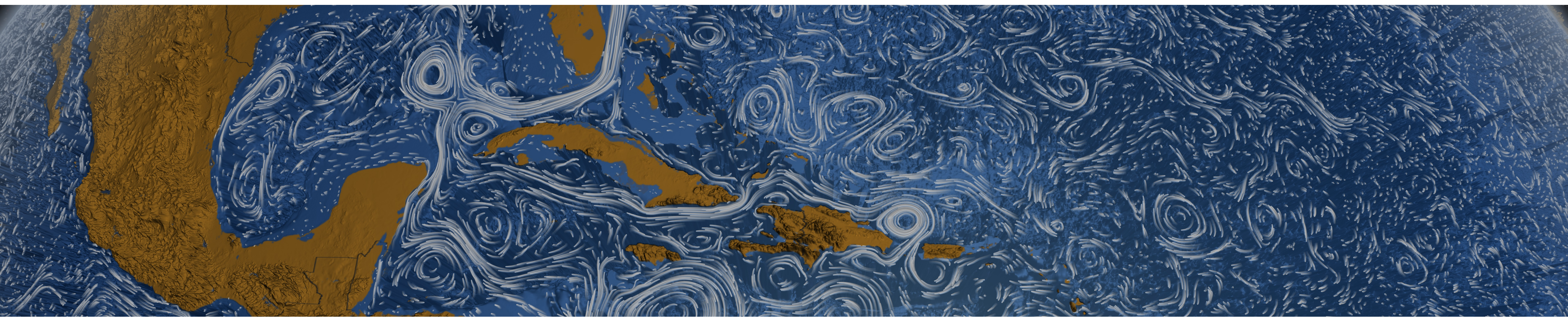
Ocean data assimilation



WAPE/MOCIS master’s program
February 10, 2026

Emmanuel COSME
Université Grenoble Alpes, IGE, Grenoble

Scope of the lecture

Texte du titre

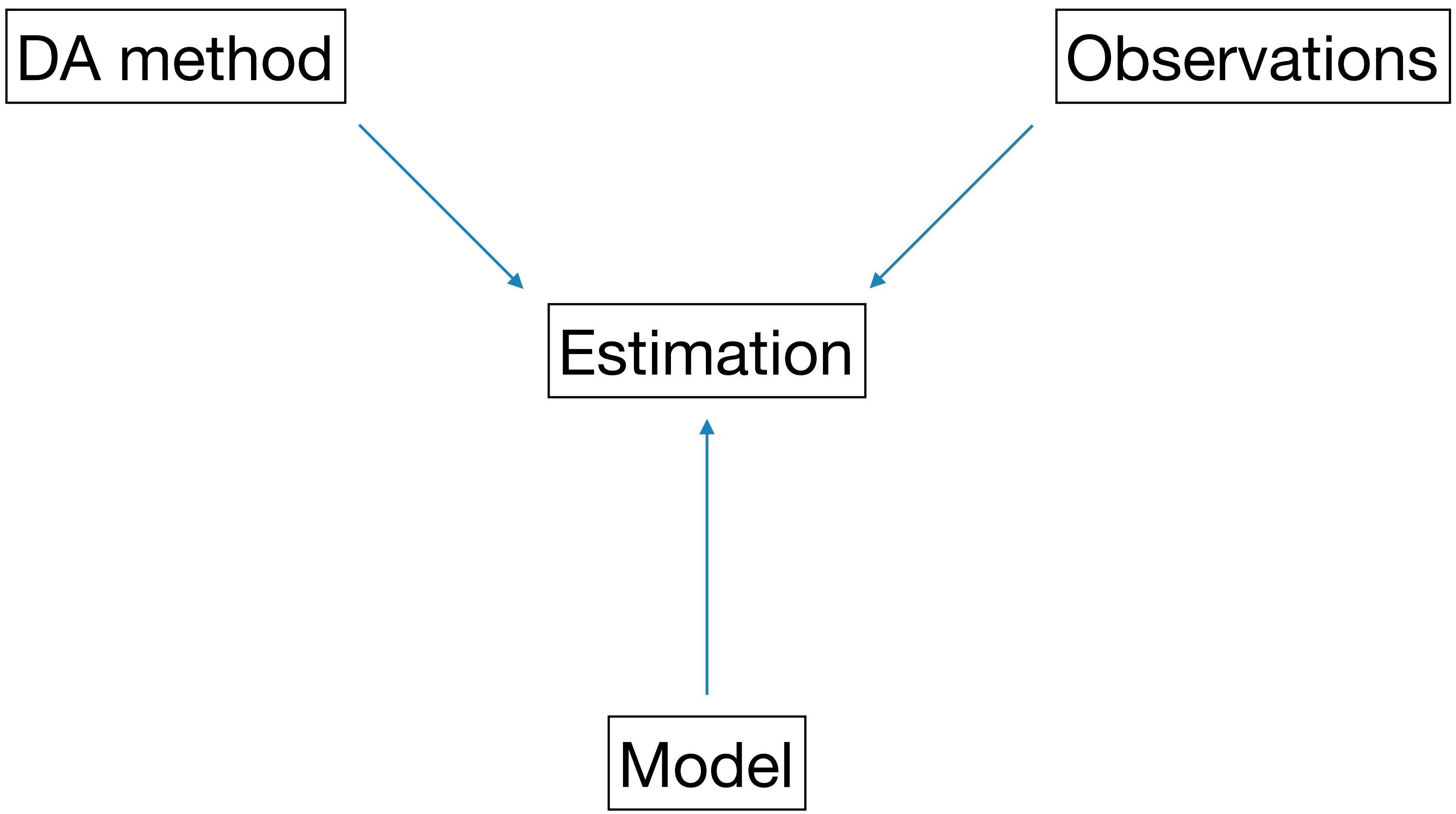
This DA lecture mostly deals with physical oceanography and the ocean circulation, but does not address:

- ocean wave forecasting
- tidal/storm surge forecasting
- ocean chemistry and water quality
- Fish, whales, sharks, jellyfish…

The slides are designed to be more or less "self-sufficient"
==> wordy sometimes, not extremely fluent

Motivation for data assimilation

Texte du titre



Motivation for data assimilation

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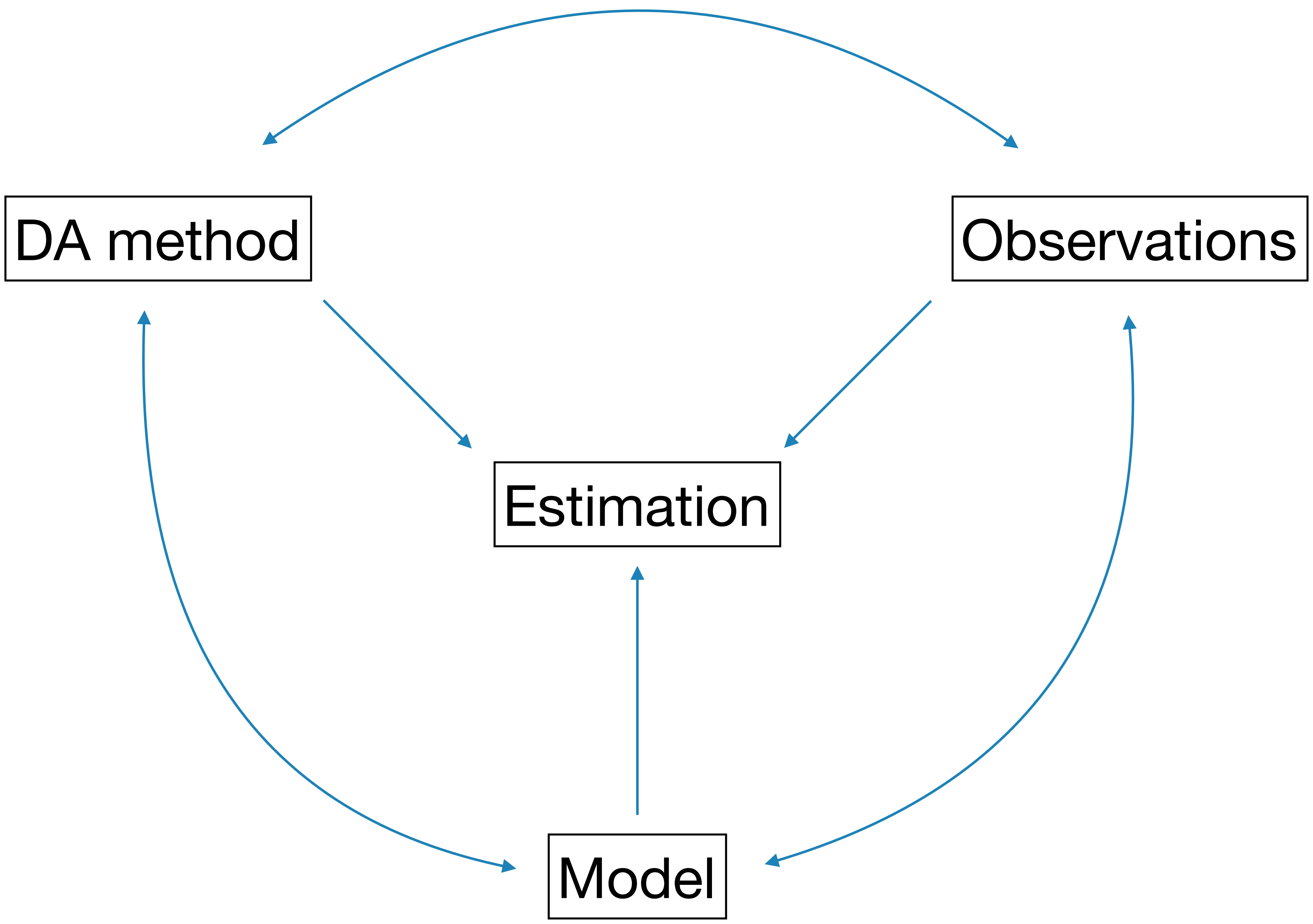

Motivation for data assimilation

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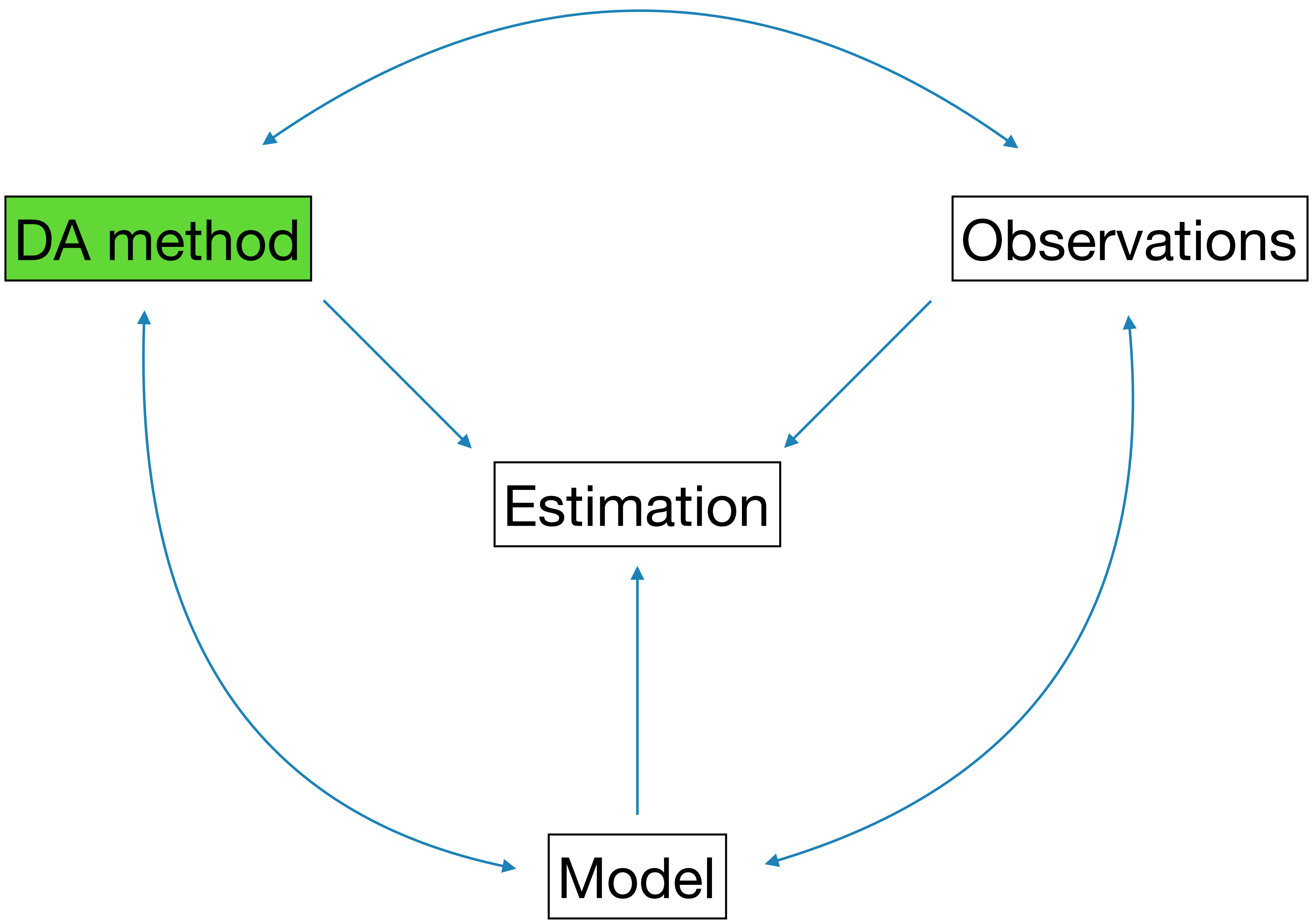

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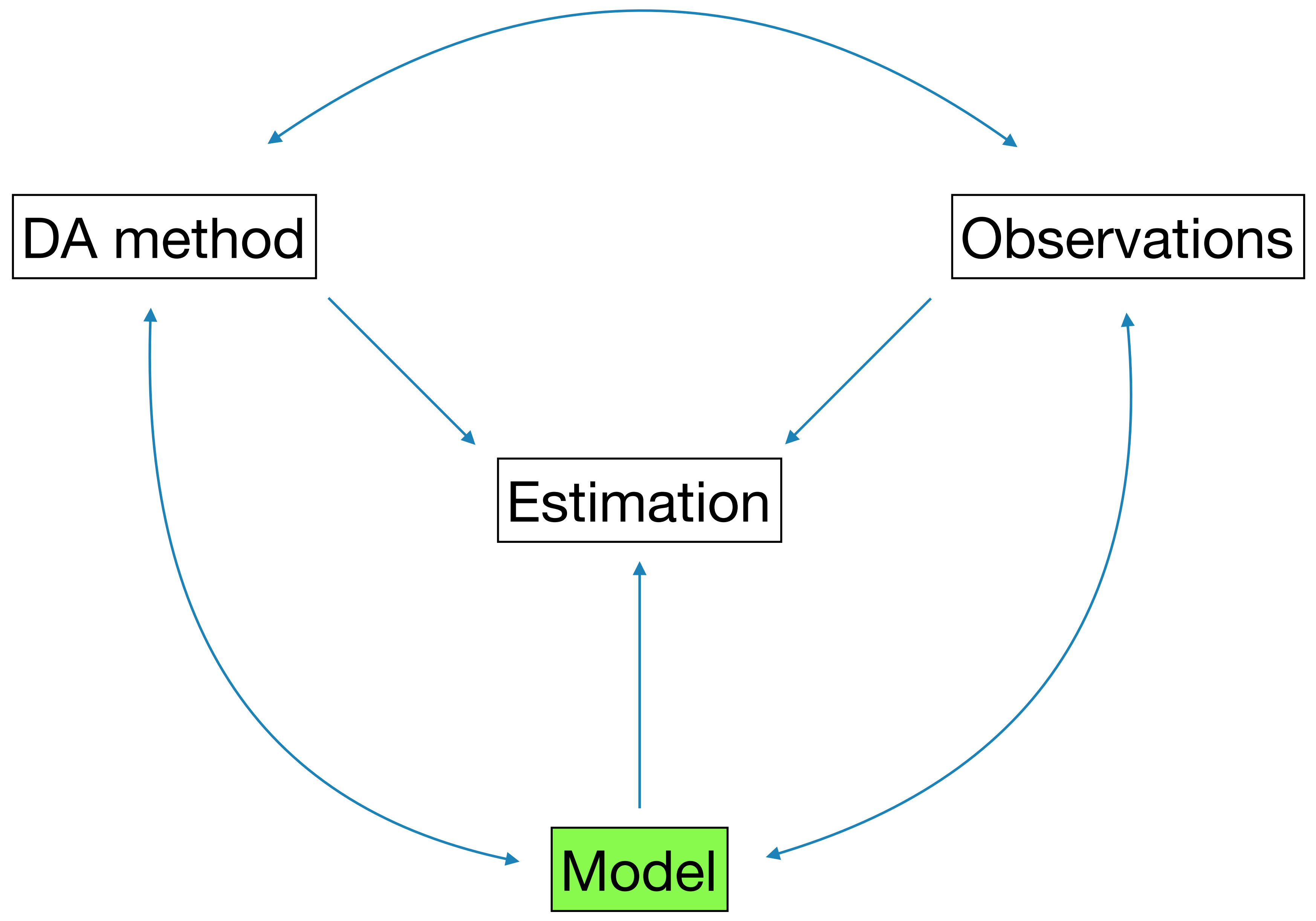

