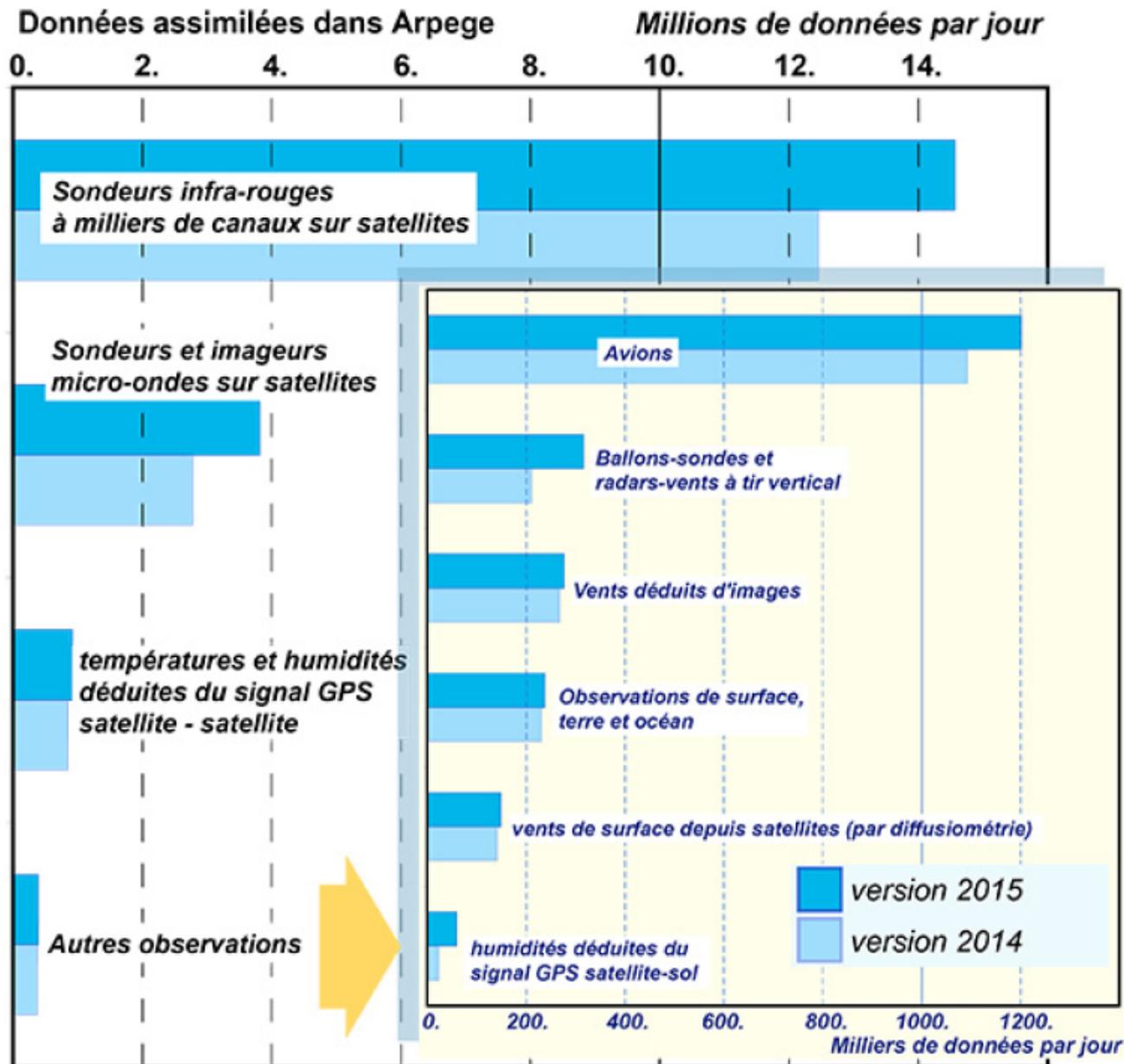
Temperature

ozone

Water vapor

©EUMETSAT, 2006

Number of observations used in ARPEGE (global DA at Météo-France)



Total ~ 20 million obs
per day

How do observations meet global NWP requirements ?

- **Surface observations**
good coverage over land, sparse coverage over sea;
observations not suited to describe upper levels.
- **Aircraft observations**
good accuracy,
but do not describe the 3D state of the atmosphere (except near airports).
- **Radiosonde data**
good accuracy, good vertical resolution,
but poor horizontal coverage over the globe.
- **Satellite data**
good horizontal coverage over the globe,
but poor vertical resolution (reduced to 1 level for satellite winds or imagers).

Radar network in France

- 30 radars (19 C-band, 5S, 6X)

every 15 minutes, at 1 km resolution.

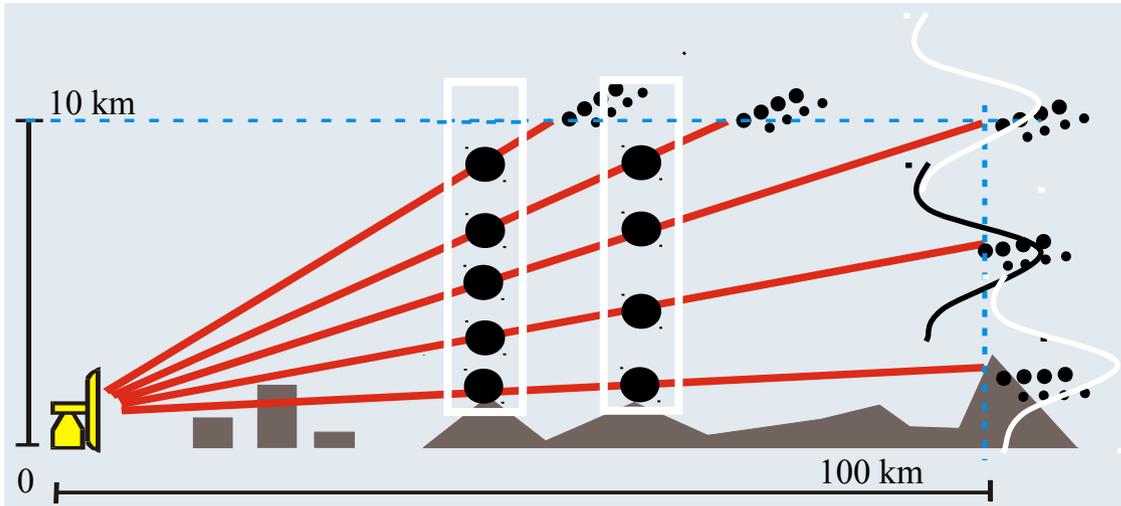
- Observations :

reflectivities Z (related to precipitation) ;

radial winds V_r (doppler effect) :

the emitted microwave signal returns to the radar with a modified frequency, when the target is moving.

=> invert Doppler equation to obtain a wind observation.



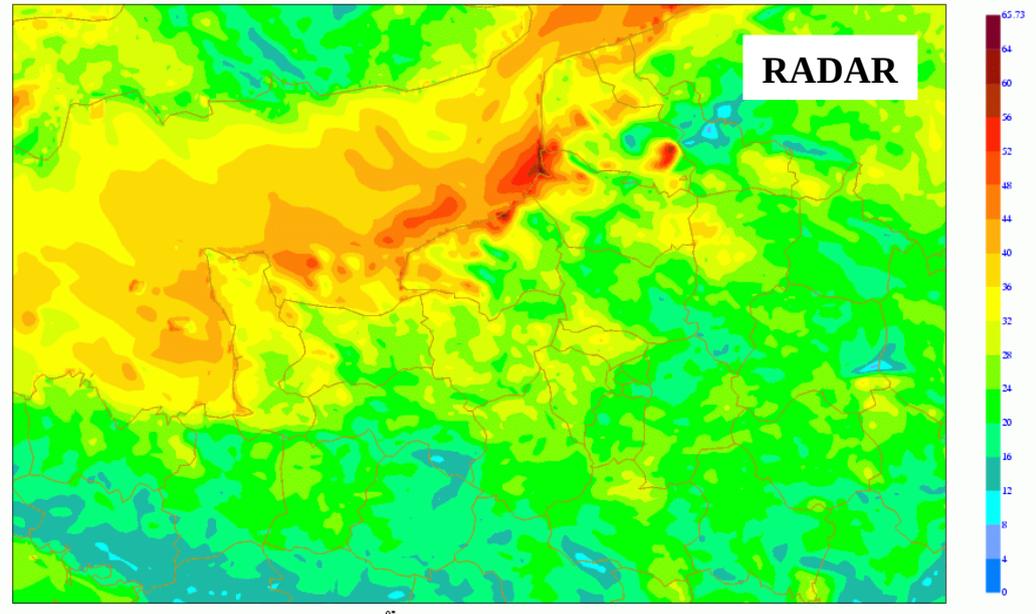
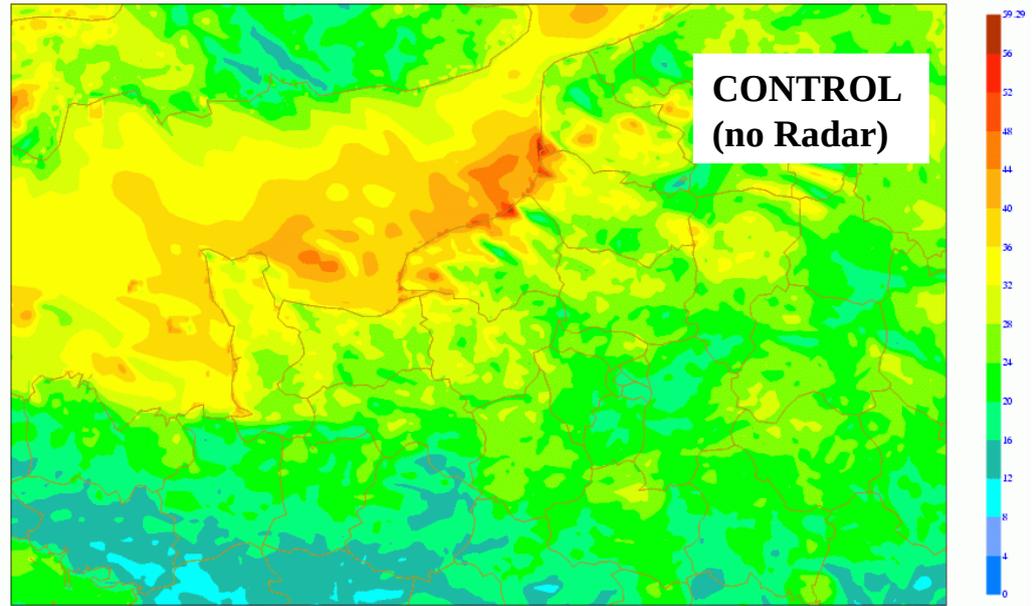
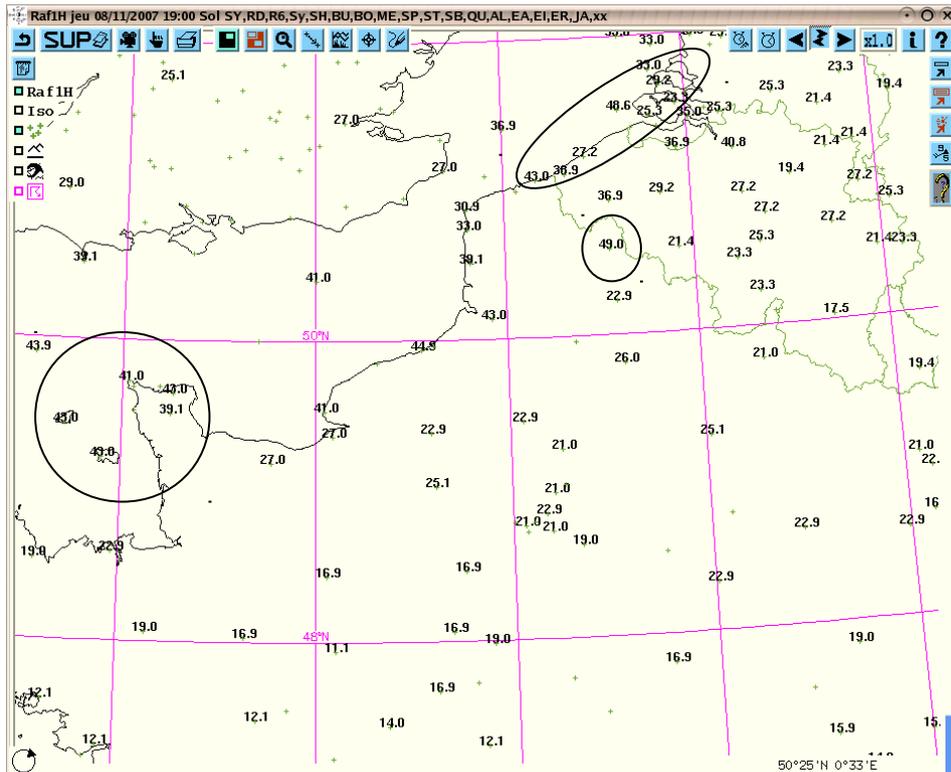
**Observations assimilated
as vertical profiles,
after estimating the pixel altitude**

Pixel altitude is computed using
a constant refractivity index along the path
(effective radius approximation)

Assimilation of radar radial winds

Wind gust at 10 m (kt)
Forecast +1h (19 UTC)

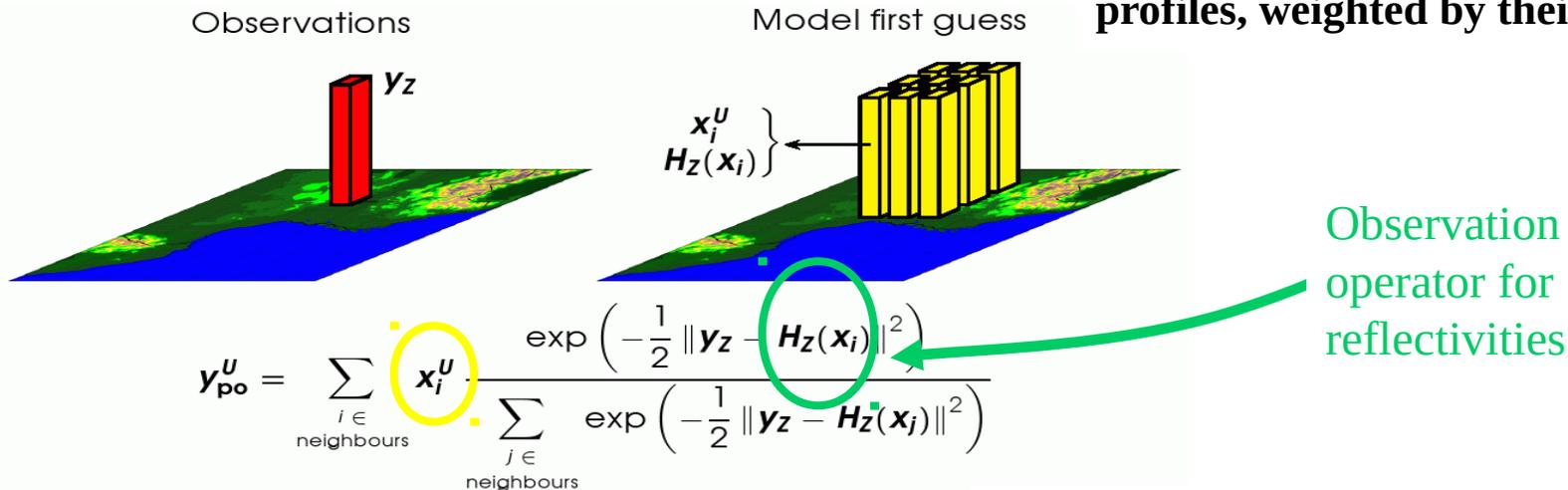
OBS



Bayesian inversion of reflectivity profiles

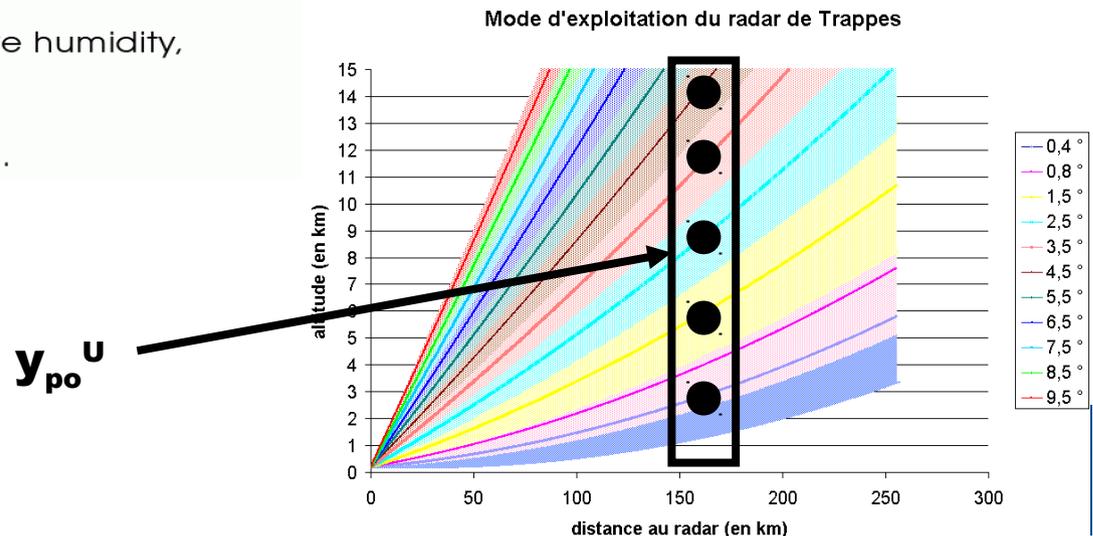
1. **Compute model reflectivities** from model relative humidity (RH) profiles.
2. **Estimate likelihoods of model reflectiv.,** by comparing them with obs. reflectivities.
3. Compute (pseudo-)observed RH profile, to be assimilated : **average of model RH profiles, weighted by their likelihoods.**

Caumont, 2006: use model profiles in the neighborhood of each observation (in 3 steps)

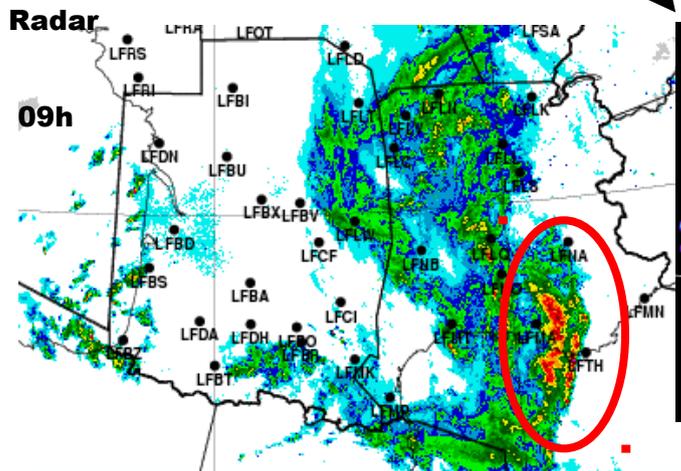
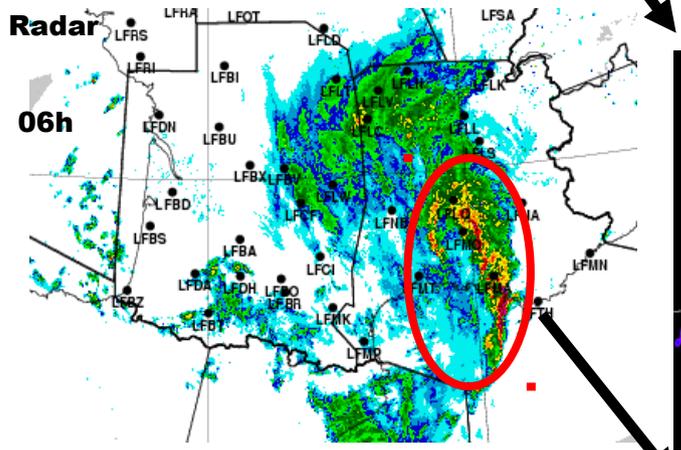
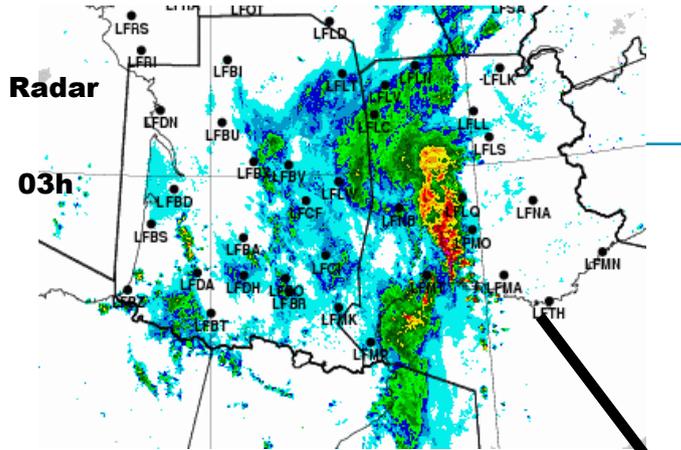


y_{po}^u : column of pseudo-observed relative humidity,
 y_z : column of observed reflectivities,
 x_i^u : column of relative humidity,
 $H_z(x_i)$: column of simulated reflectivities.