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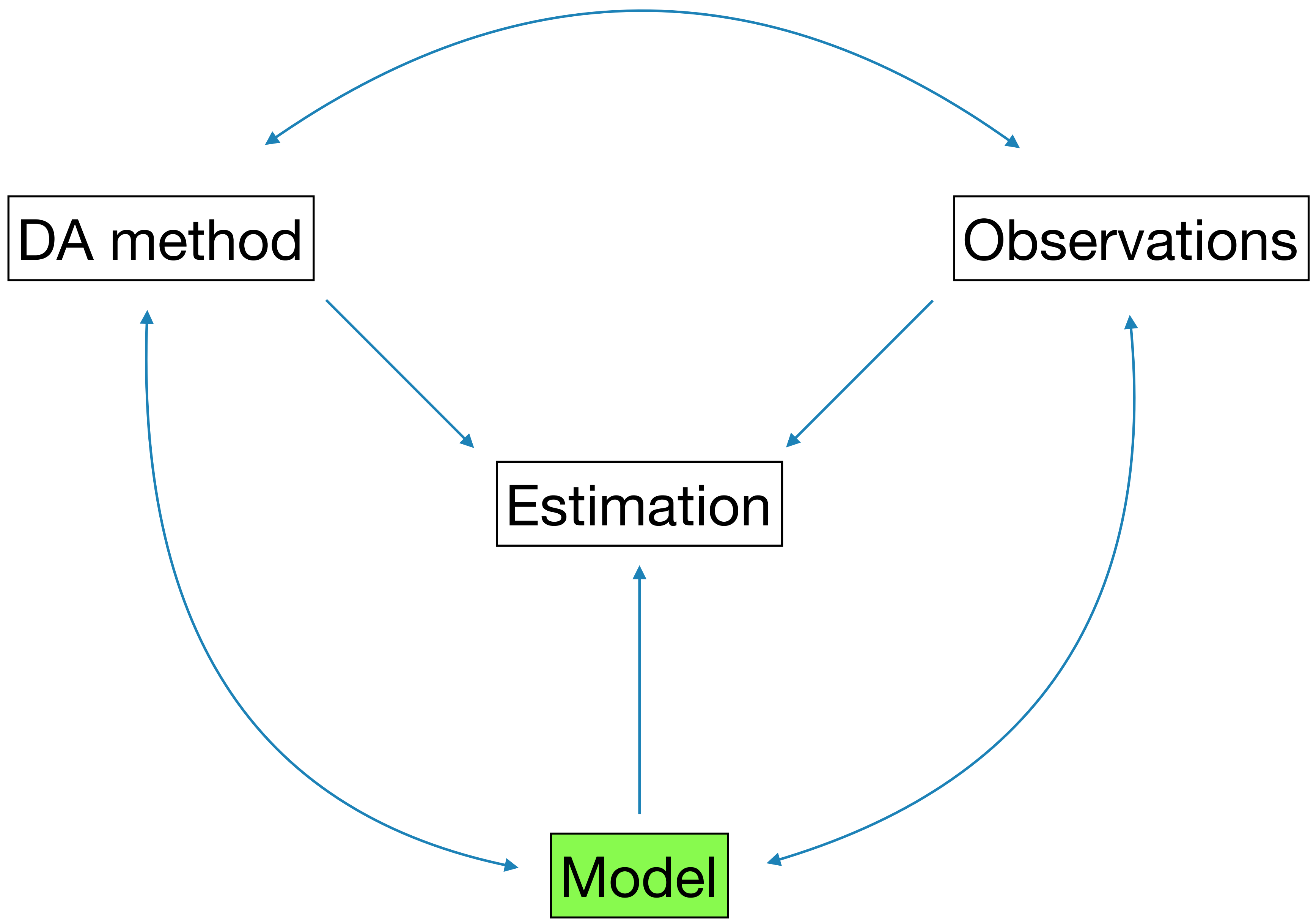
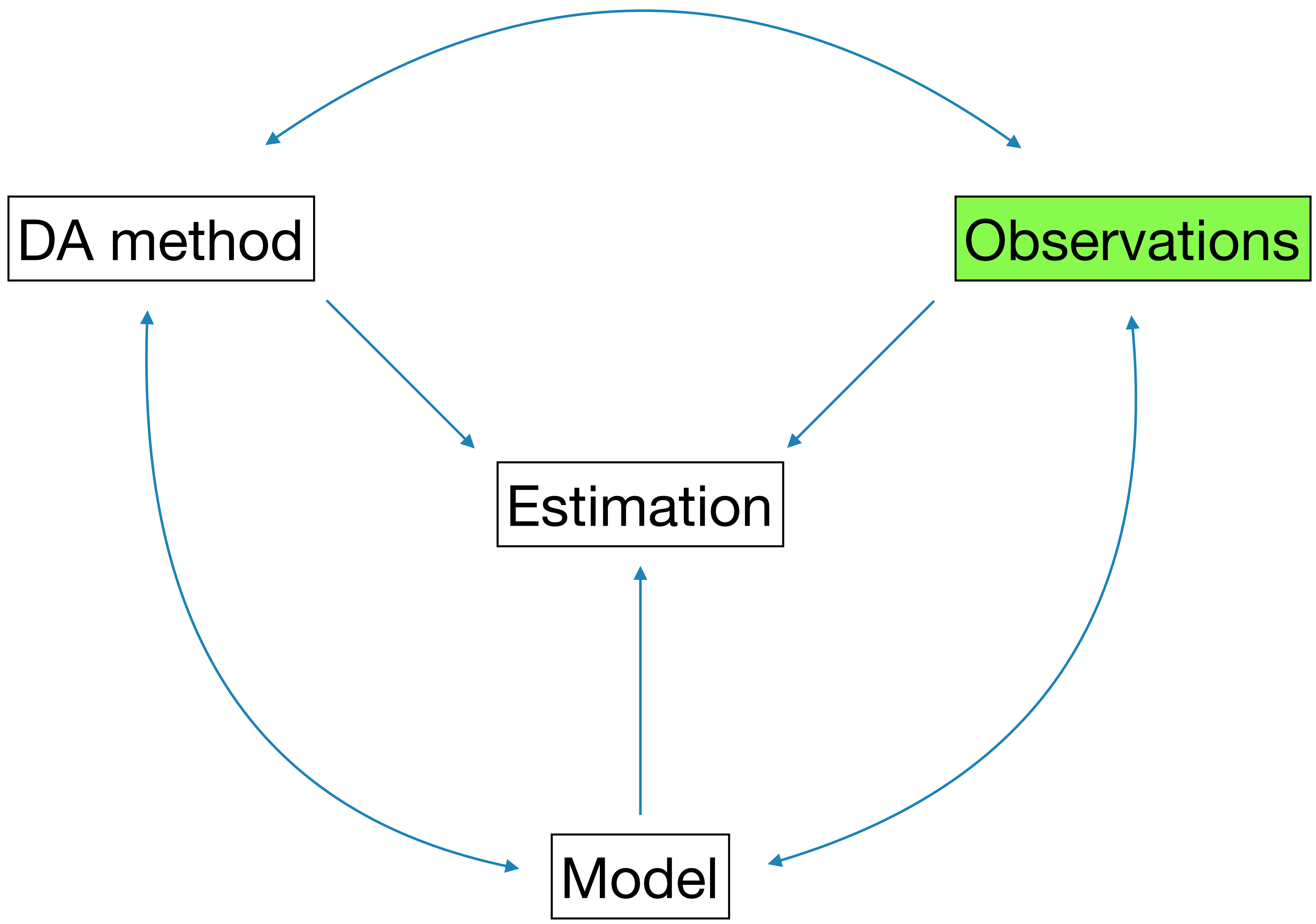


"Operational" approach

Motivation for data assimilation

Texte du titre



"Observation-centered" approach

Outline

Texte du titre

1. Atmospheric vs oceanic data assimilation
   - 1.1. History and culture
   - 1.2. Observations
   - 1.3. Dynamics and models
2. "Model-centered" data assimilation
   - 2.1. Operational oceanography
   - 2.2. Ocean models
   - 2.3. Observations of the ocean
   - 2.4. Ensemble Kalman filter implementations
3. "Observation-centered" data assimilation
   - 3.1. Assimilation of images
   - 3.2. Altimetric products and the SWOT mission
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   - 3.4. Mapping balanced motions with 4DVar
   - 3.5. Eddy/wave separation with a 4DVar technique

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History and culture

Meteorology:

Oceanography:

Atmospheric vs oceanic data assimilation

History and culture

Meteorology:

- strong and historical rooting in forecasting issues

Oceanography:

- Forecasting is an issue, but not the only one (importance of observation-centered DA)

Atmospheric vs oceanic data assimilation

History and culture

Meteorology:

- strong and historical rooting in forecasting issues
- the most advanced field for high-dim. DA

Oceanography:

- Forecasting is an issue, but not the only one (importance of observation-centered DA)
- less maturity than in meteorology

Atmospheric vs oceanic data assimilation

History and culture

Meteorology:

- strong and historical rooting in forecasting issues
- the most advanced field for high-dim. DA
- Dedicated manpower

Oceanography:

- Forecasting is an issue, but not the only one (importance of observation-centered DA)
- less maturity than in meteorology
- much less manpower

Atmospheric vs oceanic data assimilation

History and culture

Meteorology:

- strong and historical rooting in forecasting issues
- the most advanced field for high-dim. DA
- Dedicated manpower
- DA is culturally accepted

Oceanography:

- Forecasting is an issue, but not the only one (importance of observation-centered DA)
- less maturity than in meteorology
- much less manpower
- DA is always questioned

Atmospheric vs oceanic data assimilation

Illustration: maps of SSH

If a user needs a time series of global maps of Sea Level pressure, what will her choice be?

Atmospheric vs oceanic data assimilation

Illustration: maps of SSH

If a user needs a time series of global maps of Sea Level pressure, what will her choice be?

An ECMWF reanalysis, probably.

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Atmospheric vs oceanic data assimilation

Illustration: maps of SSH

If a user needs a time series of global maps of Sea Level pressure, what will her choice be?

An ECMWF reanalysis, probably.

If a user needs a time series of global maps of SSH, what will her choice be?

Atmospheric vs oceanic data assimilation

Illustration: maps of SSH

If a user needs a time series of global maps of Sea Level pressure, what will her choice be?

An ECMWF reanalysis, probably.

If a user needs a time series of global maps of SSH, what will her choice be?

DUACS products are the most widely used by oceanographers. Until 2024, they were made from nadir altimeter data with a space-time linear interpolation.

Atmospheric vs oceanic data assimilation

Observations

Meteorology:

Oceanography:

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Atmospheric vs oceanic data assimilation

Observations

Meteorology:

- Large number of observations

Oceanography:

- Comparatively small number of observations

Atmospheric vs oceanic data assimilation

Observations

Meteorology:

- Large number of observations
- Satellite observations are 3D

Oceanography:

- Comparatively small number of observations
- Satellite observations are 2D

Atmospheric vs oceanic data assimilation

Observations

Meteorology:

- Large number of observations
- Satellite observations are 3D
- Very often, observation operators are complex

Oceanography:

- Comparatively small number of observations
- Satellite observations are 2D
- Very often, observation operators are simple

Atmospheric vs oceanic data assimilation

Dynamics and models

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Atmospheric vs oceanic data assimilation

Dynamics and models

Phenomenon	Length scale L	Velocity scale U	Time scale T
Atmosphere:			
Sea breeze	5–50 km	1–10 m/s	12 h
Mountain waves	10–100 km	1–20 m/s	Days
Weather patterns	100–5000 km	1–50 m/s	Days to weeks
Prevailing winds	Global	5–50 m/s	Seasons to years
Climatic variations	Global	1–50 m/s	Decades and beyond
Ocean:			
Internal waves	1–20 km	0.05–0.5 m/s	Minutes to hours
Coastal upwelling	1–10 km	0.1–1 m/s	Several days
Large eddies, fronts	10–200 km	0.1–1 m/s	Days to weeks
Major currents	50–500 km	0.5–2 m/s	Weeks to seasons
Large-scale gyres	Basin scale	0.01–0.1 m/s	Decades and beyond

Atmospheric vs oceanic data assimilation

Dynamics and models

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The scales particularly relevant for weather predictions and important for climate require more/finer observations in the ocean.

Phenomenon	Length scale L	Velocity scale U	Time scale T
Atmosphere:			
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Atmospheric vs oceanic data assimilation

Dynamics and models

* The scale of eddies is set by the Rossby radius of deformation:

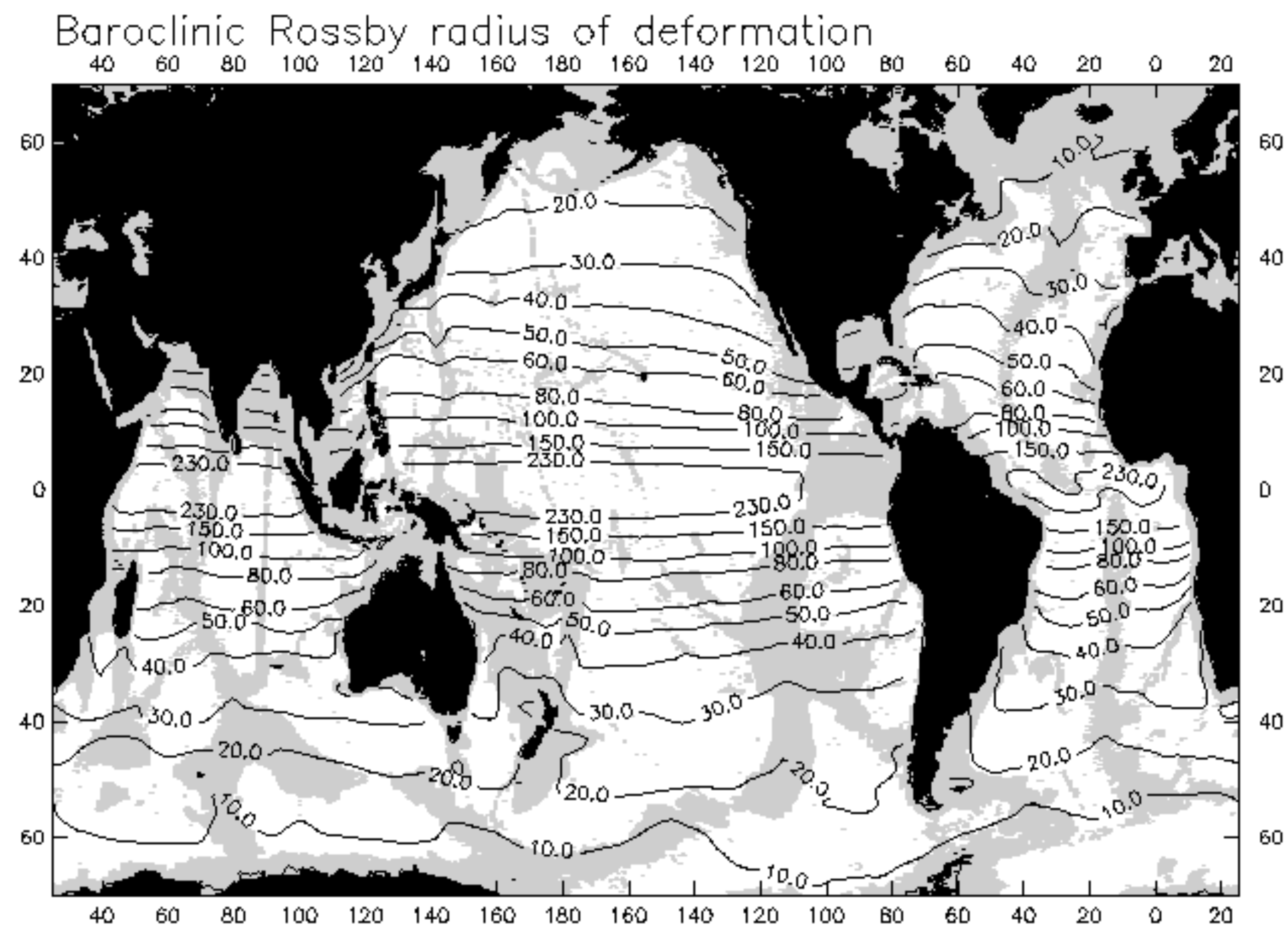
$$L_\rho = \frac{NH}{2\Omega}$$

N: Brunt-Vaïsala frequency

H: layer thickness

Ω: Earth rotation

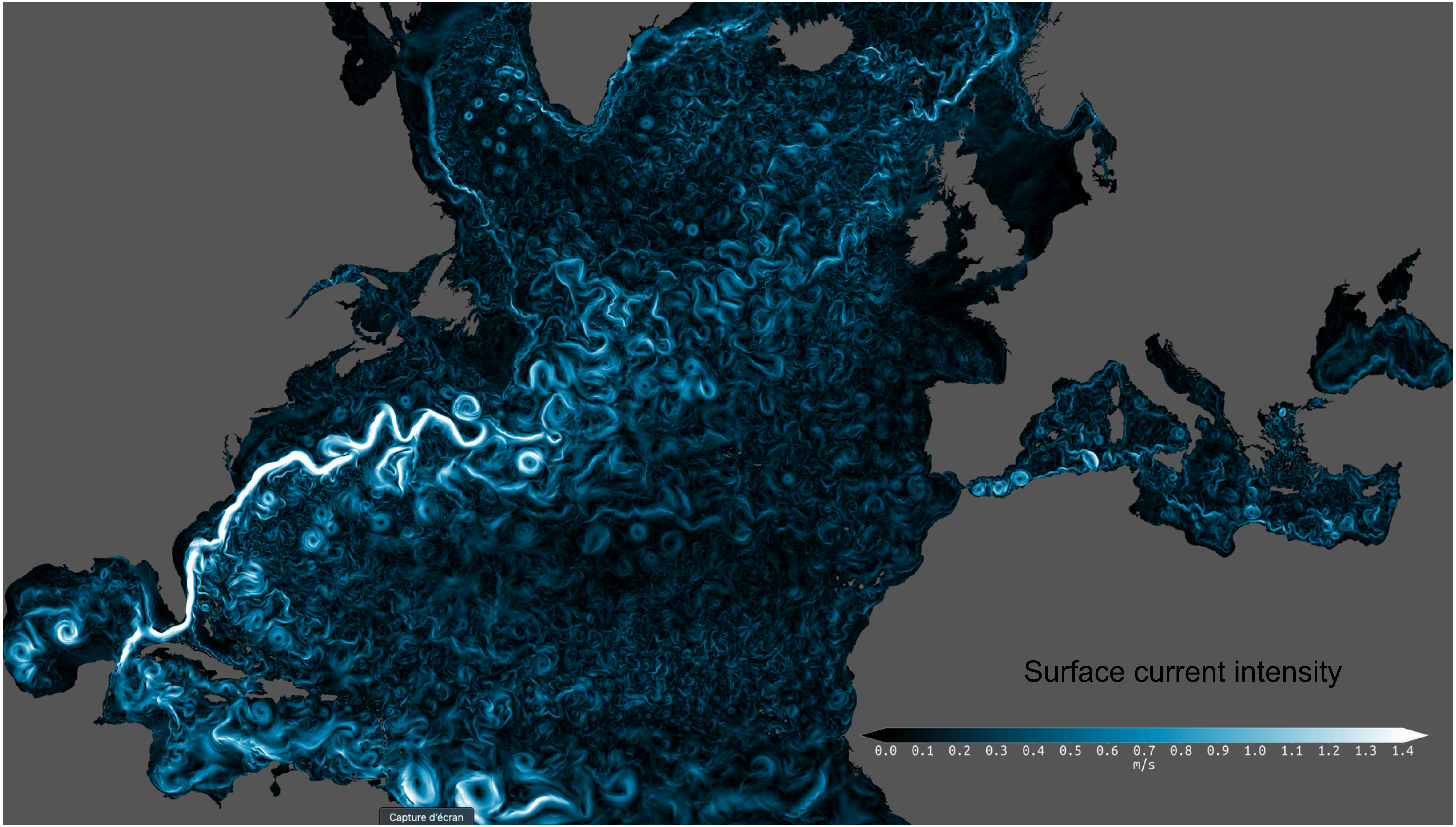
* ~30 km in the ocean, ~1000 km in the atmosphere
* Ocean weather simulations require high resolution models!



(Chelton et al, 1998)

Atmospheric vs oceanic data assimilation

Dynamics and models



https://github.com/ocean-next/eNATL60

Outline

Texte du titre

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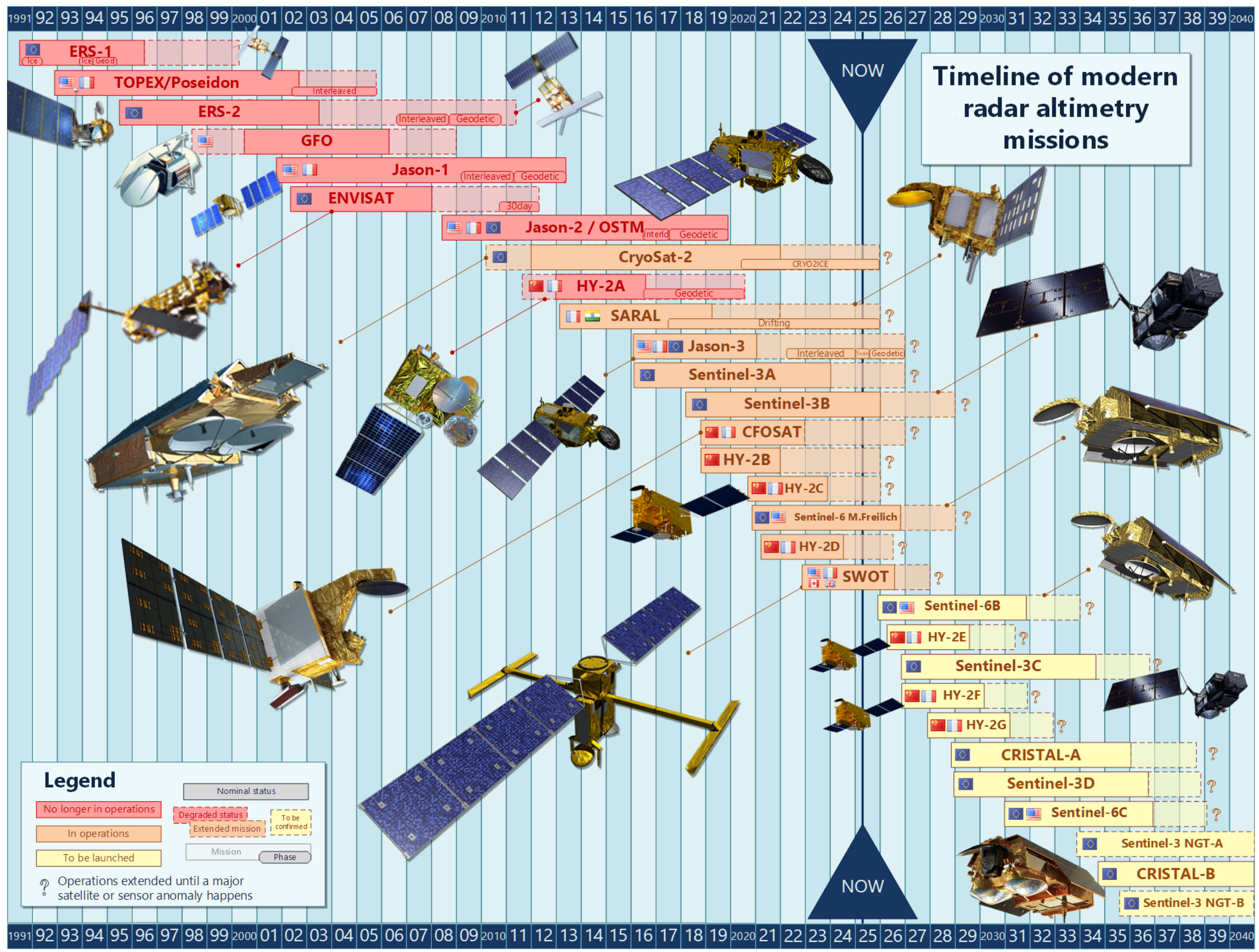
Operational oceanography

Use of data assimilation

Operational oceanography started about 25 years ago.



Operational oceanography

Use of data assimilation

The main goal is real-time monitoring and prediction of the state of the ocean, including:

- Currents (shipping, sea operations, regattas…)
- Primary production (marine resources, fishing)
- Sea ice (shipping)
- Temperature (climate, weather forecasting…)

Like weather forecast centers, OO centers provide useful information to scientists: reanalyses, targeted forecasts for field campaigns, etc.

Operational oceanography

Mercator Ocean International

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Mercator Ocean International:

- The French center of OO;
- Created in 1995;
- Located in the area of Toulouse, about 50 agents;
- officially appointed by the European Commission on 11 November, 2014 to implement and operate the Copernicus Marine Service (CMEMS).
- Development in collab with research labs
- http://www.mercator-ocean.fr/

Operational oceanography

Mercator Ocean International

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MERCATOR OCEAN INTERNATIONAL

À propos Services Océanographie opérationnelle Actualités & Agenda Ressources Presse

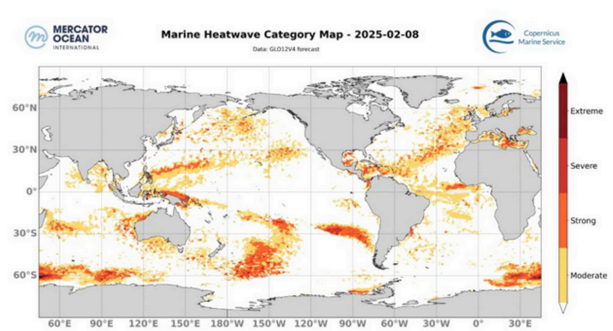
Marine heatwave forecasts – 8 February

Andreia Carvalho

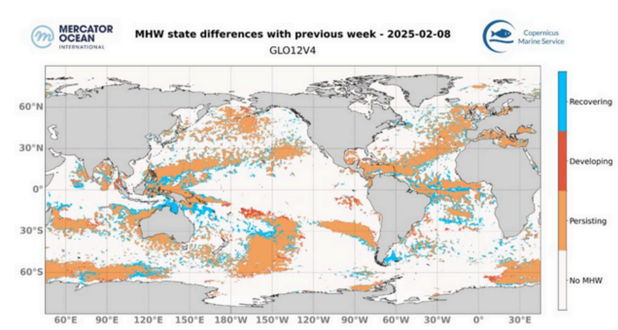
Marine Heatwave Bulletin

The marine heatwave bulletin provides forecasts and analysis of marine heatwave events across the globe and throughout the year. Used datasets include observations (satellite sea surface temperature maps) and numerical model analyses (assimilating satellite and in situ observations) to derive marine heatwave forecasts for a 10-day period. [1] **This week's forecasts were produced using as a comparison the marine heatwave situation on 28/01/2024.**

Forecasts for 8 February



Marine heatwave categories for 8 February 2025 (global ocean). GLO12. Source: Mercator Ocean International



Category and geographical extent differences for 8 February 2025 (global ocean). GLO12. Source: Mercator Ocean International

Rechercher

Rechercher...

Catégories

Sélectionner une catégorie

Étiquettes

2D3D 7e continent climat CMEMS CMEMS workshop CNES CNRS Connaissance de l'environnement océanique Copernicus copernicus marine Copernicus Marine Service Croissance Bleue Decade Collaborative Centre Digital Twin Ocean DTO EU Green week Global Ocean Week 2016 GOW LEGOS Livre marée noire Météo France NAOS Ocean Day Ocean Decade ocean governance Ocean prediction Ocean Science océan digital Partenariat Pierre Bahurel platique pollution Press 2015 RUTW satellite satellites Sea Plastics Sentinel Sentinel-3A Toulouse UN Ocean Decade Usine Nouvelle wakashio Workshop Copernicus

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Outline

Texte du titre

Ocean models

Primitive equations

Conservation of:

$$\frac{\partial u}{\partial t} + \boxed{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}} = fv - \frac{1}{\rho}\frac{\partial p}{\partial x} + K_u\frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial v}{\partial t} + \boxed{u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}} = -fu - \frac{1}{\rho}\frac{\partial p}{\partial y} + K_v\frac{\partial^2 v}{\partial z^2}$$

- momentum

Nonlinear terms

$$-\frac{\partial p}{\partial z} = \rho g$$

$$\mathrm{div}\ \vec{u} = 0$$

- Mass

$$\rho\frac{DS}{Dt} = \mathrm{div}\ (K_S \mathrm{grad}\ S)$$

- Salt

$$\rho C_v\frac{DT}{Dt} = \mathrm{div}\ (K_T \mathrm{grad}\ T)$$

- Temperature

$$\rho = \rho(T, S, p)$$

- Equation of state

\+ auxiliary conditions

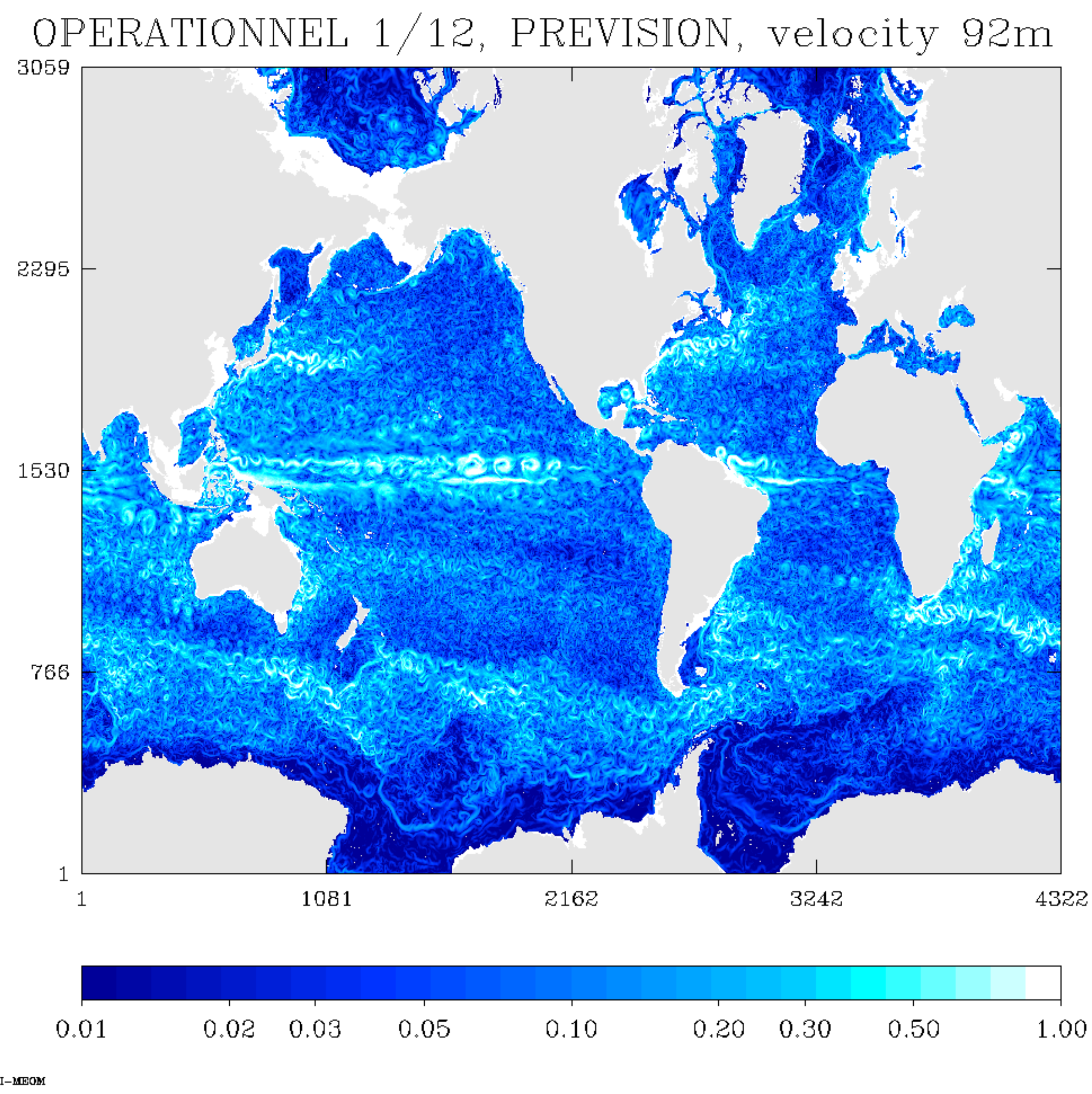
Ocean models

Discretization

- Mercator operational model: NEMO 1/12°
- Number of gridpoints:

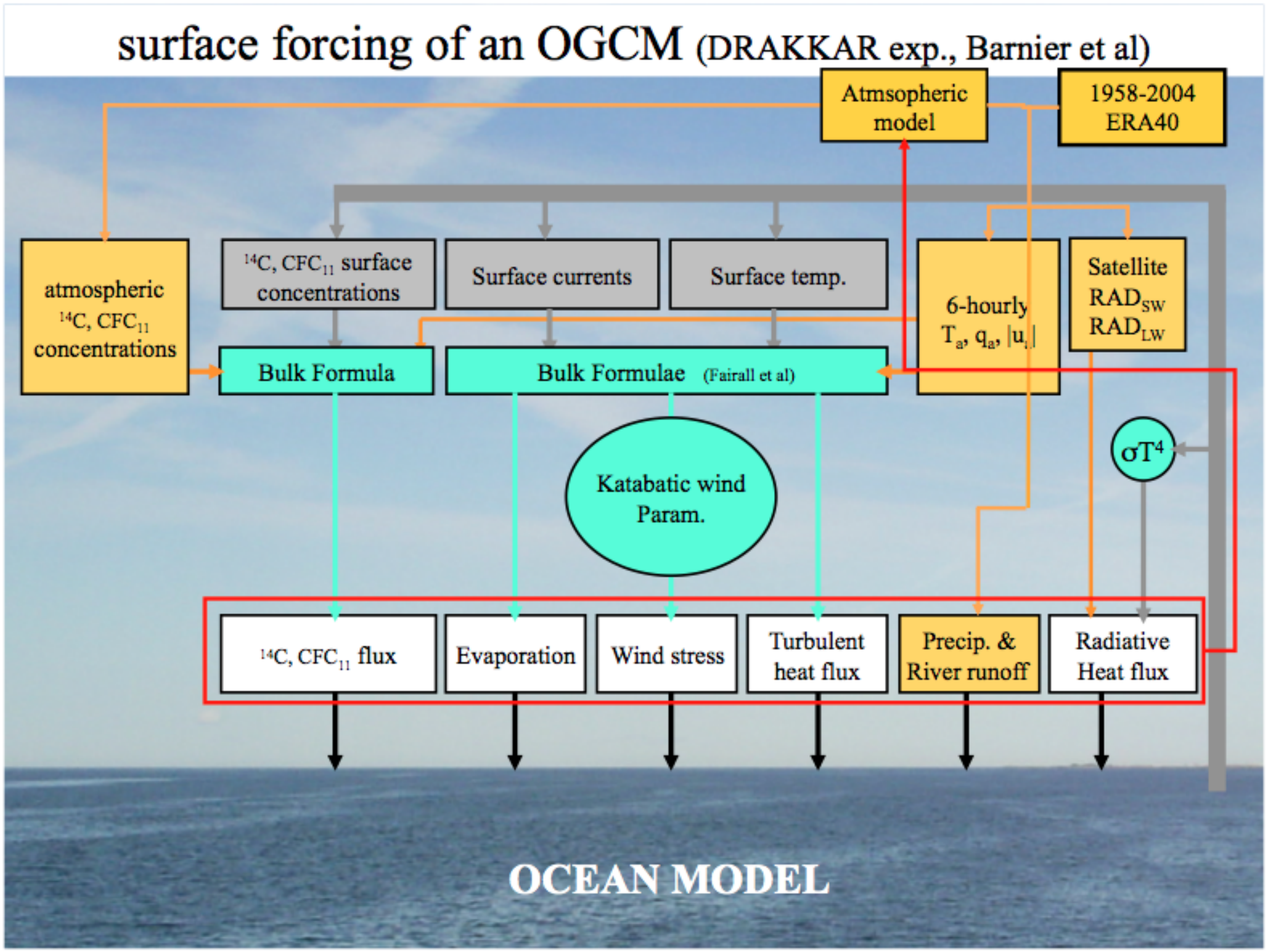
$$4322 \times 3059 \times 75 \sim 10^9$$

- 1 year of simulation costs 414 Gb memory, 90000 CPU hours, 1Tb storage (daily outputs)