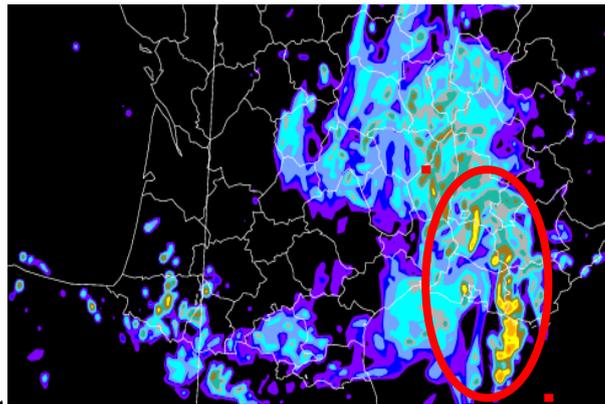
Coherence between the inverted profile and the precipitating cloud that the model is able to create



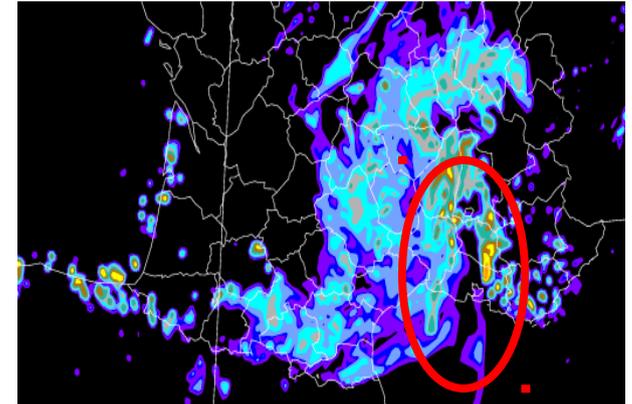
Example of High Precipitation Event (South-East of France) :
comparison of 3h forecasts between runs of
REFL (assimilation of reflectivities) and CONTROL.
Line of heavy precipitation is well analysed in REFL run.



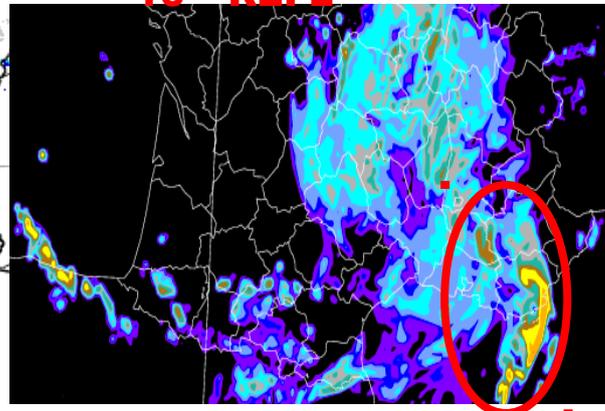
r3 - REFL



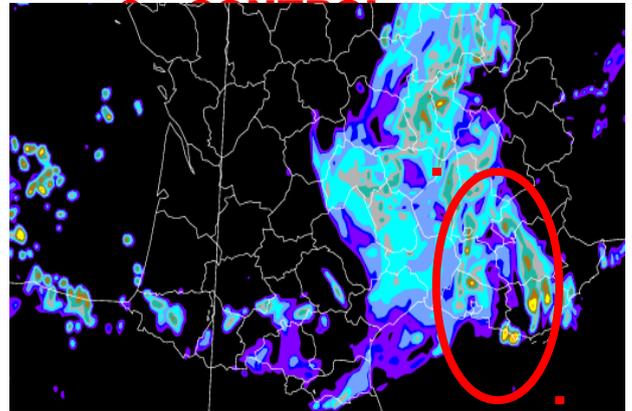
r3 - CONTROL



r6 - REFL



r6 - CONTROL

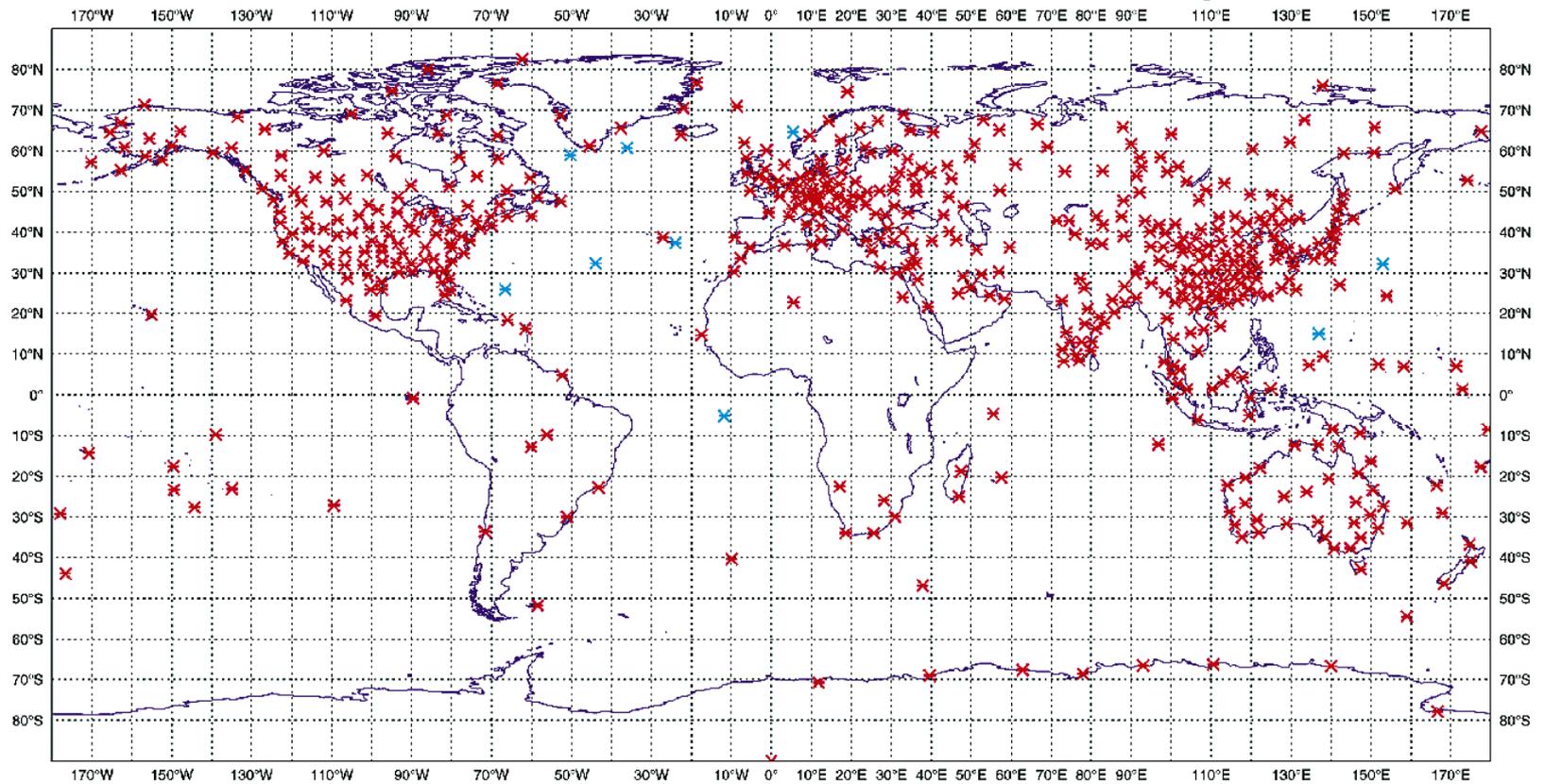


Error covariances : estimation and modelling

How can we estimate error covariances ?

- The **true atmospheric state** is never (exactly) known.
- Use **observation-minus-background** departures to estimate some average variances and correlations of **R** and **B**, using assumptions on spatial structures of errors.
- Use an **ensemble** to simulate the error evolution and to estimate space- and time-dependent background error structures.
- Use **covariance modelling** to filter out sampling noise and other uncertainties in the ensemble.

Radiosonde observation network



Covariances of innovations

- Innovations = observation-background departures :

$$\begin{aligned}\mathbf{y}^o - H(\mathbf{x}^b) &= \mathbf{y}^o - H(\mathbf{x}^t) + H(\mathbf{x}^t) - H(\mathbf{x}^b) \\ &\sim \mathbf{e}^o - \mathbf{H}\mathbf{e}^b\end{aligned}$$

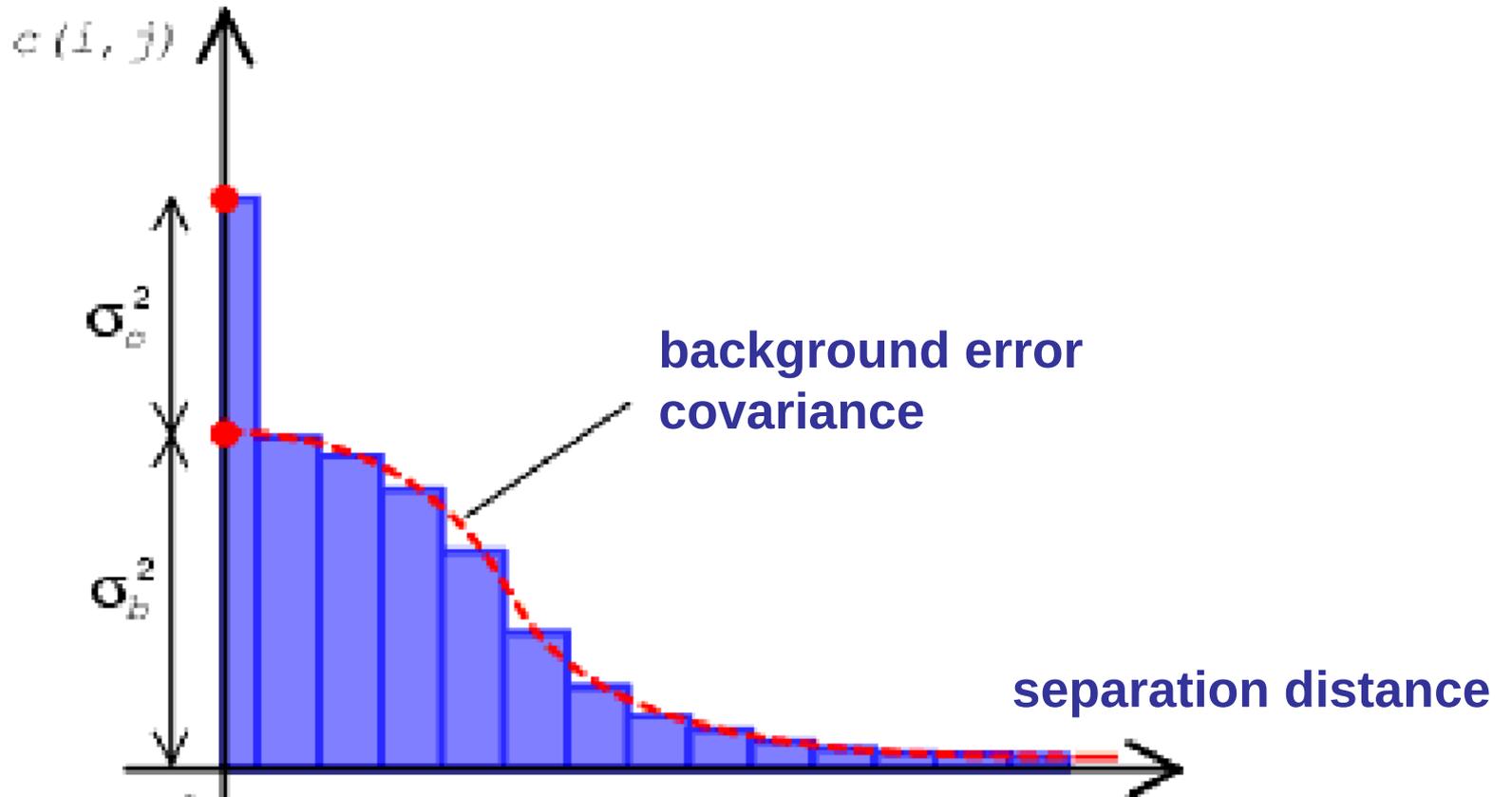
- Innovation covariances :