Ocean models

Uncertainties: example of forcing conditions



Yellow: atmospheric

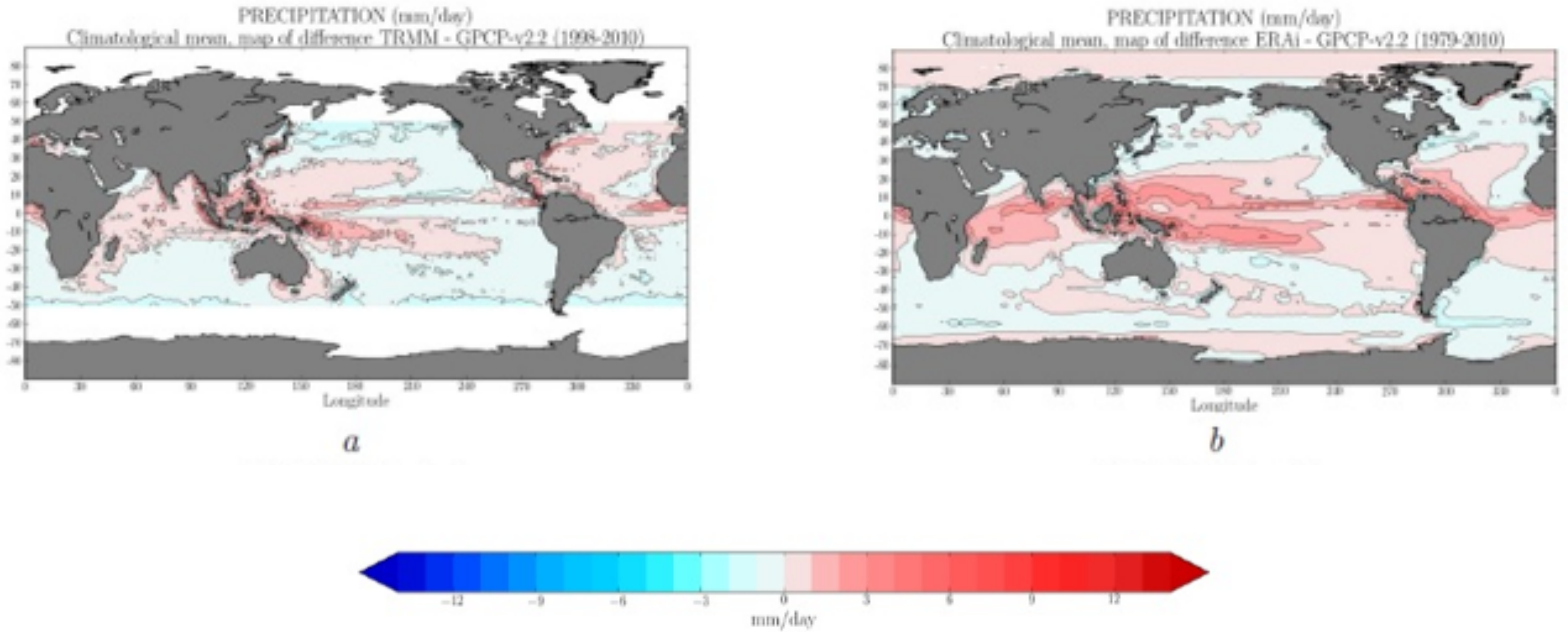
Grey: oceanic

Green: parameterizations

White: physical processes

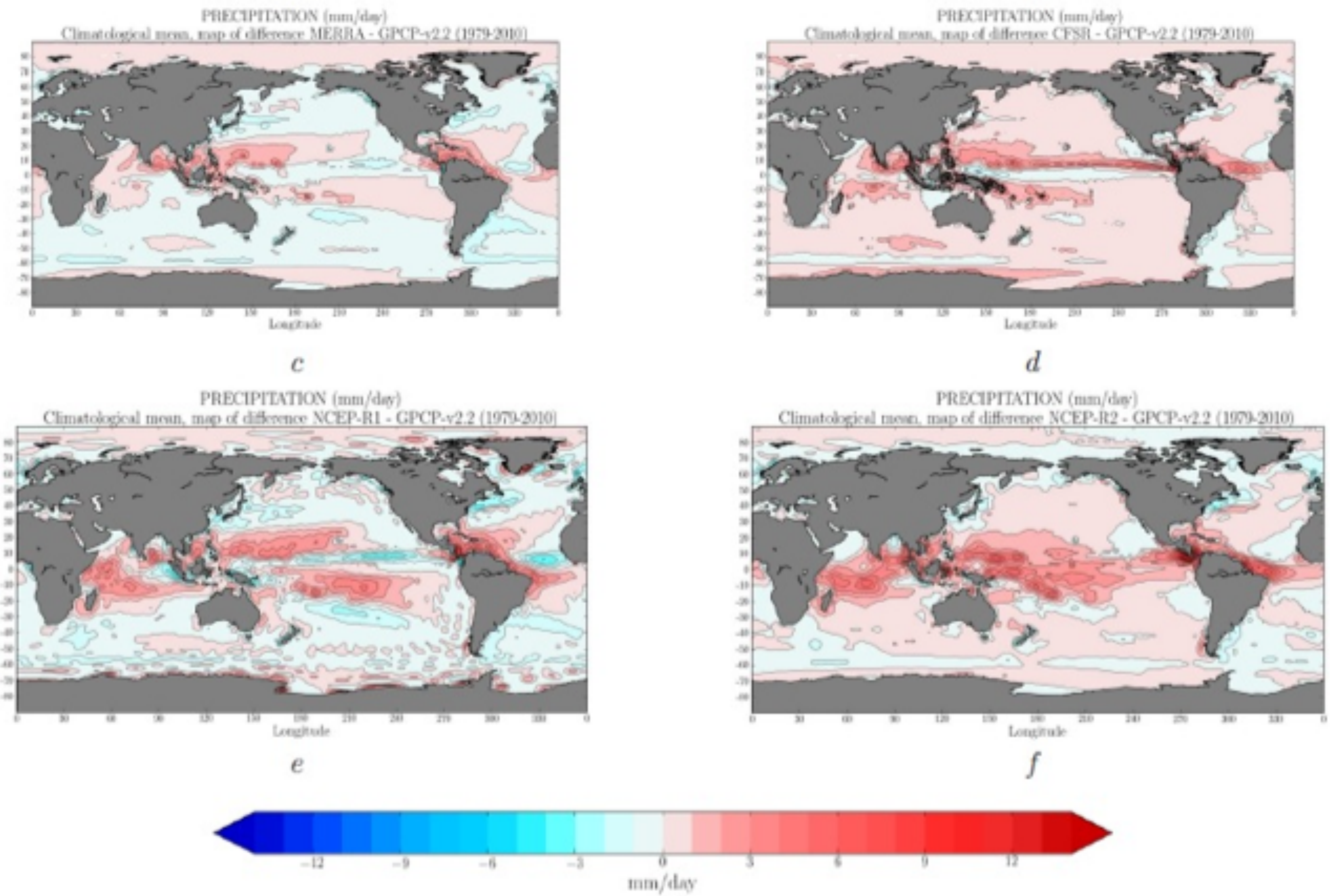
Ocean models

Uncertainties: example of forcing conditions



Ocean models

Uncertainties: example of forcing conditions



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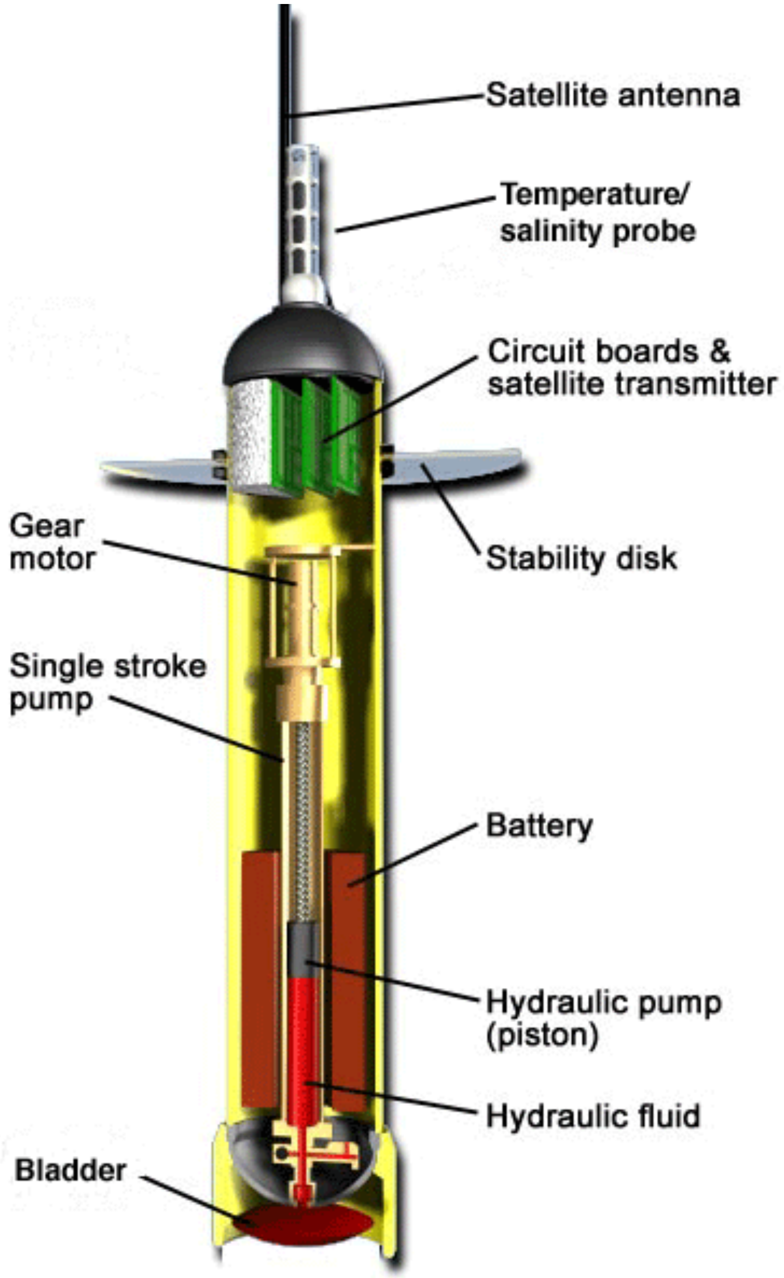
Observations of the ocean

In situ observation #1: profilers



http://www.argo.ucsd.edu/

Observations of the ocean

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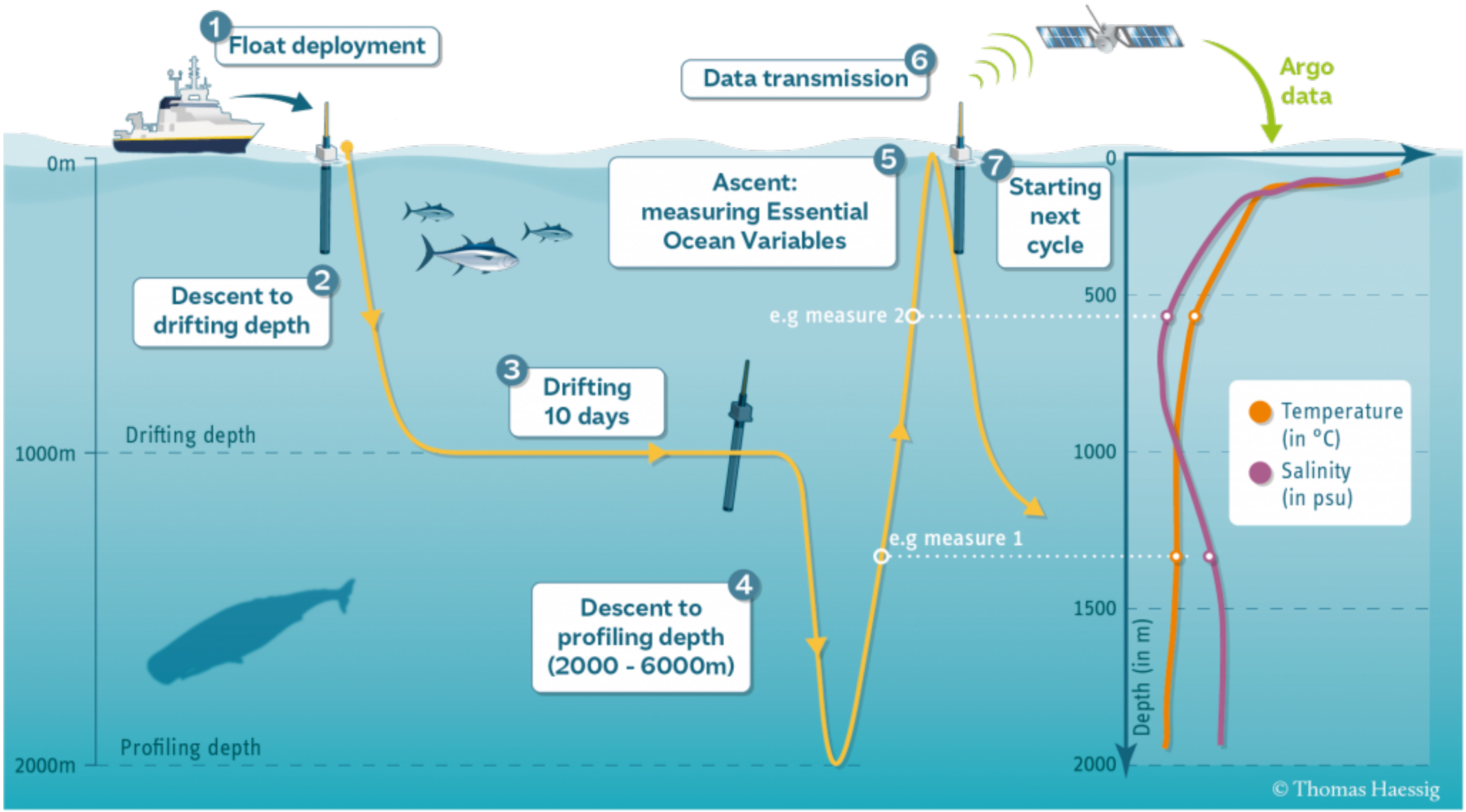


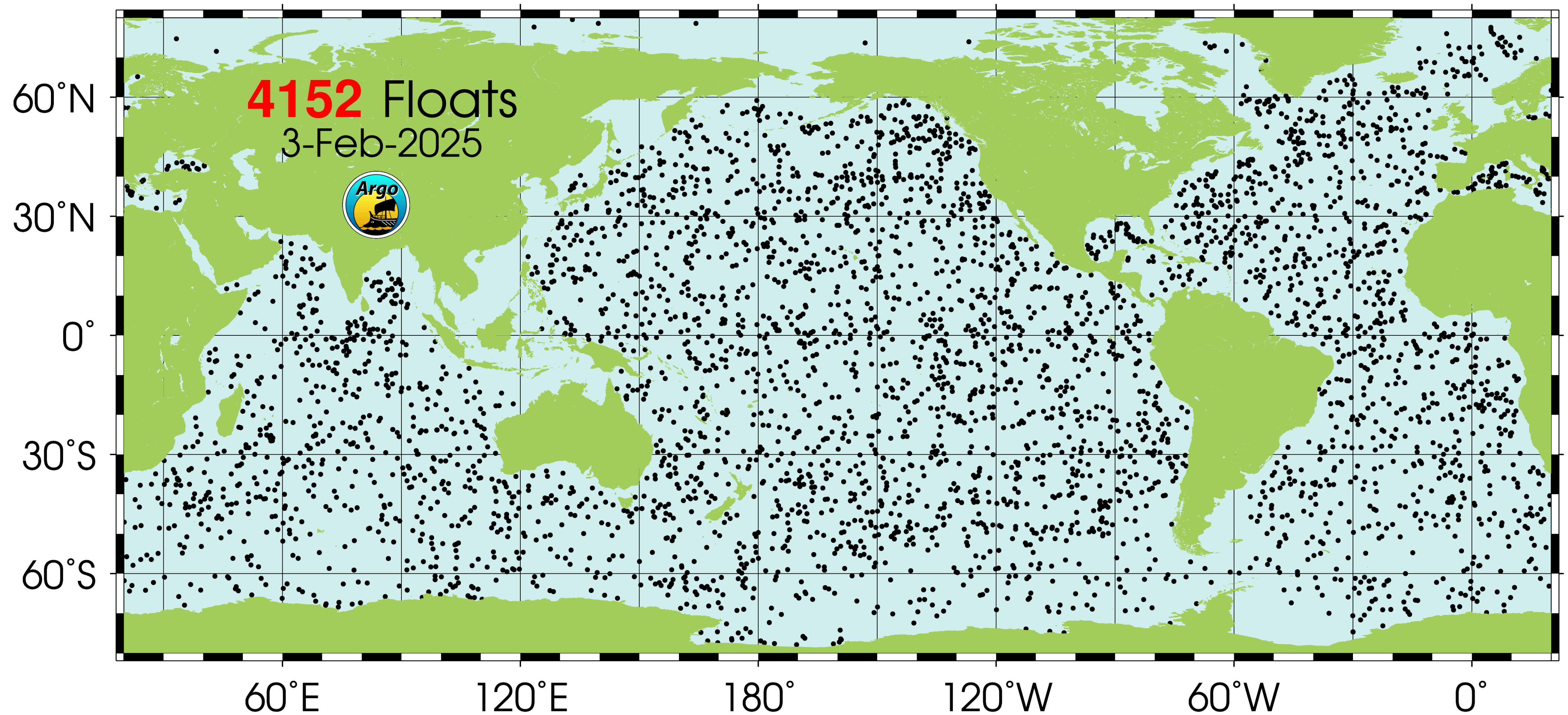
ARGO = network of profiling floats

https://argo.ucsd.edu/

Observations of the ocean

In situ observation #1: profilers

- +++ : Spatial coverage, vertical information, autonomy
- - - - : needs maintenance, some regions hard to sample, poor sampling

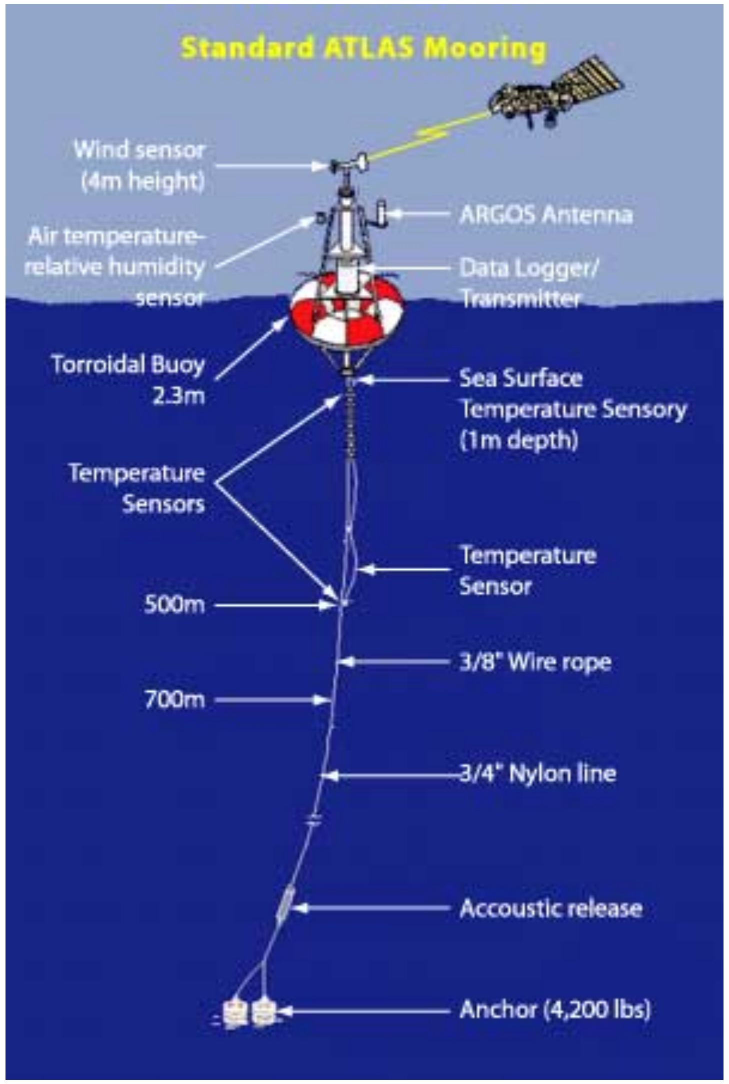


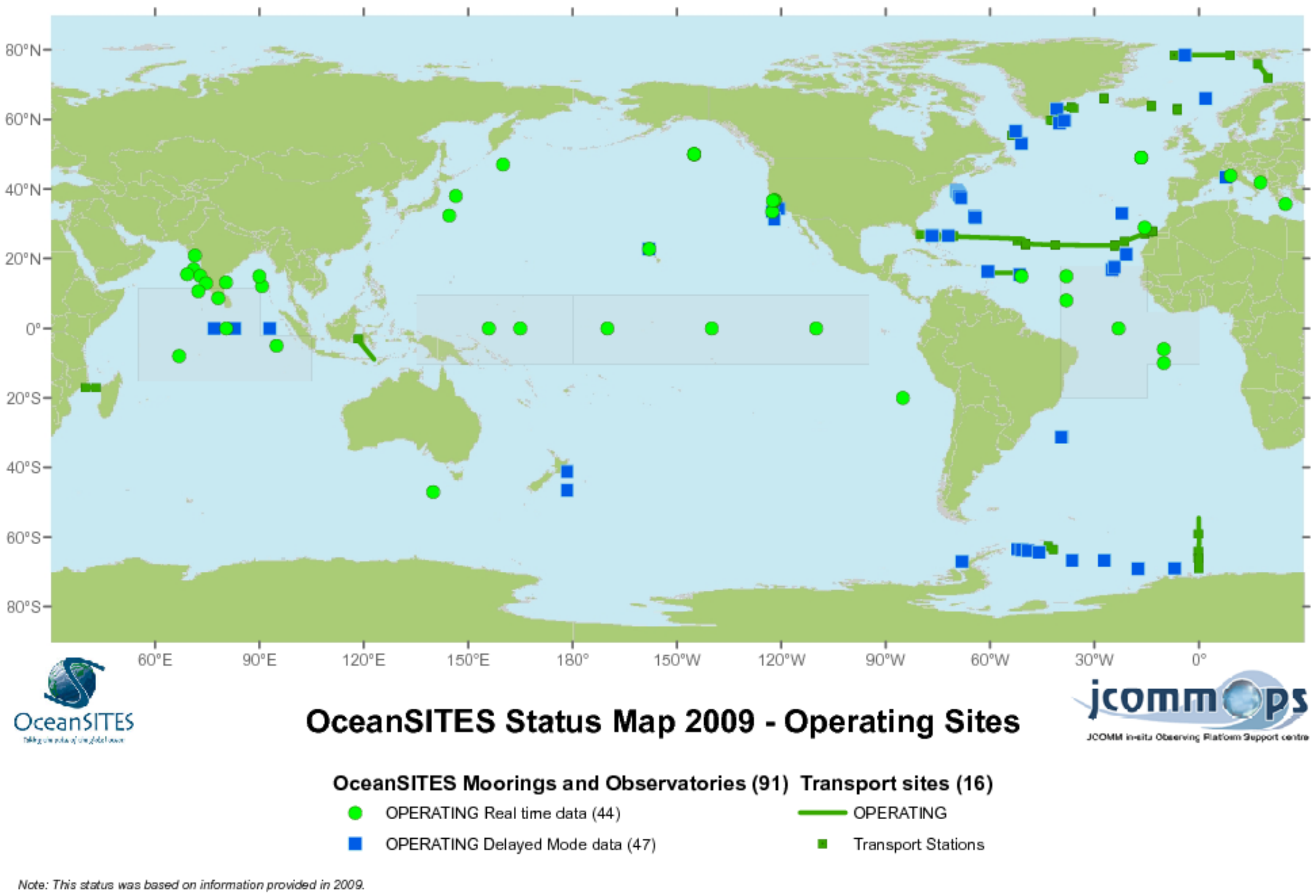
http://www.argo.ucsd.edu/

Observations of the ocean

In situ observation #2: Moorings

- +++ : time sampling, vertical information, autonomy
- - - - : expensive to build and maintain, poor spatial coverage





http://www.whoi.edu/virtual/oceansites/network/index.html

Observations of the ocean

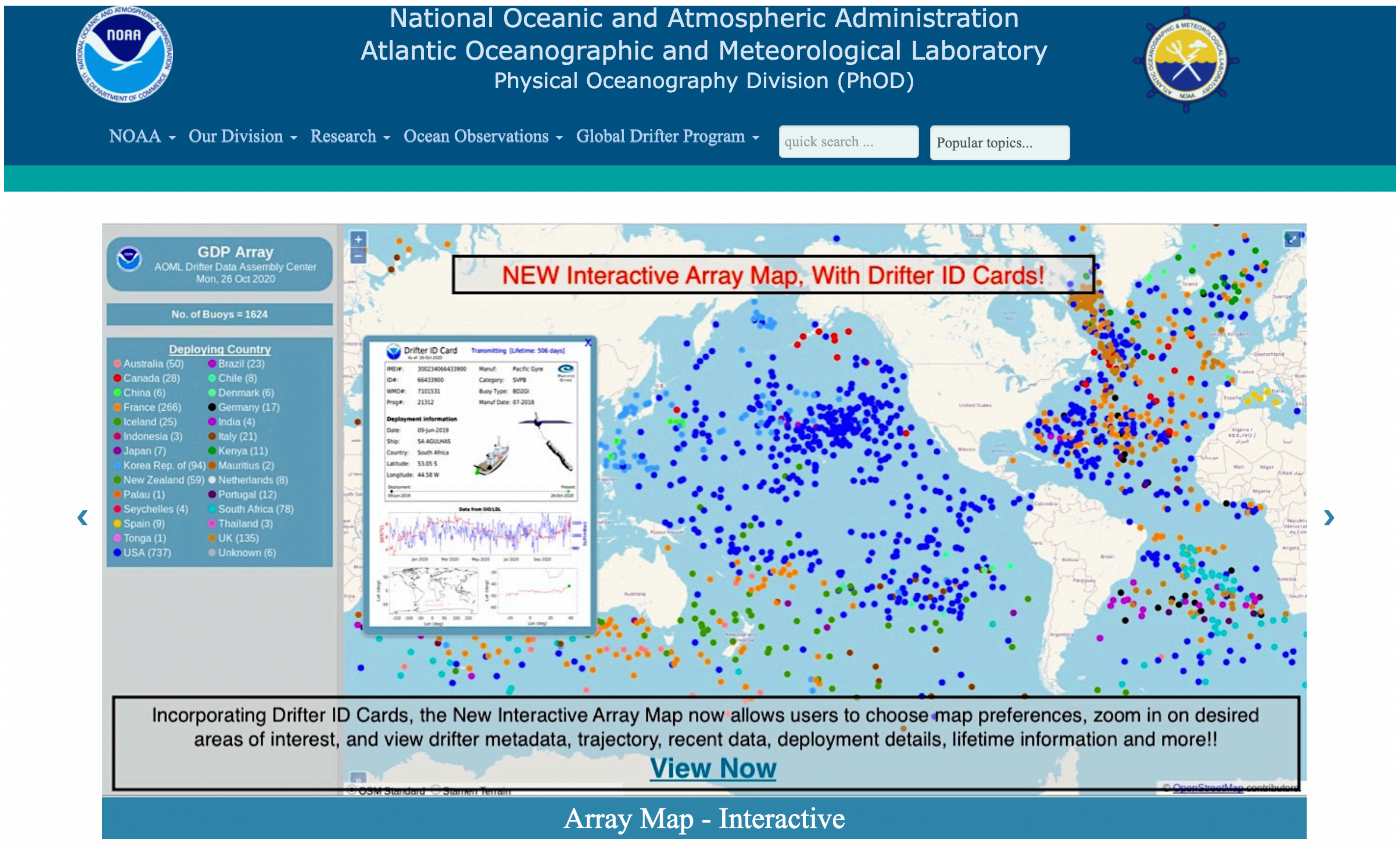
In situ observation #3: surface drifters

- +++ : Spatial coverage, autonomy
- - - - : needs maintenance, some regions hard to sample, poor sampling

A drifter measures surface temperature, pressure, currents.

Observations of the ocean

In situ observation #3: surface drifters



In fast development with the Global Drifter Program

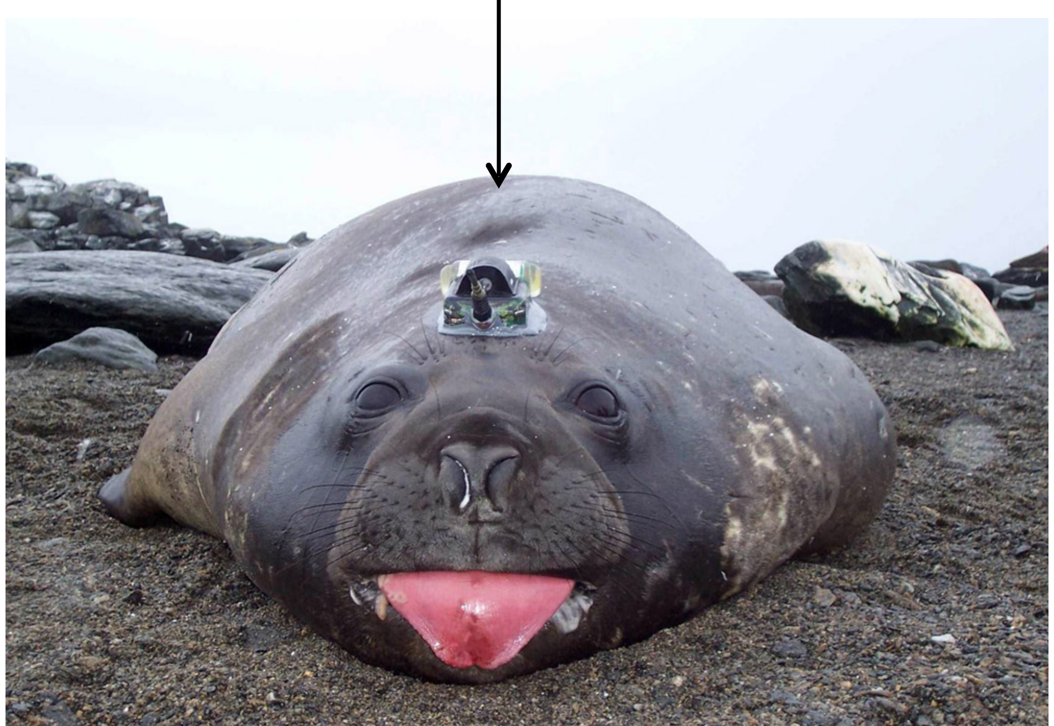
https://www.aoml.noaa.gov/phod/gdp/index.php

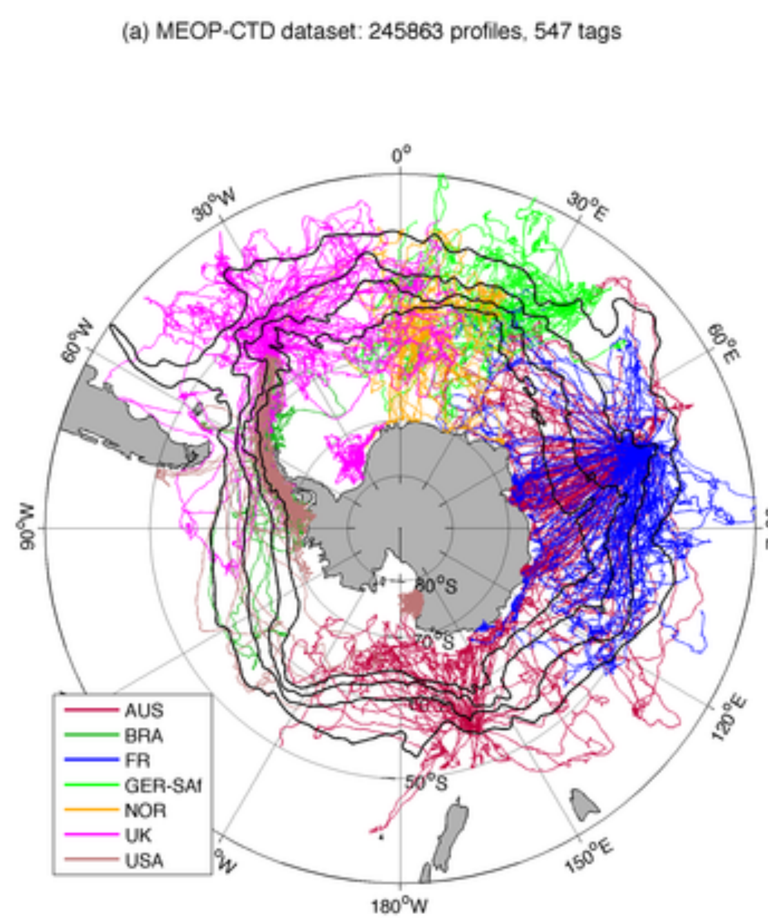
Observations of the ocean

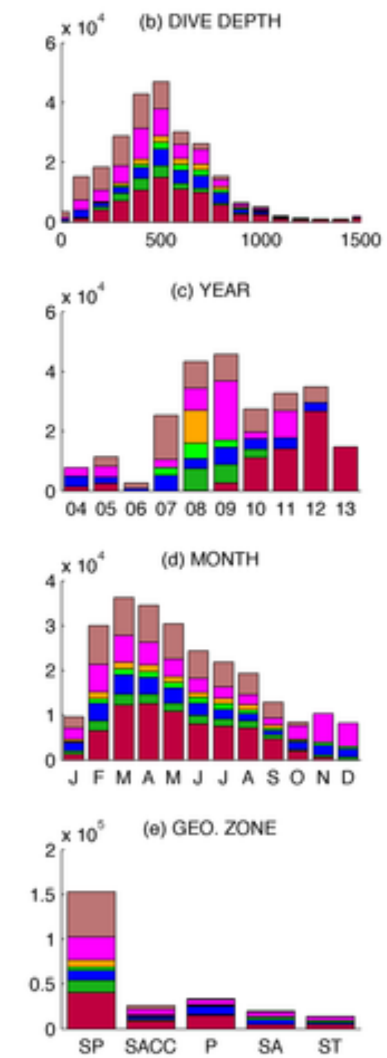
In situ observation #4: marine mammals

- +++: access to poorly observed area, vertical information
- - - - : limited spatial and temporal coverage

A miniaturized CTD (Conductivity-Temperature-Depth) probe



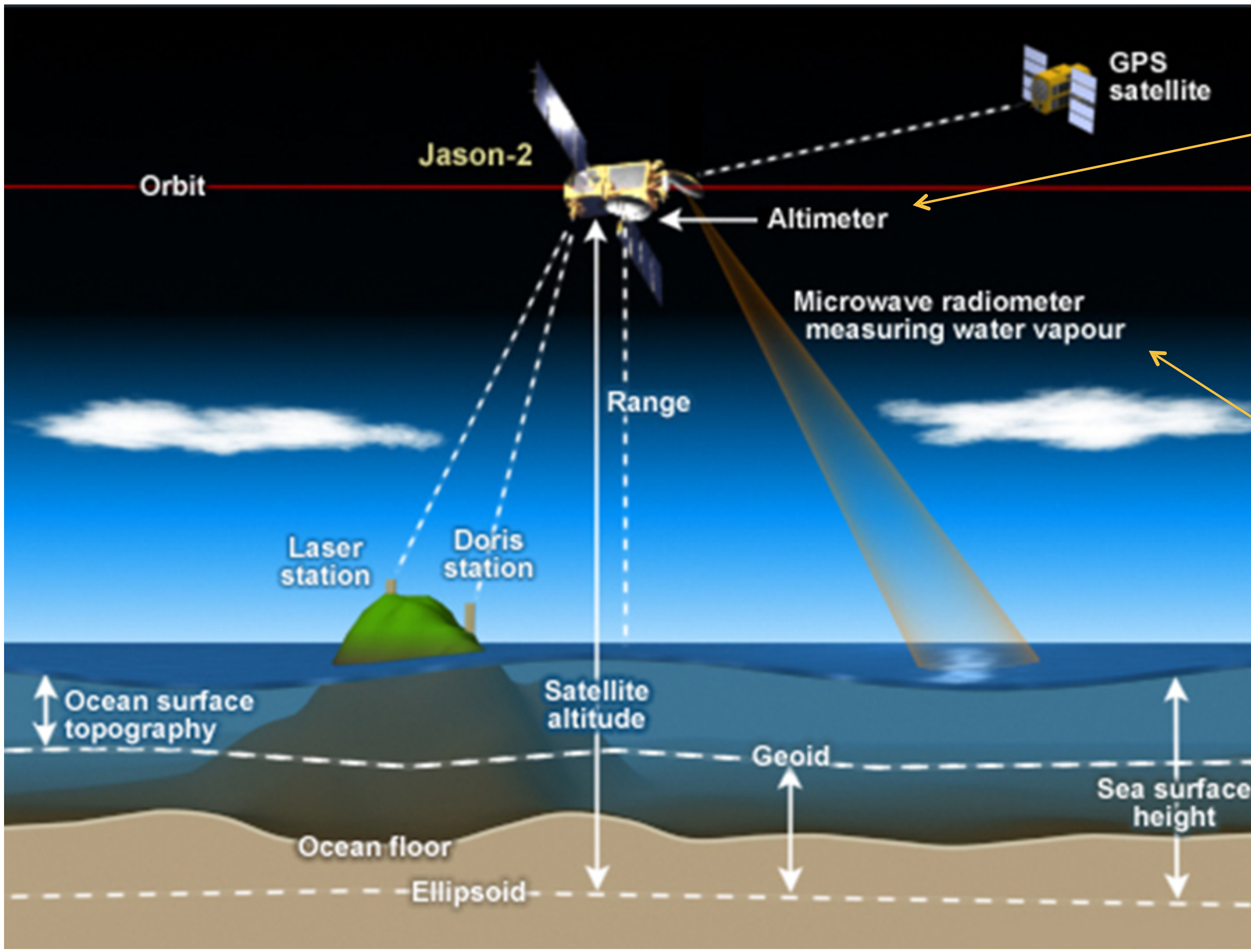




Sample poorly observed areas!