$$E[(\mathbf{y}^o - H(\mathbf{x}^b))(\mathbf{y}^o - H(\mathbf{x}^b))^T] = \mathbf{R} + \mathbf{H}\mathbf{B}\mathbf{H}^T$$

assuming that $E[\mathbf{e}^o (\mathbf{H}\mathbf{e}^b)^T] = \mathbf{0}$.

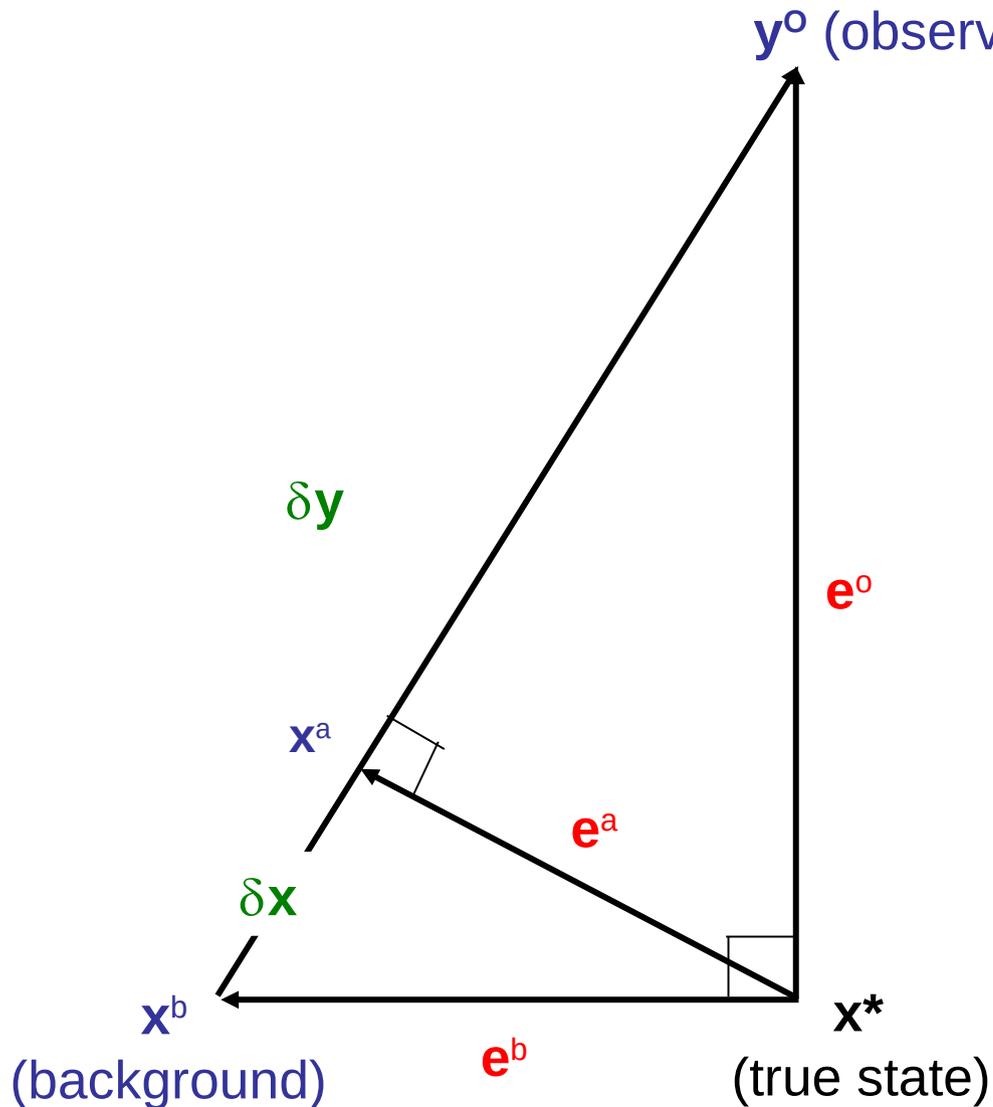
(e.g. Hollingsworth and Lönnberg 1986).

Covariances of innovations



$$E[(\mathbf{y}^o - H(\mathbf{x}^b))(\mathbf{y}^o - H(\mathbf{x}^b))^T] = \mathbf{R} + \mathbf{H} \mathbf{B} \mathbf{H}^T$$

Covariances of analysis residuals



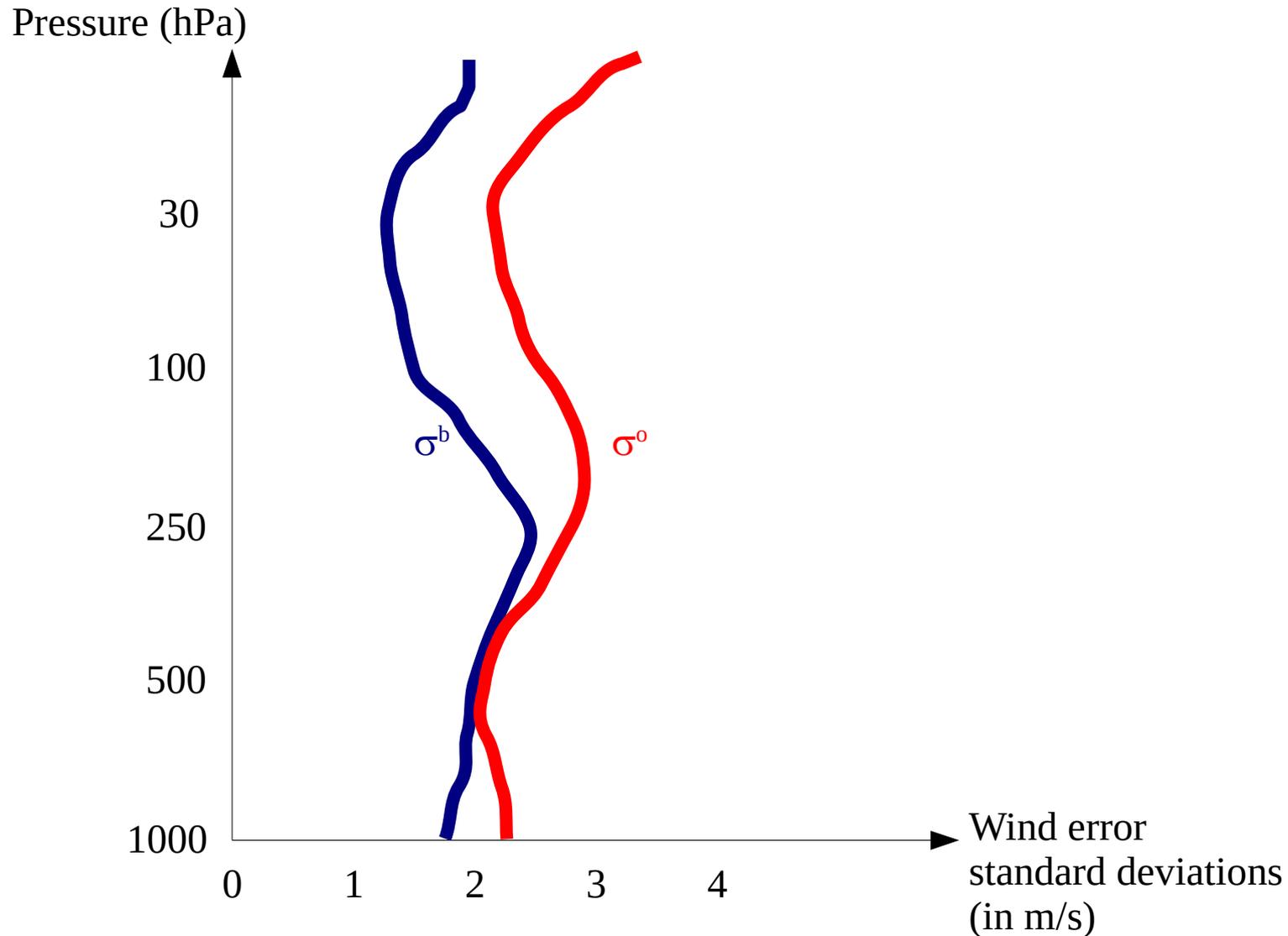
$$\delta \mathbf{y} = \mathbf{y}^o - H(\mathbf{x}^b) \quad (\text{innovation})$$

$$\mathbf{H} \delta \mathbf{x} = H(\mathbf{x}^a) - H(\mathbf{x}^b) \quad (\text{increment})$$

$$E[\mathbf{H} \delta \mathbf{x} \delta \mathbf{y}^T] = \mathbf{H} \mathbf{B} \mathbf{H}^T$$

$$E[(\mathbf{y}^o - H(\mathbf{x}^a)) \delta \mathbf{y}^T] = \mathbf{R}$$

Vertical profiles of standard deviations of background errors and observation errors