Observations of the ocean

Satellite observation #1: altimetry



Radar altimeter (emitter & antenna)

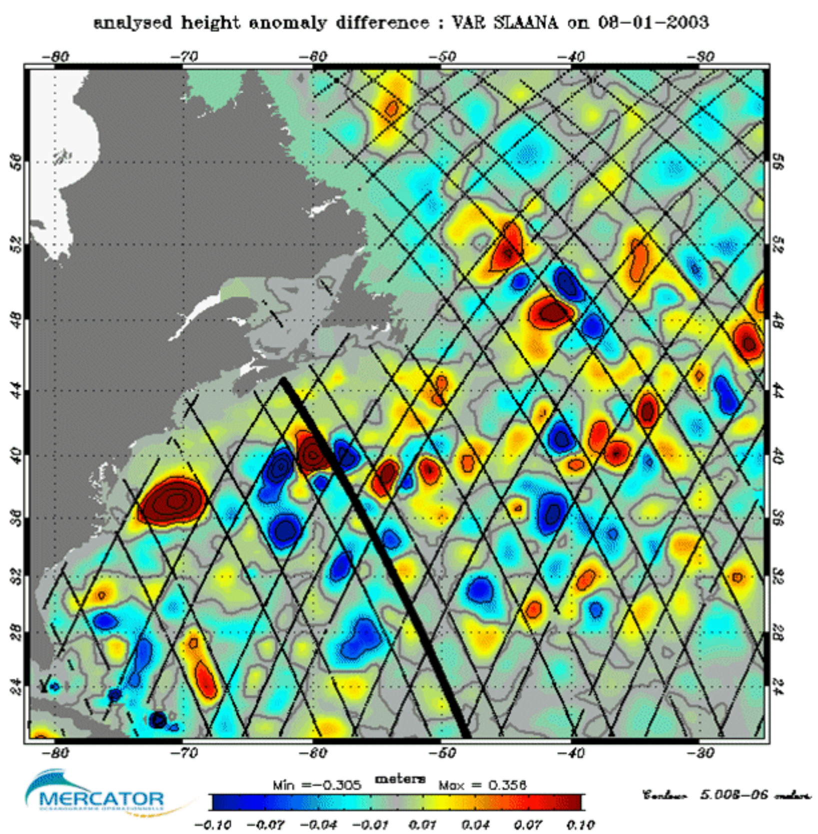
For atmospheric corrections

Height of the satellite: ~1340 km

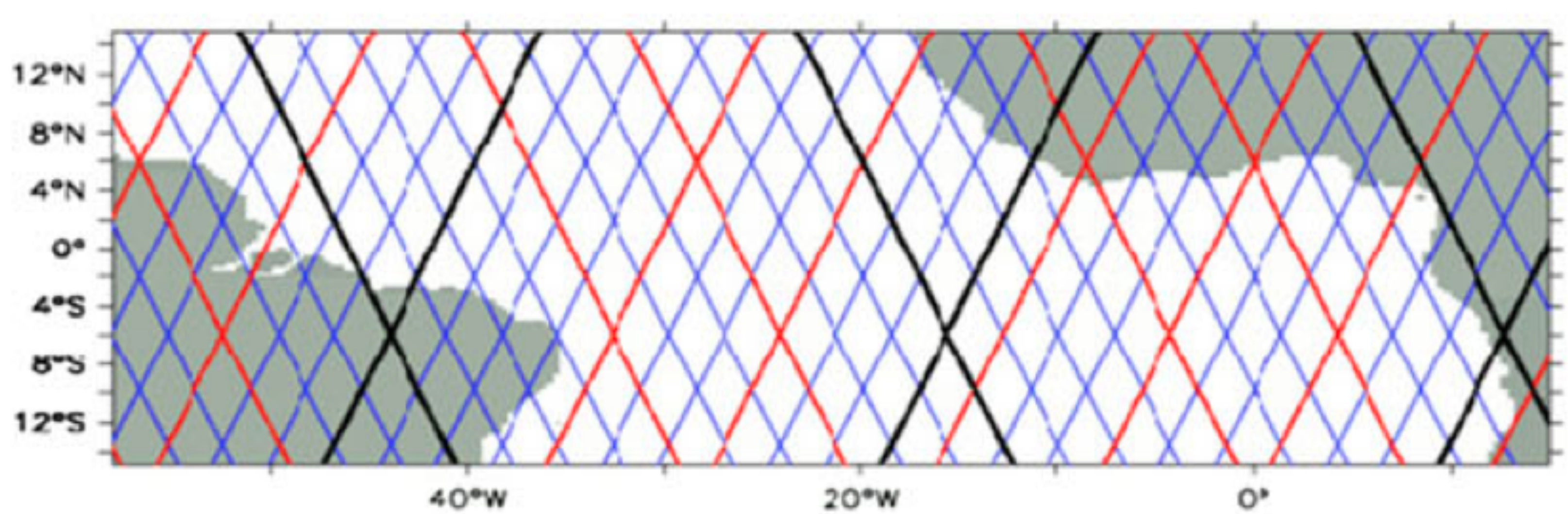
https://www.aviso.altimetry.fr/en/home.html

Observations of the ocean

Satellite observation #1: altimetry



Orbit of Jason: Cycle of 10 days.



Orbit-1 (Jason)

H=1336km i=66°

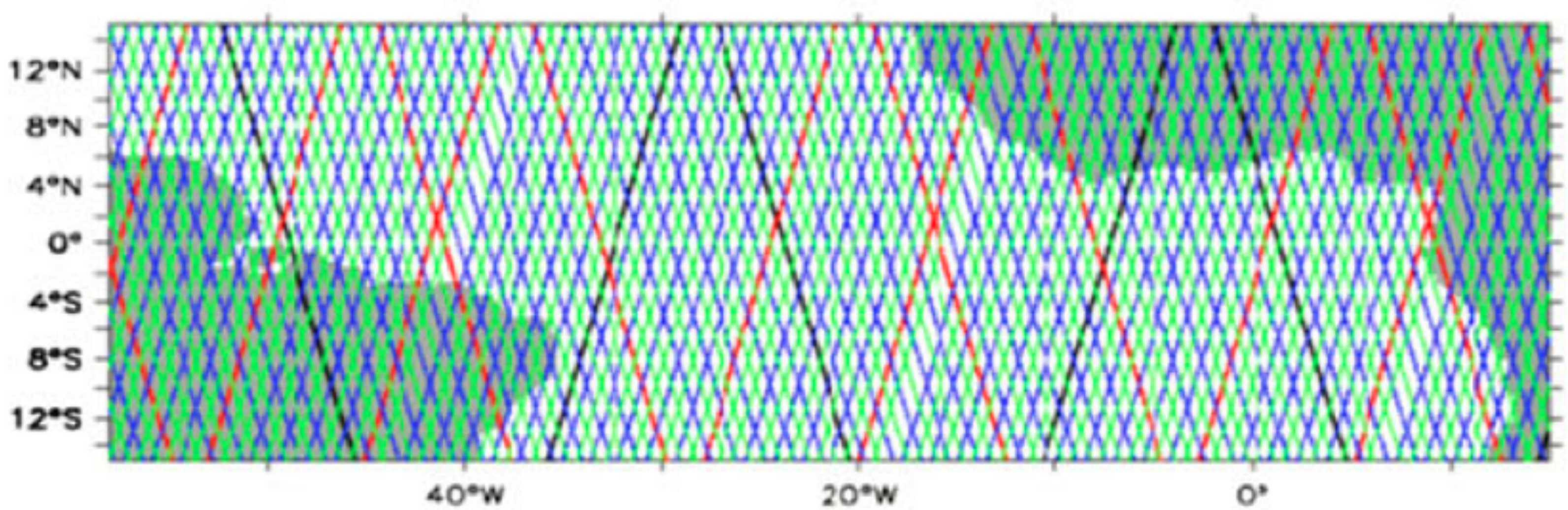
(sub-)cycles (days) : **0.9** 3.3 9.9

Observations of the ocean

Satellite observation #1: altimetry

Orbit of Envisat and Saral:

Cycle of 35 days



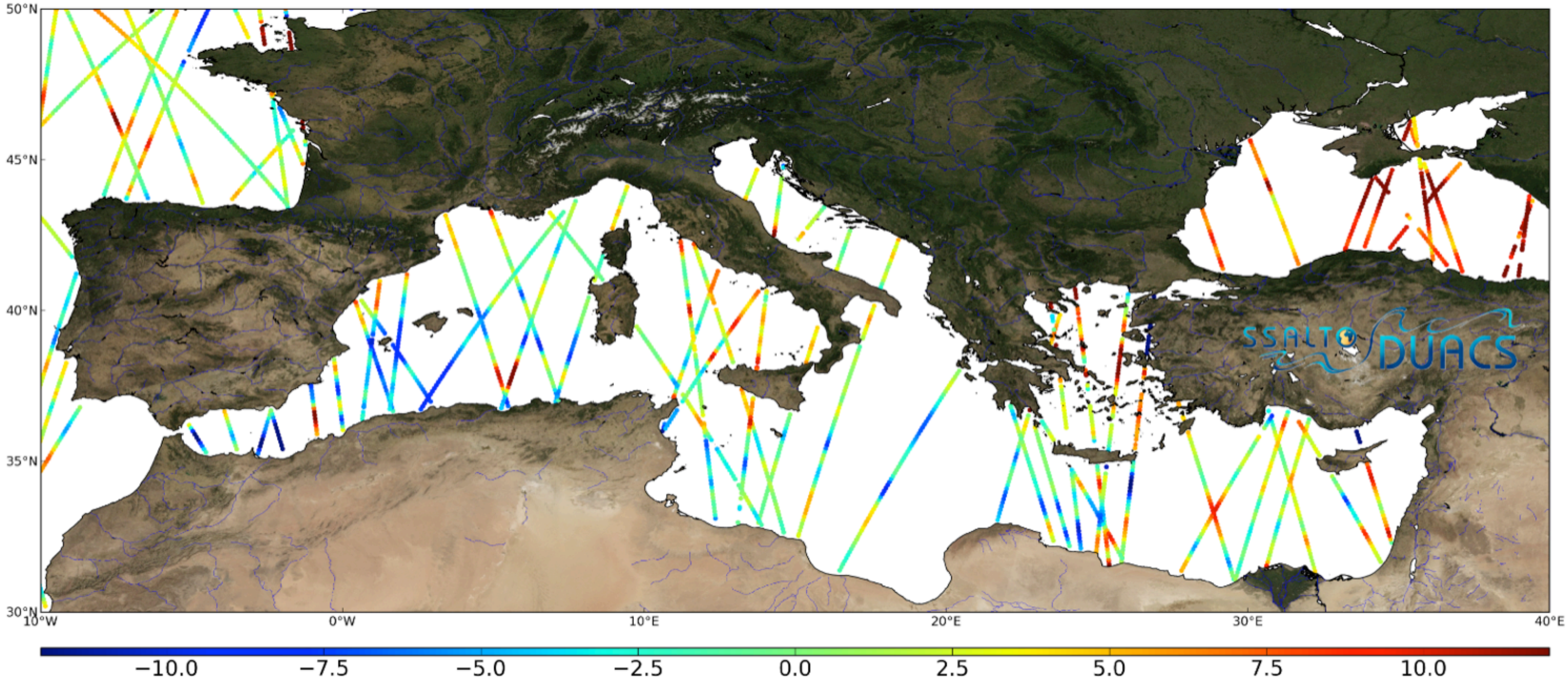
Orbit-3 (Envisat, Saral

H=782km i=98°

(sub-)cycles (days) : **1.0** **3.0** **17.5** **35.0**

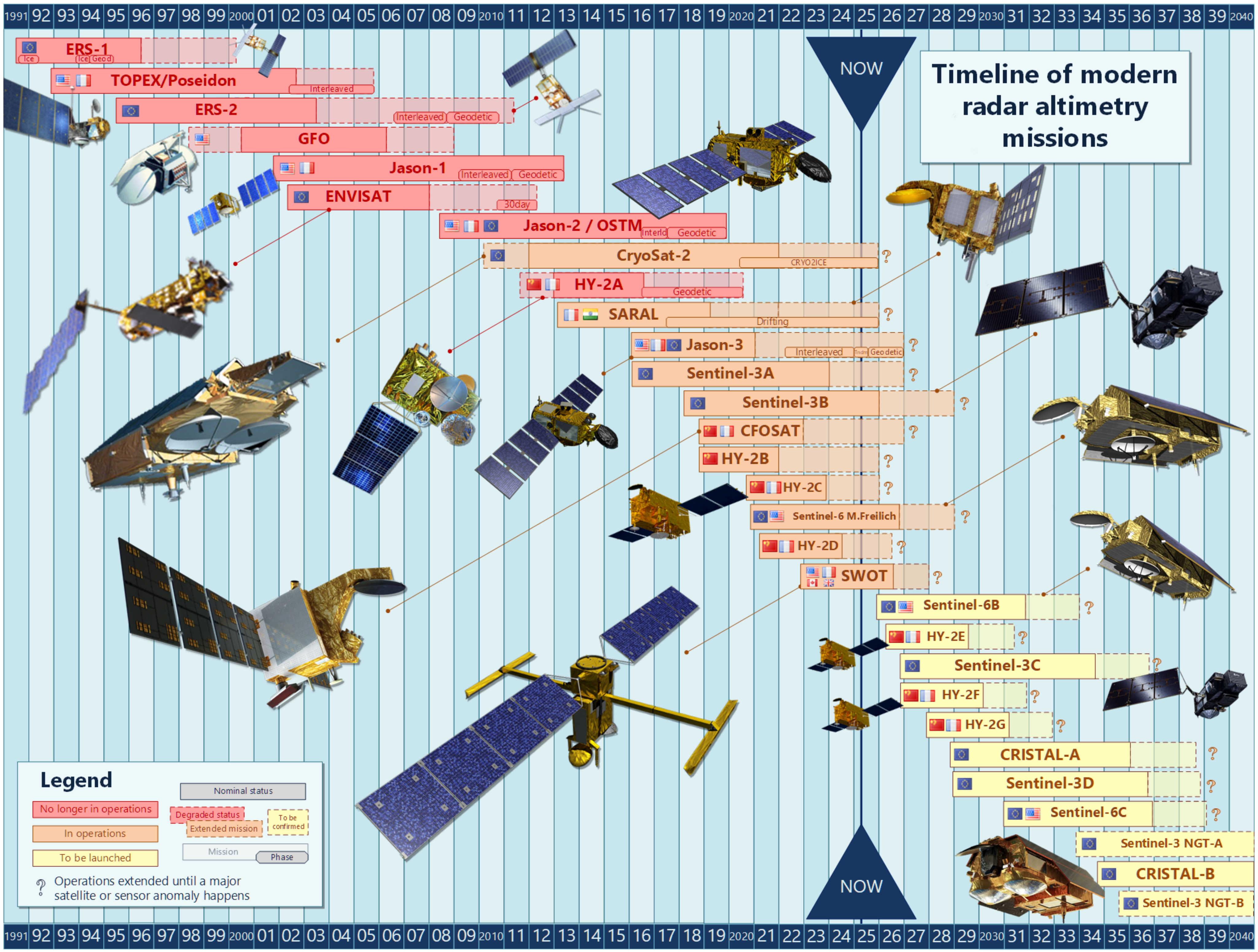
Observations of the ocean

Satellite observation #1: altimetry



Observations of the ocean

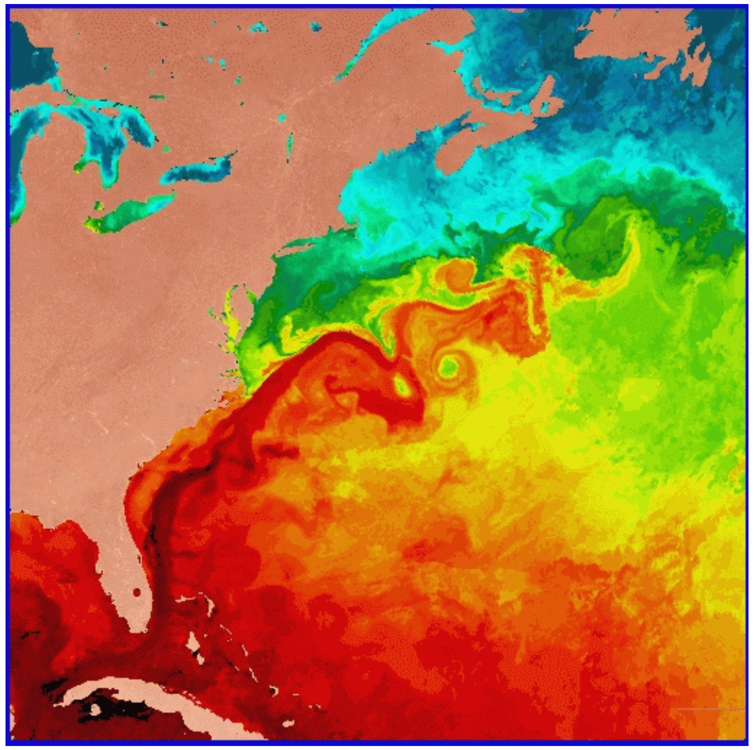
Satellite observation #1: altimetry



Observations of the ocean

Satellite observation #2: SST

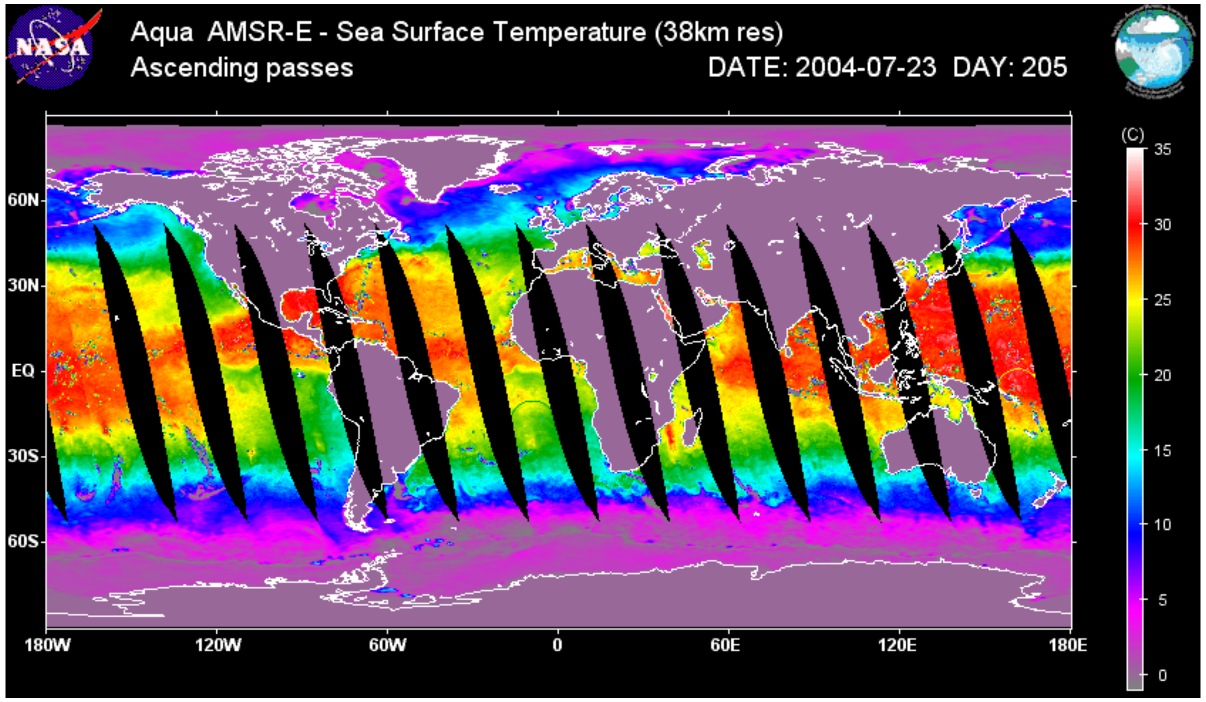
- IR radiometer (e.g. AVHRR)



- Microwave radiometer (e.g. AMSR-E)

- Both at 1-km resolution.

- MW insensitive to clouds but less sensitive and easy to calibrate.



Some IR sensors are on-board geostationary satellites (res. 5 km). Most are polar orbiting.