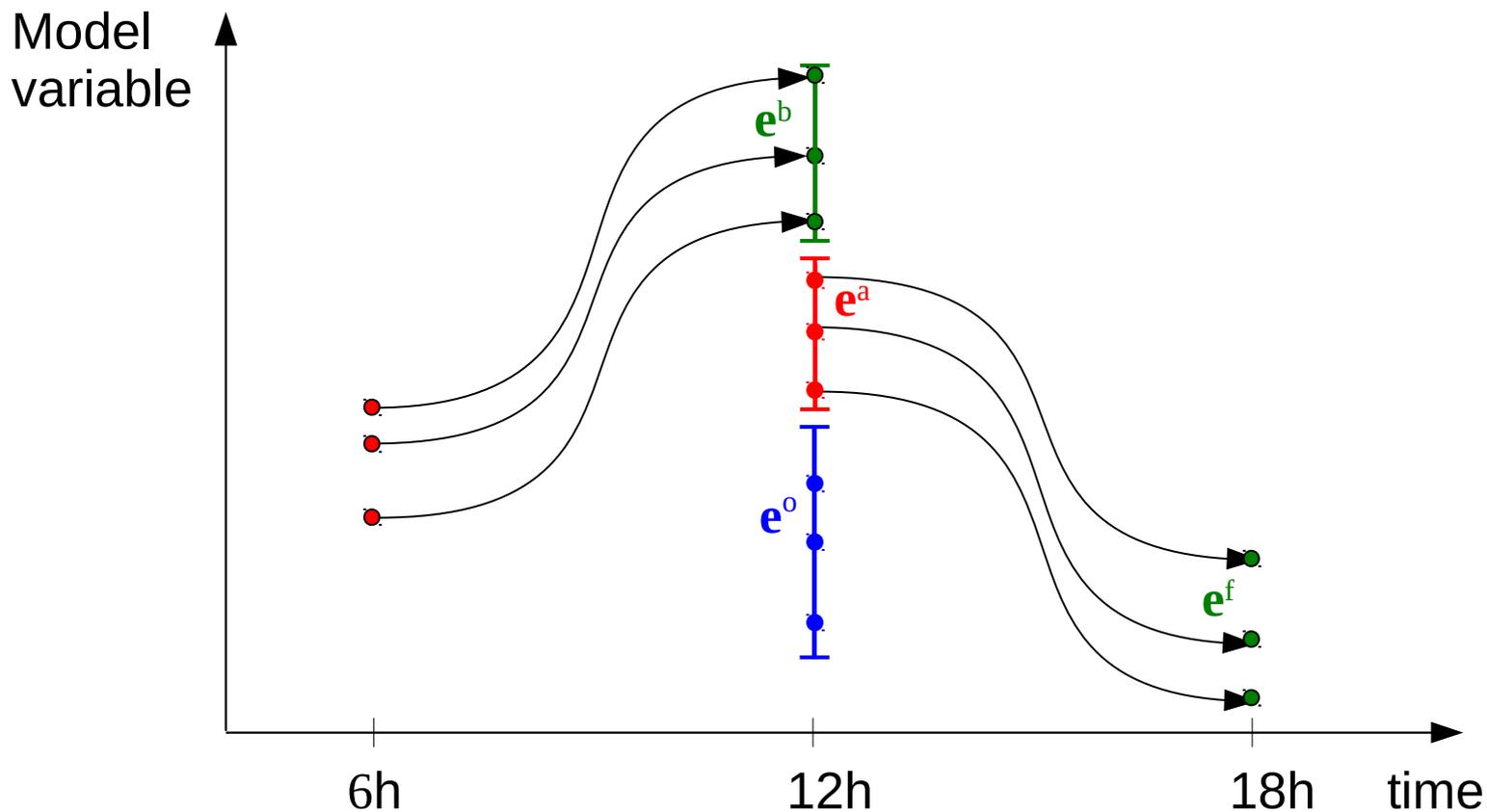


Properties of innovation methods

- Provides estimates in observation space.
- A good quality data dense network is needed.
- Assumption that observation errors are spatially uncorrelated.
- An objective source of information on **B** and **R**.
- At a given location and time, only 1 innovation value : only a single error realization is available.

=> Statistical averages (expectations) are replaced by space and time averages (ergodic assumption).

Ensemble Data Assimilation (EDA) : simulation of error cycling



$$e^a = (I - KH)e^b + Ke^0$$

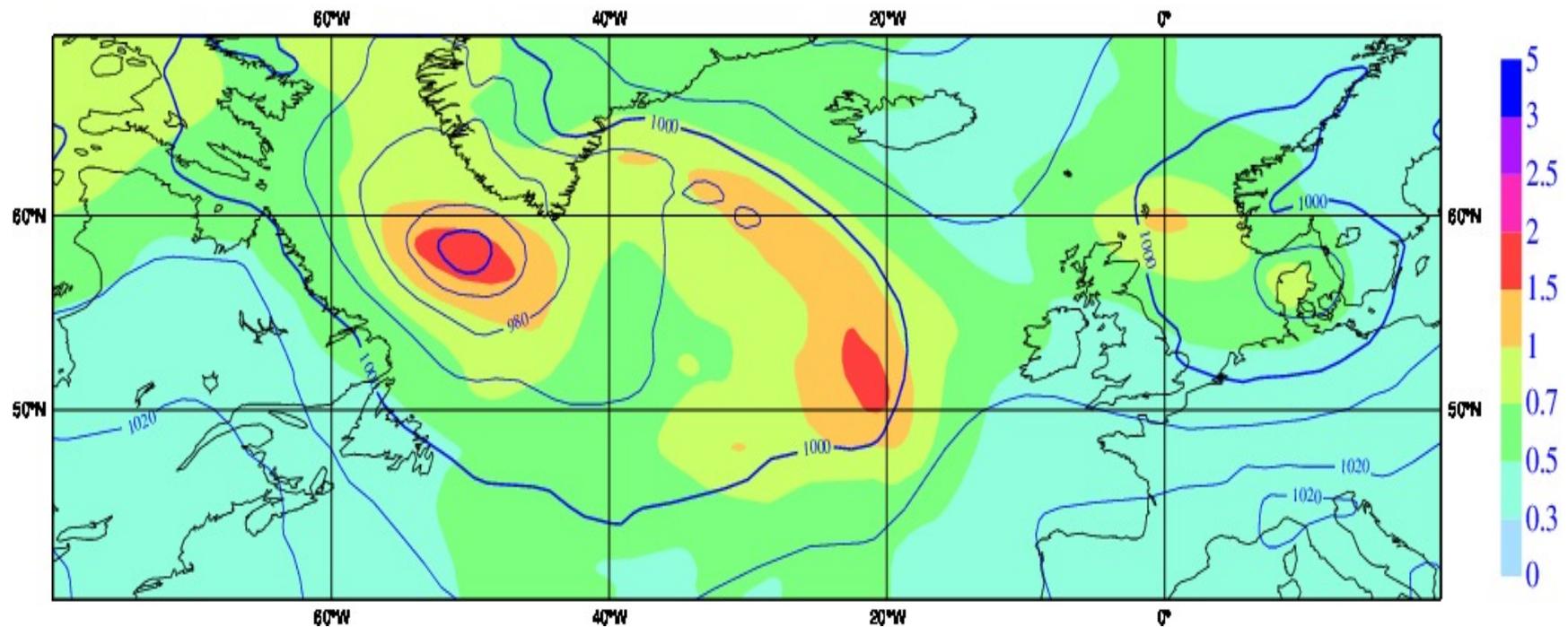
$$e^f = Me^a + e^m$$

with $e^b = e^{f-}$

and $e^0 = R^{1/2} \eta$ (random draws of R)

(e.g. Houtekamer et al 1996, Fisher 2003, Berre et al 2006 ;
ARPEGE : 25 members to estimate flow-dependent B)

Dynamics of background error variances



Standard deviations of surface pressure errors (hPa)
(superimposed with MSLP analysis (hPa)).

Modelling and filtering covariances

- Huge size of **B** : model it with operators which are sparse and/or of small size.
- Sampling noise, and other uncertainties. => Spatio-temporal filtering.
- Factorisation : $\mathbf{B} = \mathbf{B}^{1/2} \mathbf{B}^{\top/2}$
 $\mathbf{B}^{1/2} = \mathbf{L} \mathbf{S} \mathbf{C}^{1/2}$

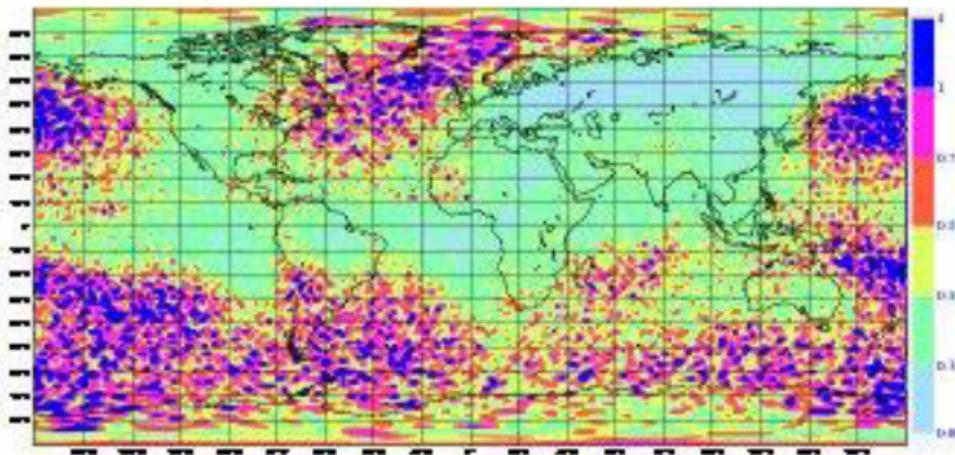
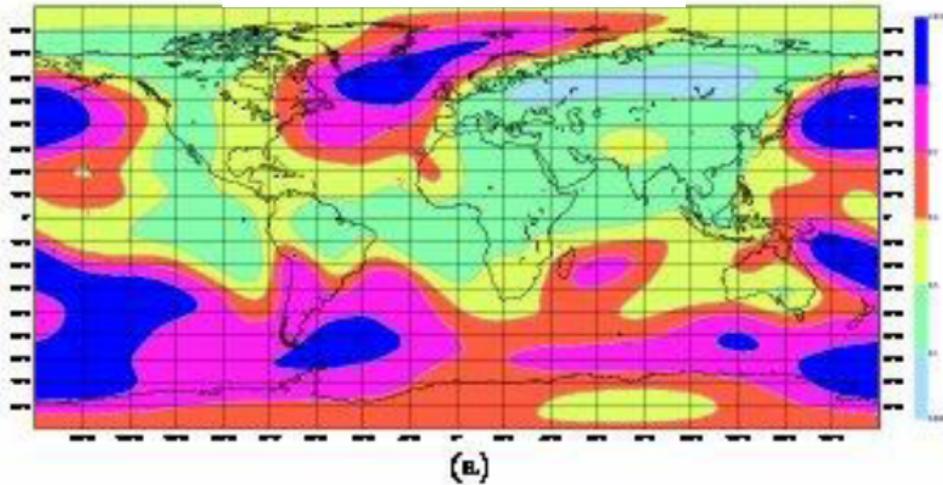
L ~ mass/wind **cross-covariances** (related to geostrophy),
including flow-dependence (non linear balances).

S flow-dependent **standard deviations** (~ expected error amplitudes),
filtered spatially.

C matrix of **3D spatial correlations** (~ spatial structures of errors),
filtered in wavelet space (block-diagonal model).

Spatial filtering of variance field

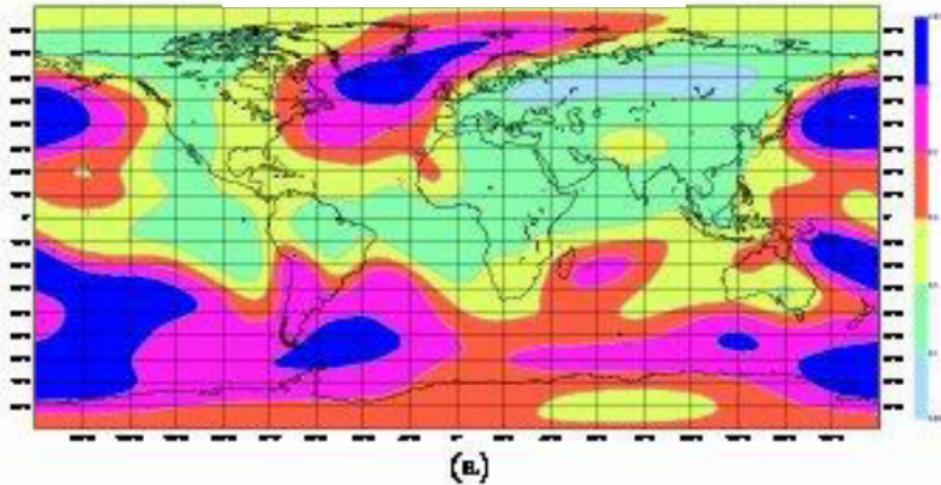
EXACT
VARIANCES



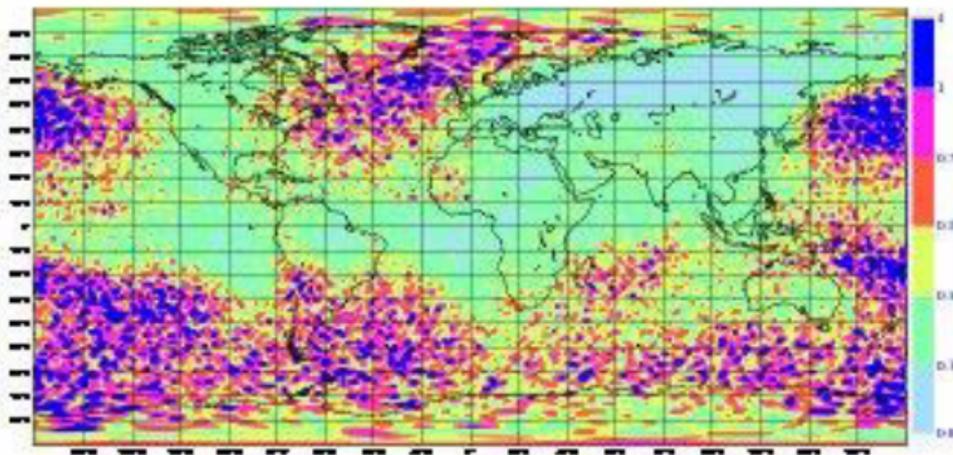
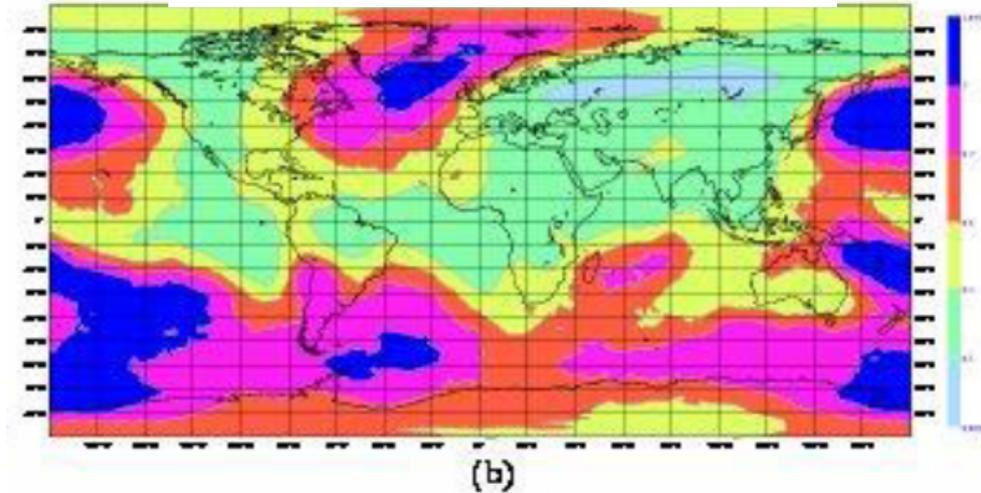
RAW ENSEMBLE
VARIANCES (N=6)

Spatial filtering of variance field

EXACT
VARIANCES



FILTERED ENSEMBLE
VARIANCES (N=6)



RAW ENSEMBLE
VARIANCES (N=6)

Low-pass filter applied to raw variances :

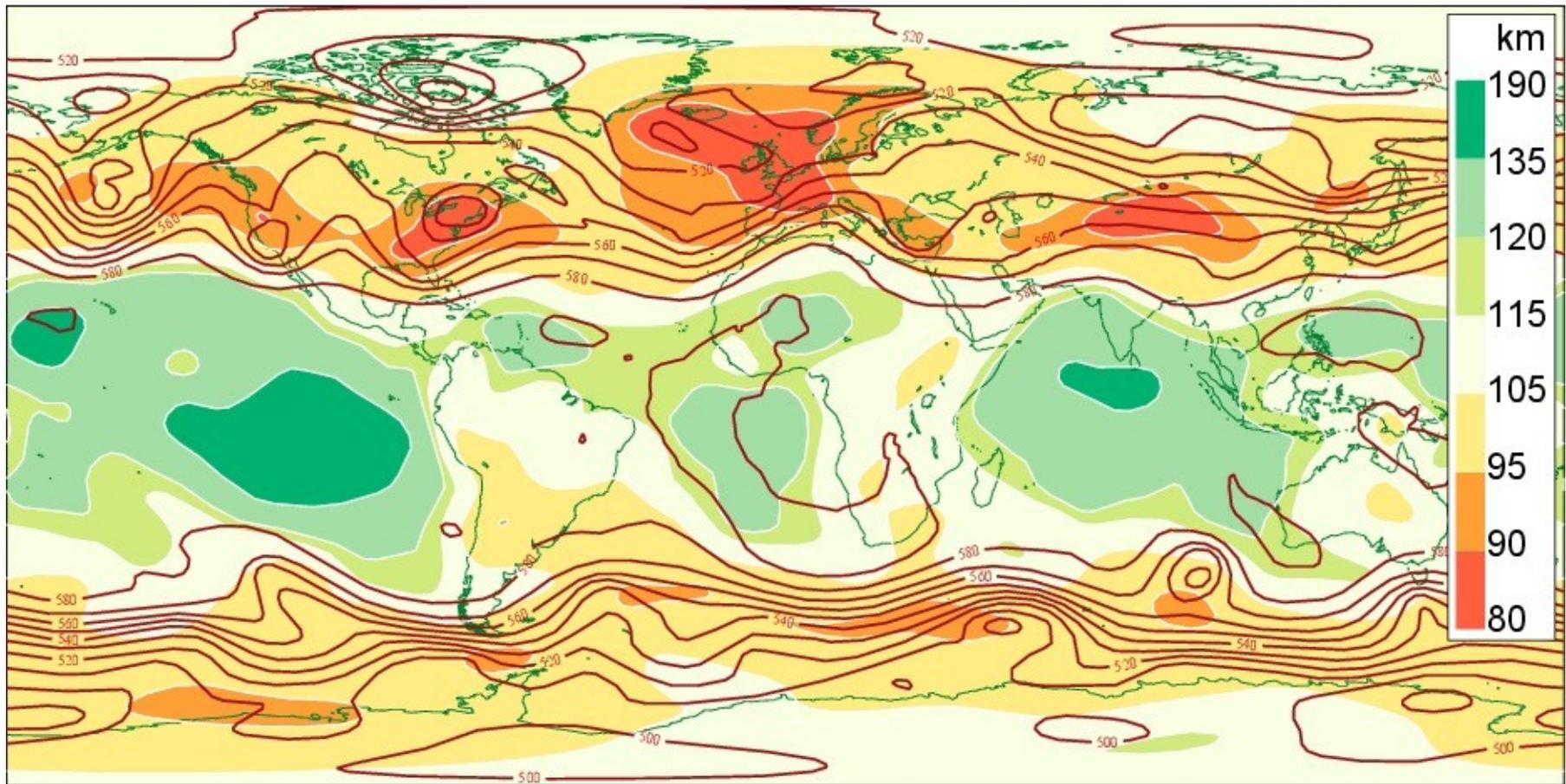
$$\mathbf{v}'_b = \mathbf{F} \mathbf{v}_b$$

with \mathbf{F} optimized in spectral space /
spatial structures of signal & noise :