Observations of the ocean

Satellite observation #2: SST

Two issues with satellite SST from the DA viewpoint:

- Cloud detection
- SST is a “skin” temperature (representation error)

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    3.5. Eddy/wave separation with a 4DVar technique

EnKF implementations

Kalman filter equations

Initialization: $\mathbf{x}_0^f$ and $\mathbf{P}_0^f$

Analysis step:

$$
\begin{aligned}
\mathbf{K}_k &= (\mathbf{H}_k\mathbf{P}_k^f)^T[\mathbf{H}_k(\mathbf{H}_k\mathbf{P}_k^f)^T + \mathbf{R}_k]^{-1}, \\
\mathbf{x}_k^a &= \mathbf{x}_k^f + \mathbf{K}_k(\mathbf{y}_k^o - \mathbf{H}_k\mathbf{x}_k^f), \\
\mathbf{P}_k^a &= (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_k^f.
\end{aligned}
$$

Forecast step:

$$
\begin{aligned}
\mathbf{x}_{k+1}^f &= \mathbf{M}_{k,k+1}\mathbf{x}_k^a, \\
\mathbf{P}_{k+1}^f &= \mathbf{M}_{k,k+1}\mathbf{P}_k^a\mathbf{M}_{k,k+1}^T + \mathbf{Q}_k.
\end{aligned}
$$

EnKF implementations

Kalman filter equations

Often too big to invert

Initialization: $\mathbf{x}_0^f$ and $\mathbf{P}_0^f$

Analysis step:

Too big to store

$$\begin{aligned}
\mathbf{K}_k &= (\mathbf{H}_k\mathbf{P}_k^f)^T[\mathbf{H}_k(\mathbf{H}_k\mathbf{P}_k^f)^T + \mathbf{R}_k]^{-1}, \\
\mathbf{x}_k^a &= \mathbf{x}_k^f + \mathbf{K}_k(\mathbf{y}_k^o - \mathbf{H}_k\mathbf{x}_k^f), \\
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Forecast step:

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\mathbf{P}_{k+1}^f &= \mathbf{M}_{k,k+1}\mathbf{P}_k^a\mathbf{M}_{k,k+1}^T + \mathbf{Q}_k.
\end{aligned}$$

Rarely that simple, and unknown

Often nonlinear in practice

EnKF implementations

EnKF forecast step

* In the forecast step, each member is advanced with the numerical model:

$$\mathbf{x}^f_{k+1,i} = M_{k,k+1}(\mathbf{x}^a_{k,i}) + \eta_{k,i}$$

EnKF implementations

EnKF analysis step

- At the analysis step, each member is corrected using observations.
- Different analysis schemes exist:
  - stochastic/deterministic,
  - algebra in observation/ensemble space,
  - Serial/batch processing of observations,
  - With/without adaptive scheme at some point,
  - etc

EnKF implementations

Flavors of EnKF: illustration

Institut des géosciences de l'environnement

Deliverable 3.1

SANGOMA

Contents

SANGOMA European project, http://www.data-assimilation.net/)

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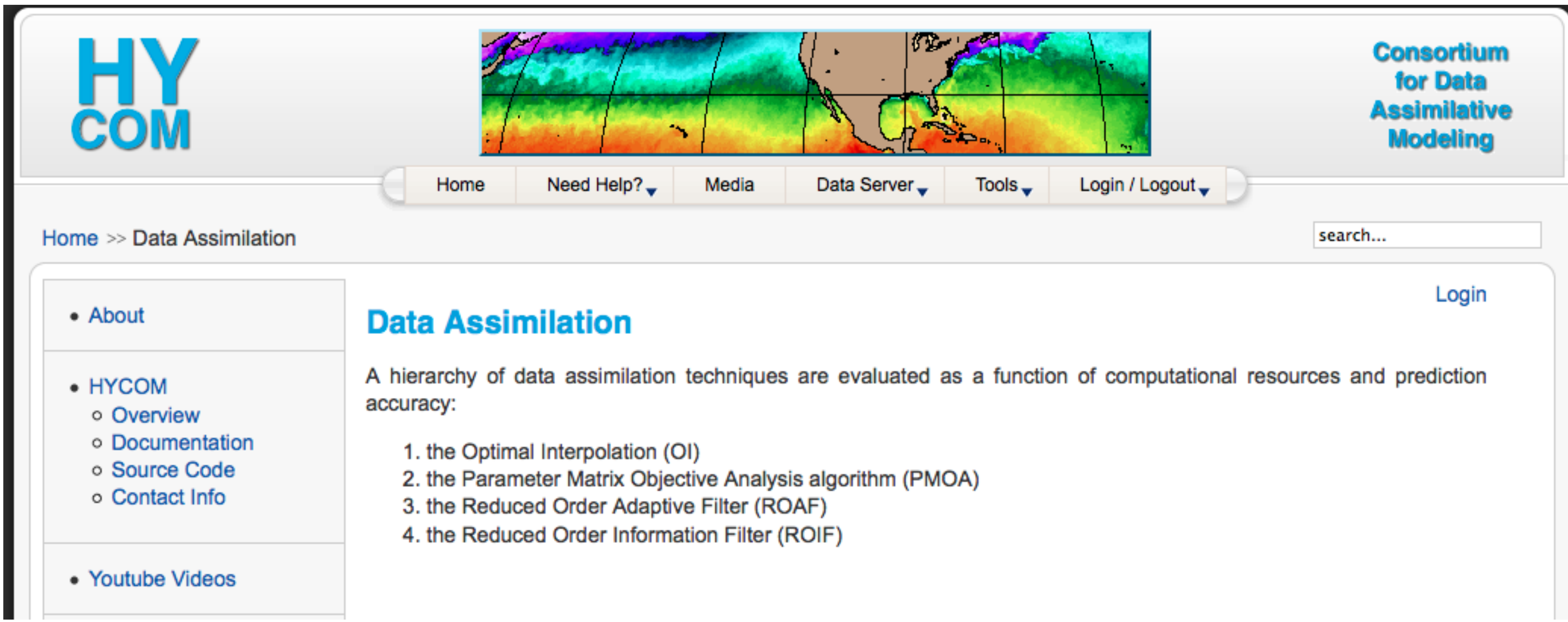
EnKF implementations

Flavors of EnKF: illustration



http://hycom.org/

EnKF implementations

Flavors of EnKF: A simple view

- OI methods
  - Forecast of 1 (mean) state
  - Analysis using statistics from a fixed ensemble
- Stochastic EnKF
  - Correction of each state with perturbed observations
- Deterministic EnKFs
  - Correction of mean and anomalies without perturbing observations

EnKF implementations

Flavors of EnKF: A simple view

- Ocean DA: O(10^6 - 10^8) variables, O(10^3 - 10^5) obs.
- Ensemble Kalman filters used in operational oceanic DA systems:
    - Ensemble OI (Mercator-Océan, France; Bureau of Meteorology, Australia; and others)
    - Deterministic EnKF (NERSC, Norway)

EnKF implementations

Flavors of EnKF: A simple view

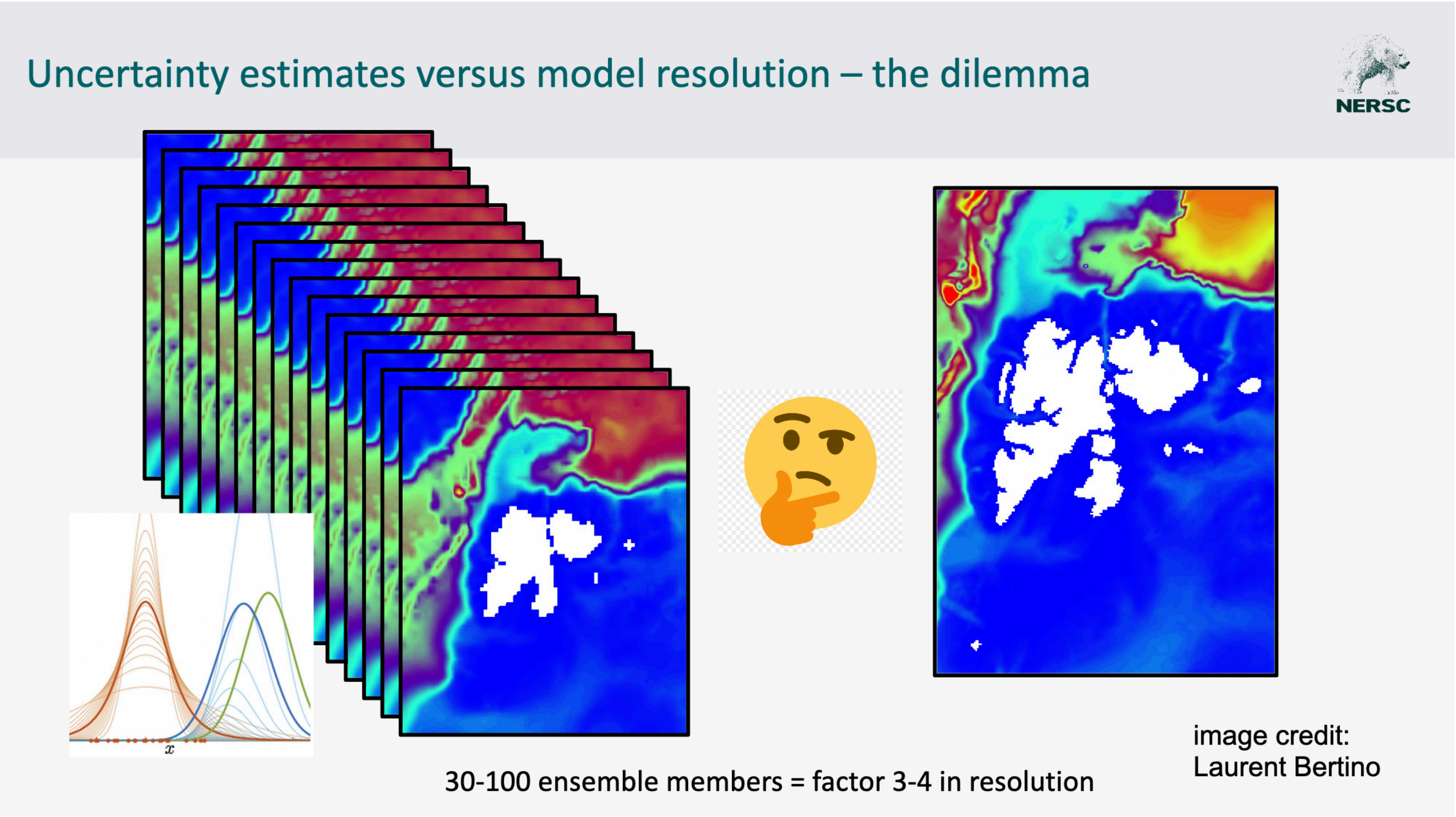


Figure from Michael Ying, NERSC

EnKF implementations

Ensemble Optimal Interpolation

- Ensemble OI:
  - Only a mean state is propagated with the model;
  - The error modes (ensemble anomalies) are the same at all analysis steps.
- - - - : no estimation of uncertainties;
- +++: computationally affordable, robust (no collapse), more “physically-based” than historical OI with analytical covariance functions.

EnKF implementations

Localization

- Localization aims at delimiting in space the impact of an observation;
- Localization is necessary for several reasons:
  - To avoid long-range corrections due to spurious long-range correlations, themselves due to the small size of the ensemble;
  - To artificially increase the rank of the covariance matrix and provide more degrees of freedom to the corrections;
  - To make computation possible in some cases.

EnKF implementations

Localization

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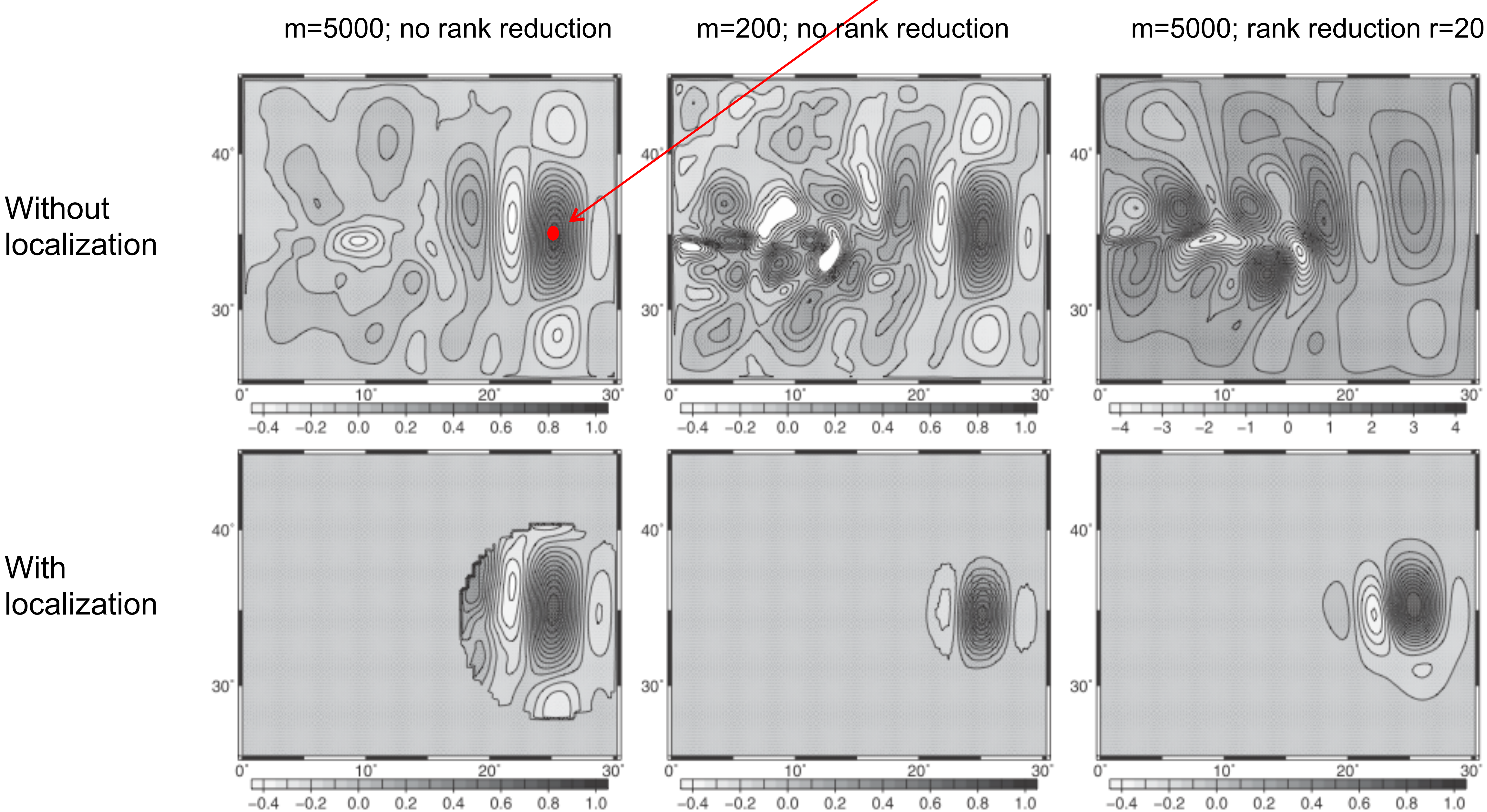
  - To make computation possible in some cases.

Short illustration of this, today

EnKF implementations
Localization

Increments in SSH due to an observation here



m=200; no rank reduction
Awkward localization

m=200; no rank reduction

m=5000; rank reduction r=20

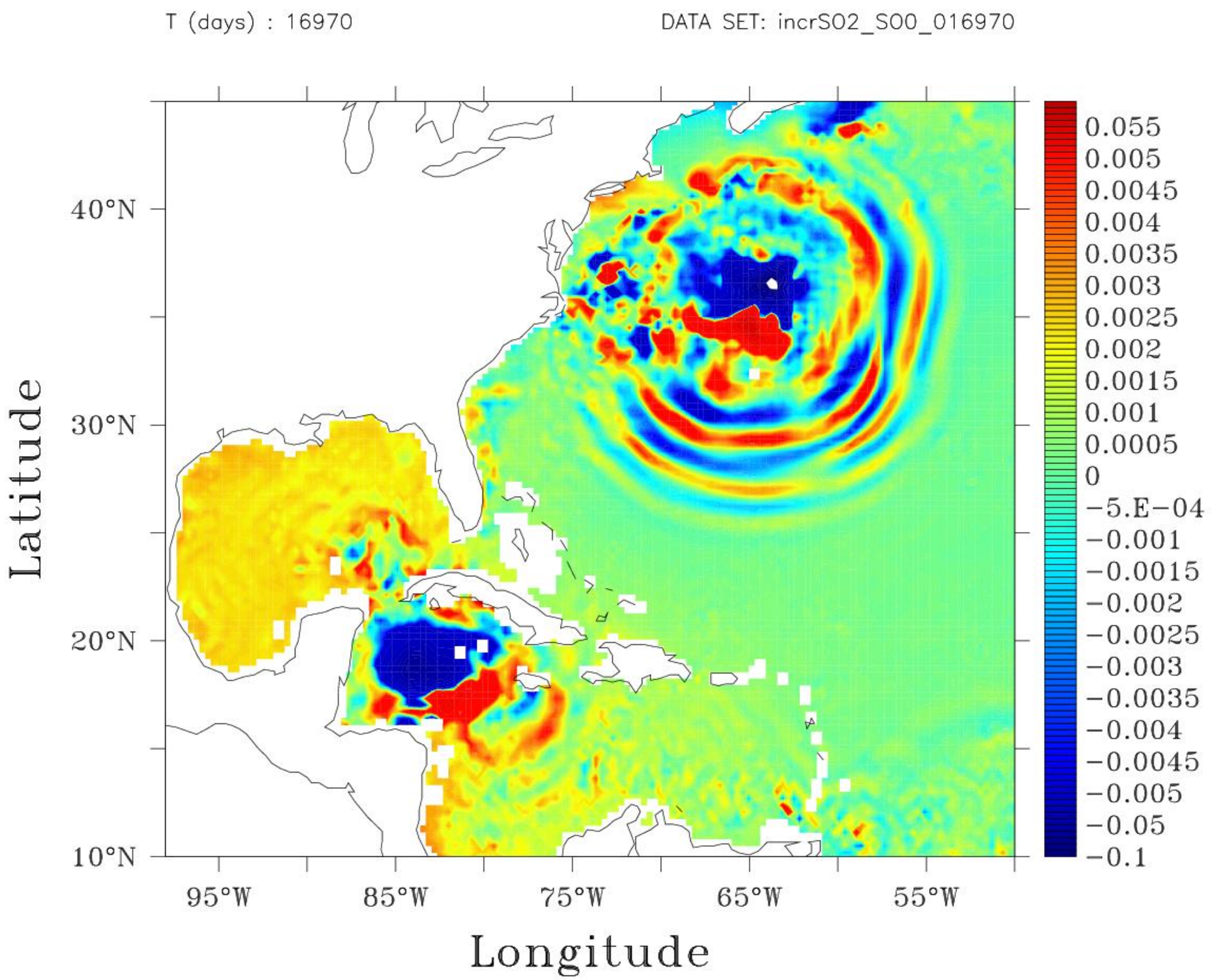
(Brankart et al, 2011)

EnKF implementations

Incremental Analysis Updating (IAU)

Model not involved during analysis: discontinuity, balance problems and shocks at restart possible.