$$F = 1 / (1 + E[\text{noise}^2]/\text{signal}^2)$$

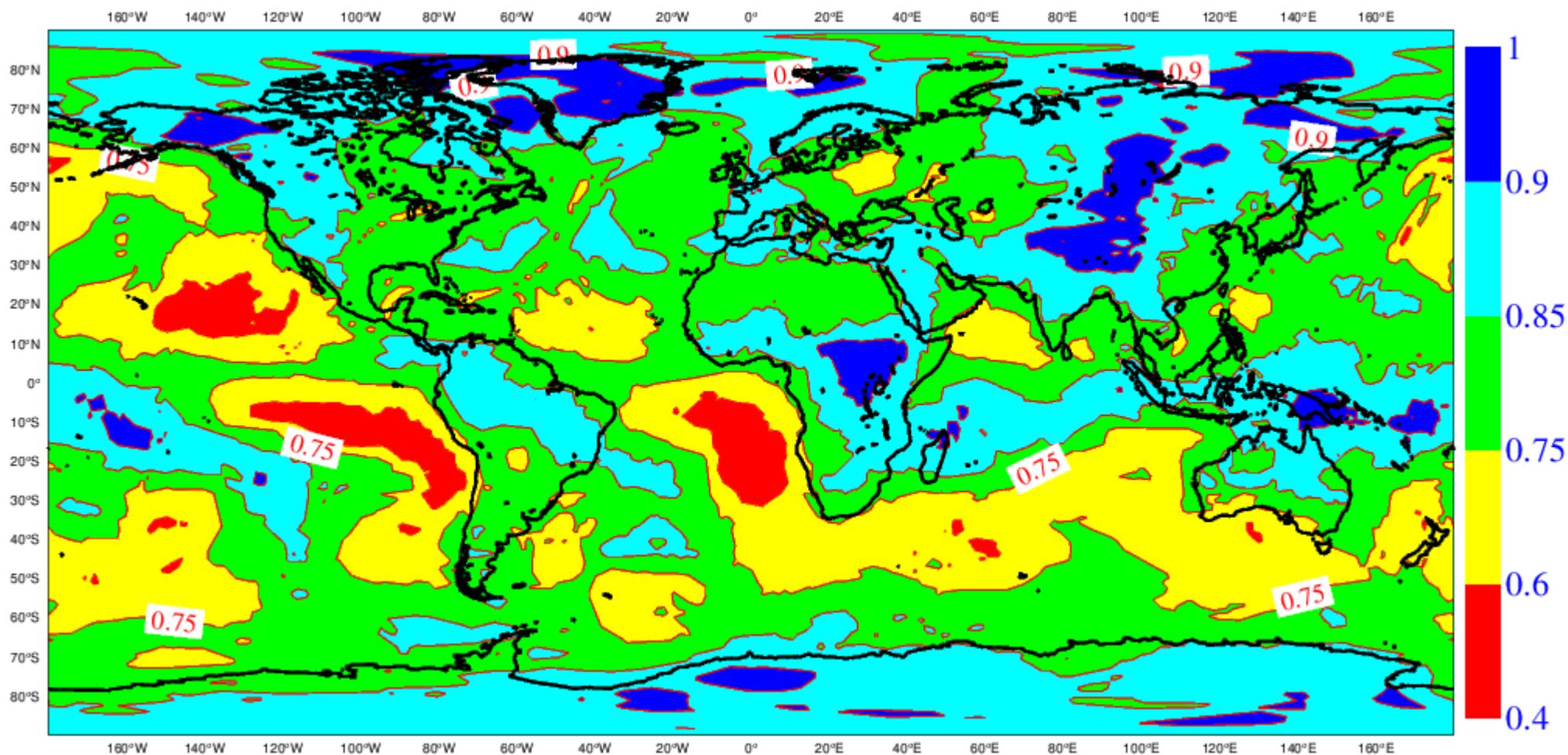
$$\text{and } E[v_b^2] = \text{signal}^2 + E[\text{noise}^2]$$

Dynamics of horizontal correlations



Horizontal length-scales (in km) of wind errors near 500 hPa,
superimposed with geopotential

Dynamics of vertical correlations



Vertical correlations of temperature errors
between 850 & 870 hPa

4D_{En}Var

Variational analysis based on a 4D Ensemble

Minimisation of $J(\underline{\delta\mathbf{x}})$ where $\underline{\delta\mathbf{x}}$ is a 4D analysis increment :

$$J(\underline{\delta\mathbf{x}}) = \underline{\delta\mathbf{x}}^T \underline{\mathbf{B}}^{-1} \underline{\delta\mathbf{x}} + (\underline{\mathbf{d}} - \underline{\mathbf{H}} \underline{\delta\mathbf{x}})^T \underline{\mathbf{R}}^{-1} (\underline{\mathbf{d}} - \underline{\mathbf{H}} \underline{\delta\mathbf{x}})$$

with $\underline{\mathbf{B}} = \underline{\mathbf{X}}^{b'} \underline{\mathbf{X}}^{b'T} \circ \underline{\mathbf{L}}$, where $\underline{\mathbf{L}}$ is a localization matrix,

$$\underline{\mathbf{X}}^{b'} = (\underline{\mathbf{x}}^{b'}_1, \dots, \underline{\mathbf{x}}^{b'}_{N^e}),$$

$$\underline{\mathbf{x}}^{b'}_{ne} = \underline{\mathbf{x}}^b_{ne} - \langle \underline{\mathbf{x}}^b \rangle / (N^e - 1)^{1/2}, \quad ne = 1, N^e.$$

$\underline{\mathbf{x}}^{b'}$ of dimension $K+1$ (time) \times M (3D variables) \times N (dim 3D).

(Liu et al, 2008, 2009 ; Buehner et al, 2010 ; Lorenc, 2012 ;

Desroziers et al 2014).

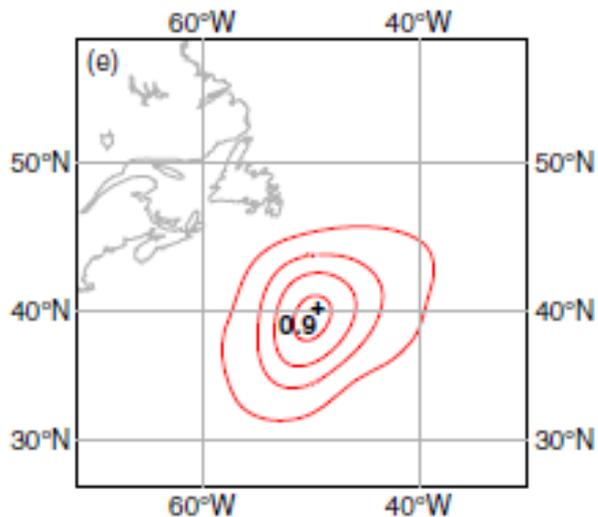
4DEnVar

Variational analysis based on a 4D Ensemble

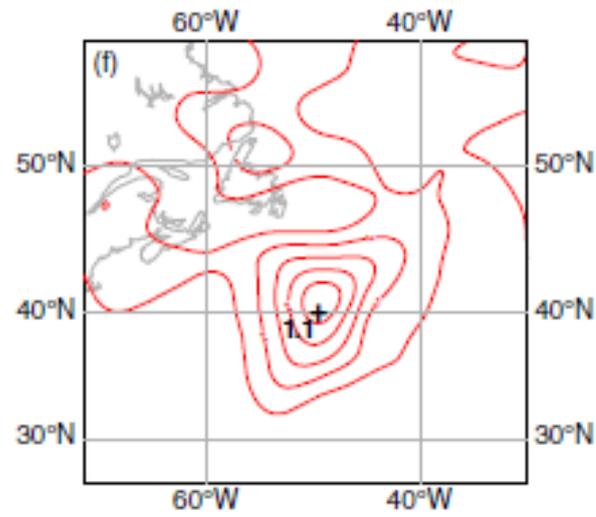
- 4D covariances from an ensemble of trajectories.
- Improved realism of 4D background error covariances (anisotropies, non linear evolution including all physical processes).
- No need to develop and maintain an adjoint model in this case.
 - Especially important for AROME.
- Pursue within the variational framework
 - Global assimilation of all available observations, distributed in space and in time.
- Introduces additional levels of parallelism (space, time, ensemble).

Covariance anisotropy and localisation

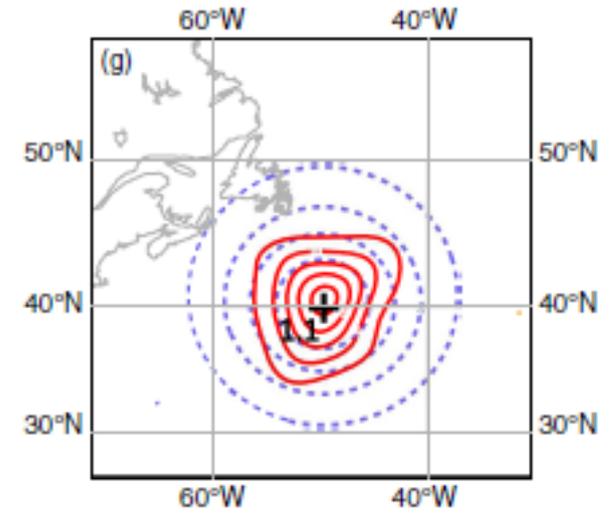
Ensemble to get information on anisotropy, but requires filtering = localisation.



Exact covariances



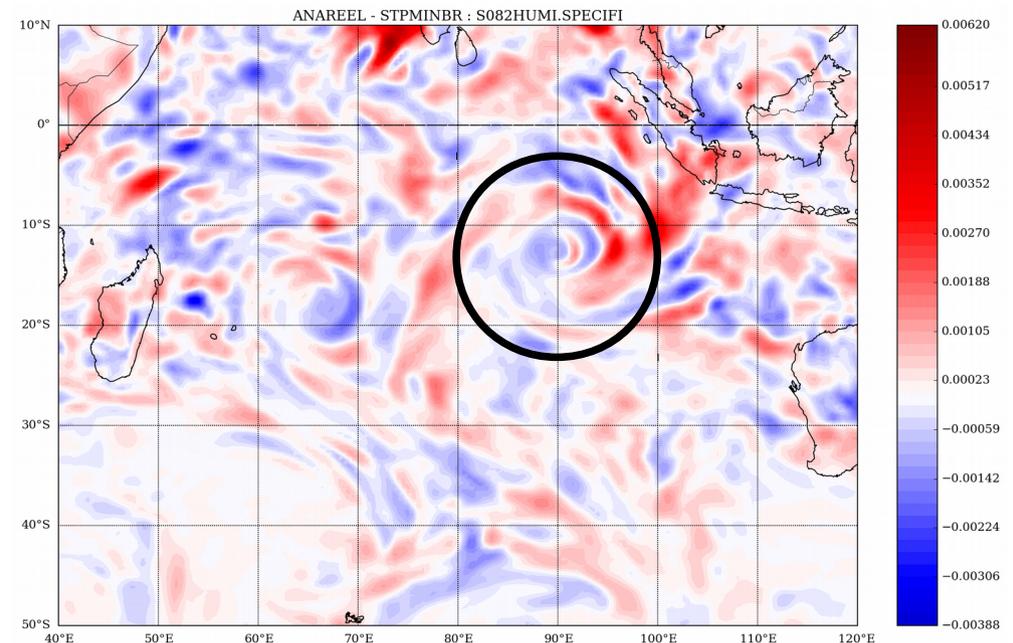
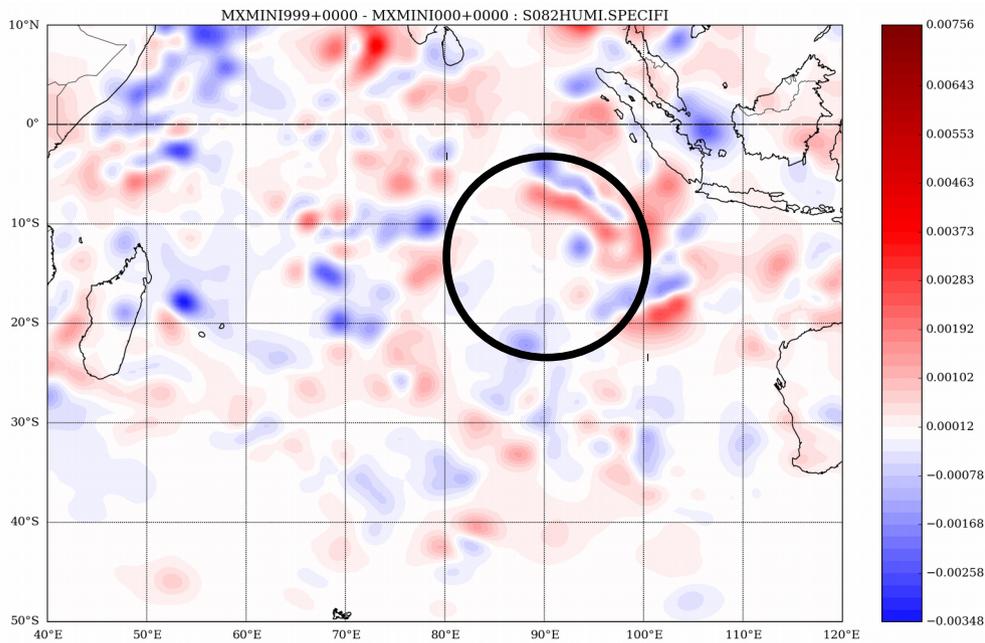
Raw covariances
(200 members)



Localised covariances
(200 members)

Flow-dependent anisotropic increments

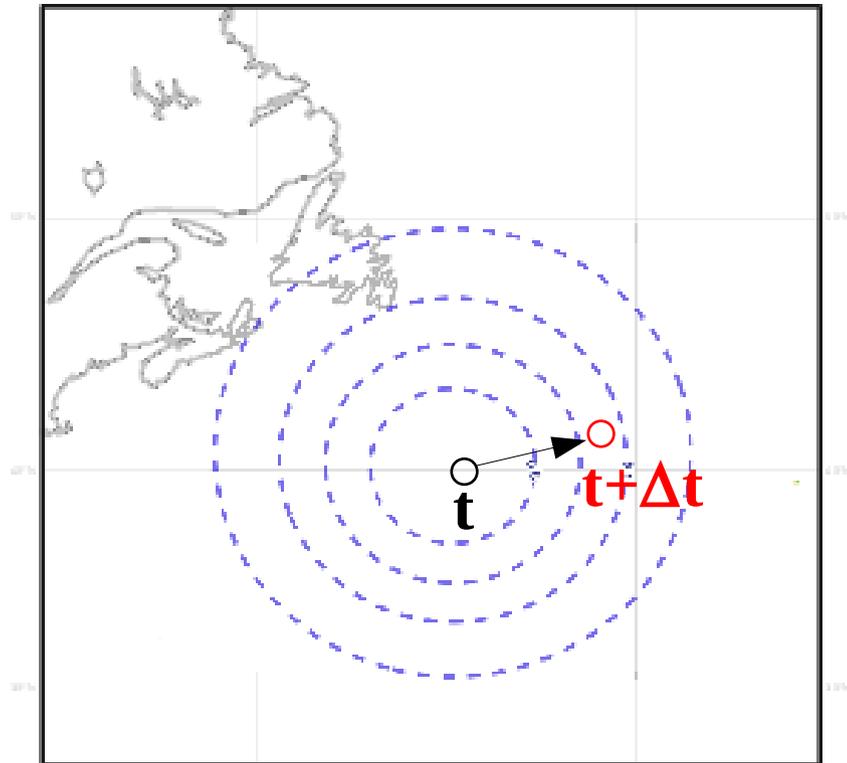
Humidity analysis increments (near 850 hPa)



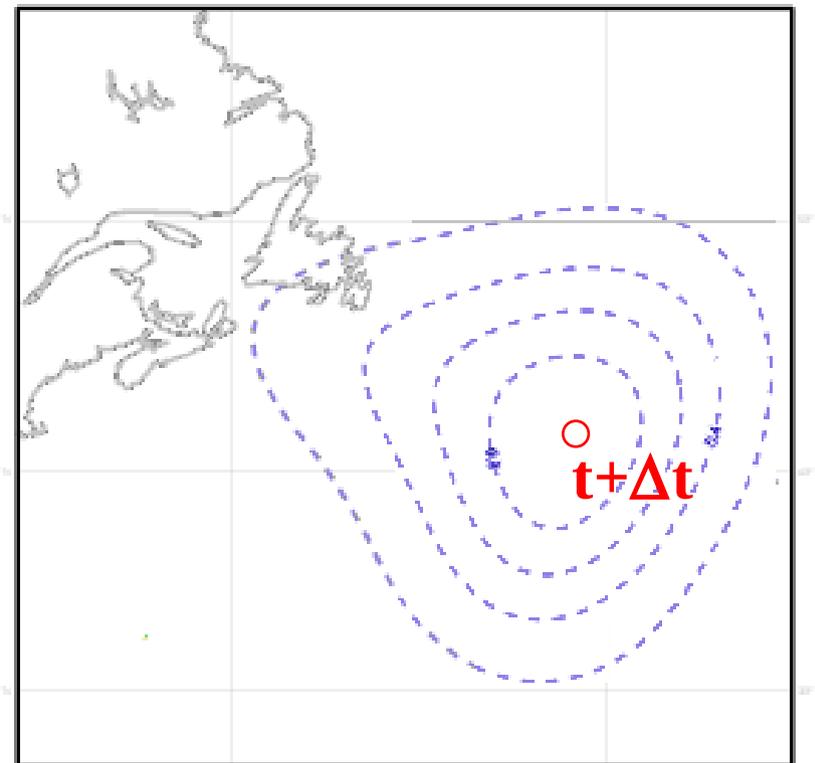
With isotropic correlations

With anisotropic correlations,
filtered by localisation

4D localisation of correlations



Static localisation :
spurious attenuation
of increments at $t+\Delta t$



Advected localisation :
consistent with the dynamics



Conclusions

Conclusions

- Data assimilation is vital for weather forecasting.
- Observations are very diverse in type, density and quality.
- 4D-Var for temporal and non linear aspects.

- Observation-background departures for estimation of average variances and correlations in **R** and **B**.
- Ensemble DA for error simulation and for covariance dynamics.

- Sampling noise issues and filtering methods.
- Towards 4DEnVar (variational assimilation based on a 4D ensemble).



Thanks for your attention

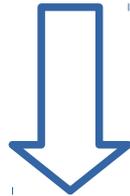
Liens principaux entre thématiques ensemblistes

Spécification des incertitudes

(observations, modèle) du système d'analyse/prévision



Assimilation d'ensemble : simulation de la propagation des erreurs au cours du "cyclage" de l'assimilation



Spécification des covariances d'erreur d'ébauche, formulations **EnVar** de l'assimilation

Prévision d'ensemble : simulation de la propagation / amplification des erreurs au cours de la prévision



Prévision probabiliste : traitement statistique des prévisions de l'ensemble

Passive remote sensing

Only natural sources of radiation (sun, earth...) are involved, and the sensor is a simple receiver, « passive ».

Atmosphere in parallel plan, no diffusion, specular surface

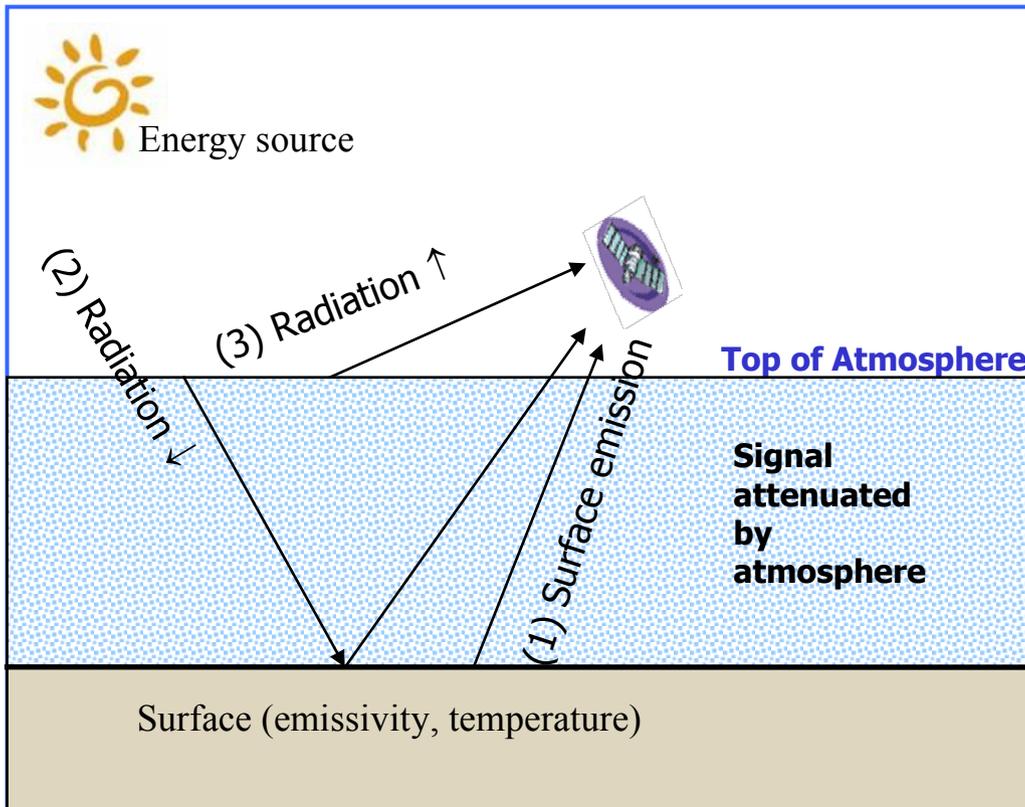
$$T(p, \nu) = \varepsilon(p, \nu) T_s \tau + (1 - \varepsilon(p, \nu)) \tau T(\nu, \downarrow) + T(\nu, \uparrow)$$



Emissivity

Radiative transfer equation, dependent on T, q :

Observation operator for satellite radiances.



Two main ingredients in weather forecasting

What will be the weather tomorrow ?

Bjerknes (1904) :

in order to a good forecast, we need to know

- the atmospheric evolution laws
(~ modelling) ;
- the atmospheric state at initial time
(~ data assimilation).