Right: spurious wave generated by the assimilation of a single observation.



(Rozier et al, 2007)

EnKF implementations

Incremental Analysis Updating (IAU)

- An empirical solution is Incremental Analysis Updating (IAU, Bloom et al, 1996)
- IAU consists in computing corrections at the analysis step, then re-running the ensemble over the forecast window, adding incrementally to each member its correction under the form of a forcing term.

Here, IAU is run from the middle of the previous forecast window to the middle of the next forecast window.

Continuity is guaranteed (perhaps at the expense of quality of the analysis).

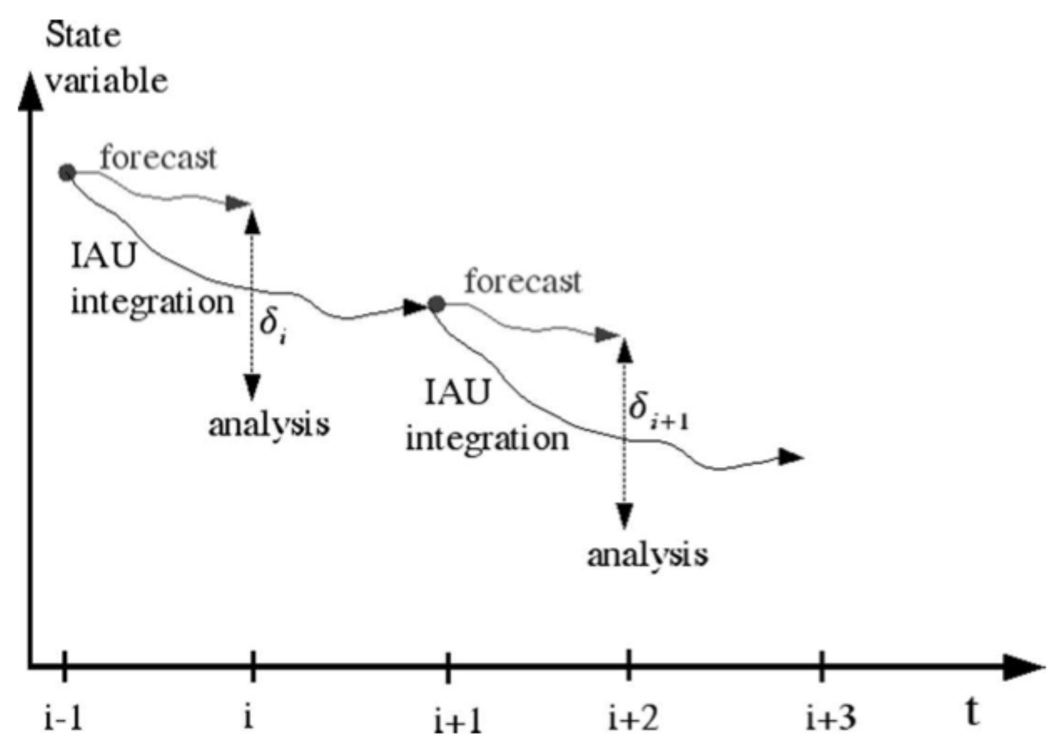


FIG. 1. IAU method from Bloom et al. (1996); δ represents the increment.

EnKF implementations

Incremental Analysis Updating (IAU)

Figure: spatially averaged zonal velocity U in the Gulf Stream zone.

Black: free run

Red: EnOI

Green: EnOI with IAU

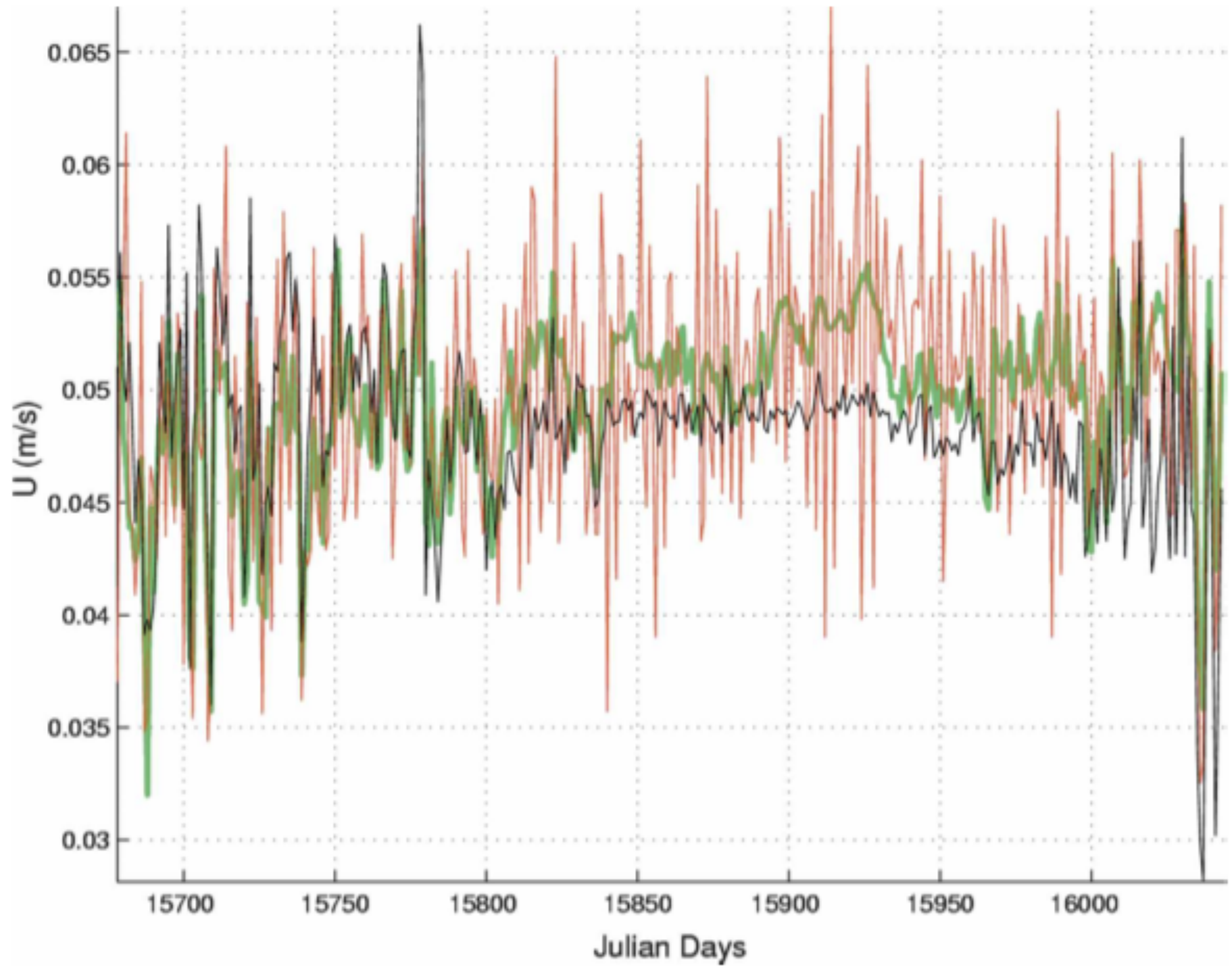


FIG. 12. Same as in Fig. 11, but at a 55-m depth (model depth level 5) from Julian day 15678 (4 Dec 1992) to 16038 (5 Dec 1993): black line represents FREE run, red line represents INT run, and green line represents IAU run.

(Ourmières et al, 2005)

EnKF implementations

Bogus

- Some quantities must be conserved. Example: mass.

$$\mathrm{div}\ \mathbf{u} = 0$$

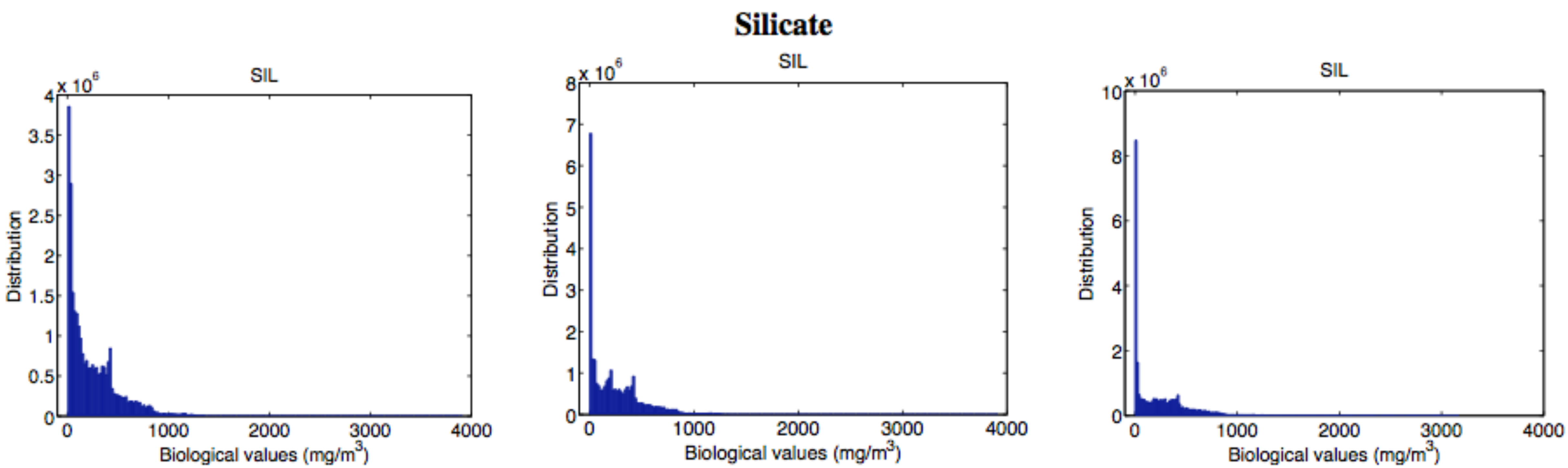
- Bogus: a fictitious observation of div **u**, equal to 0.

- Bogus can be used in regions where the assimilation makes things worse…

EnKF implementations

Gaussian anamorphosis

- Sometimes the distribution of some variables does not follow a Gaussian law;



Distribution of silicate at 3 different dates (over a large oceanic domain)

(Simon et al, 2009)

EnKF implementations

Gaussian anamorphosis

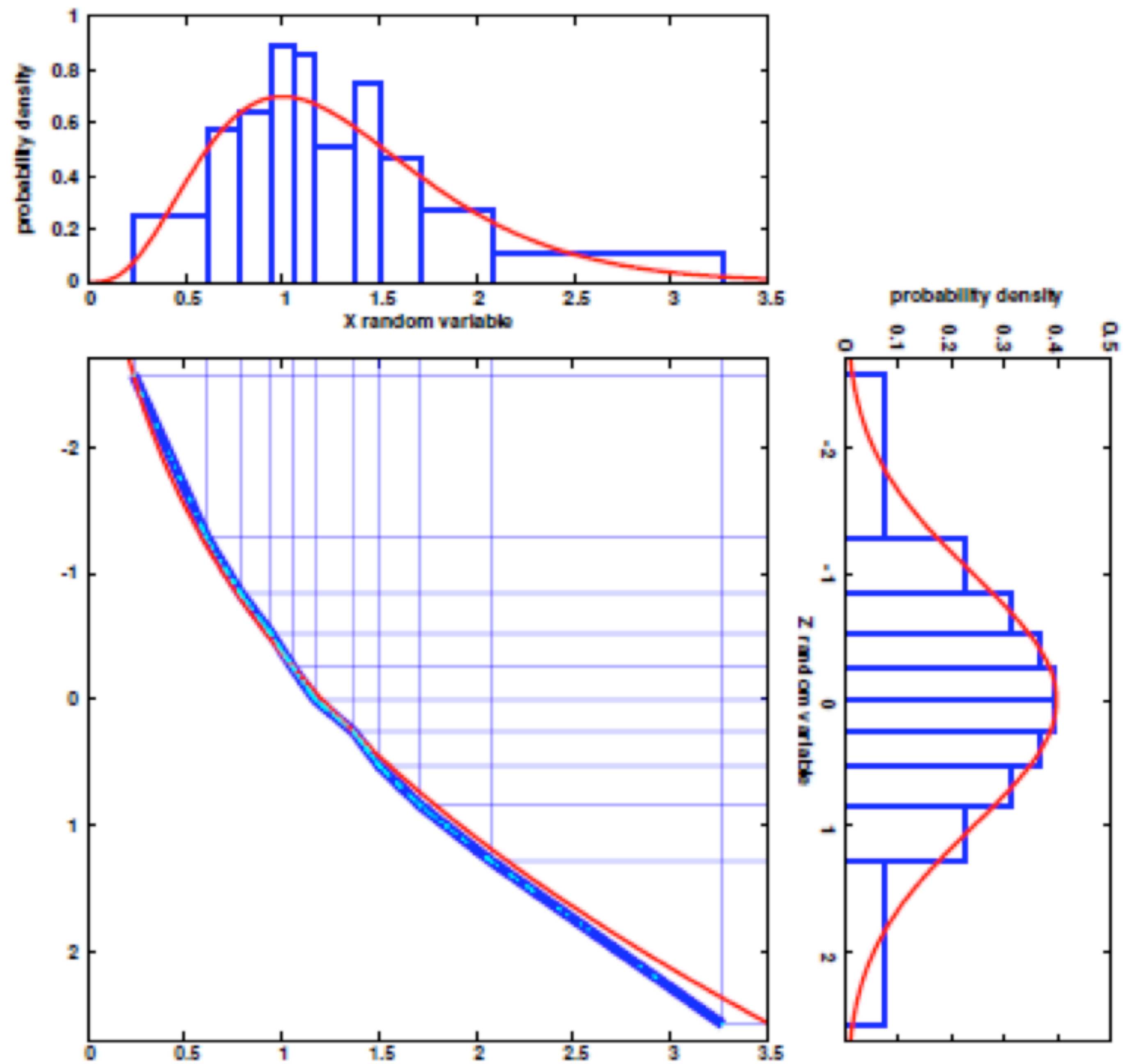
- Sometimes the distribution of some variables does not follow a Gaussian law;
- But the EnKFs work better with Gaussian variables;
- Gaussian anamorphosis: transformation of a distribution into a Gaussian distribution.

(Bertino et al, 2003)

EnKF implementations

Gaussian anamorphosis

- The transformation can be analytical or empirical;
- On the opposite figure, the transformation is empirical;
- Such transformation can be performed on each variable individually.



(Béal et al, 2010)

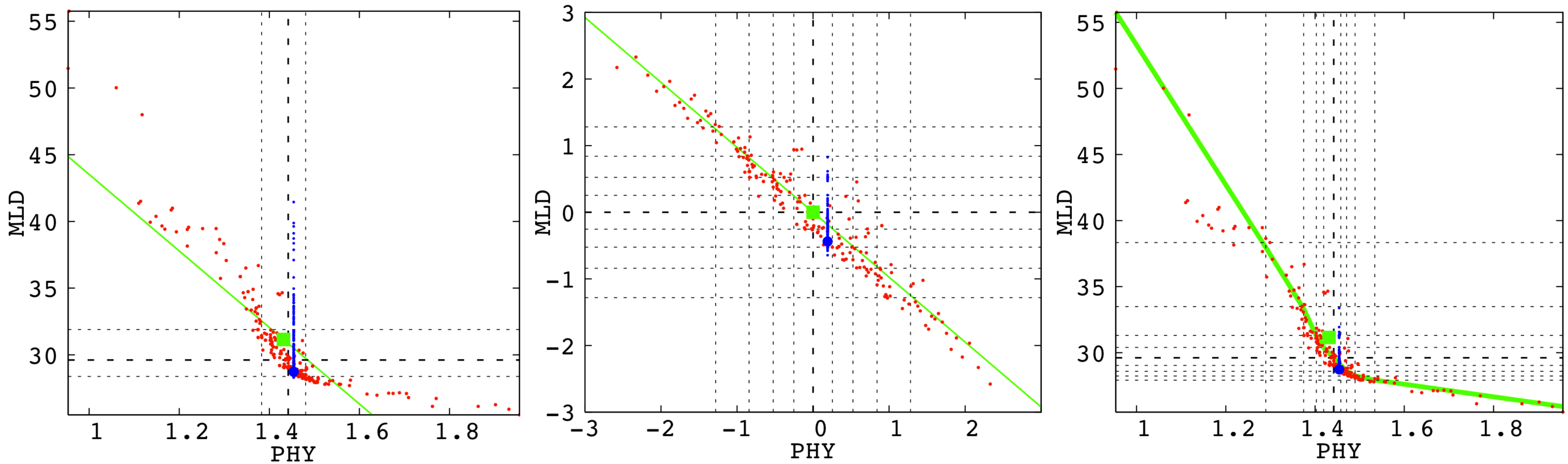
EnKF implementations

Gaussian anamorphosis

- After transformation, the EnKF analysis is performed;
- Then, the physical variables are retrieved by the inverse transformation.

EnKF implementations

Gaussian anamorphosis



Obs. update at BATS station (65○W-32○N) using a perfect PHY observation. Prior ensemble (red), mean (green square), linear regression line (thin green line), truth (big blue dot), posterior ensemble (blue dots). Left: EnKF analysis; Middle: analysis in the transformed state space; Right: Anamorphosis-EnKF posterior. The thick green line on the right is the transformation of the thin green line on the middle.

(Béal et al, 2010)

EnKF implementations

About the observation error covariance matrix

$$\mathbf{P}^f = \mathbf{S}^f {\mathbf{S}^f}^T$$

- The EnKF correction is either calculated with (using a serial processing of observations)

$$\delta\mathbf{x} = \mathbf{S}^f (\mathbf{H}\mathbf{S}^f)^T \left[(\mathbf{H}\mathbf{S}^f)(\mathbf{H}\mathbf{S}^f)^T + \mathbf{R}\right]^{-1} (\mathbf{y} - \mathbf{H}\mathbf{x}^f),$$

- Or, with $\Gamma = (\mathbf{H}\mathbf{S}^f)^T \mathbf{R}^{-1} (\mathbf{H}\mathbf{S}^f)$

$$\delta\mathbf{x} = \mathbf{S}^f \left[\mathbf{I} + \Gamma\right]^{-1} (\mathbf{H}\mathbf{S}^f)^T \mathbf{R}^{-1} (\mathbf{y} - \mathbf{H}\mathbf{x}^f).$$

EnKF implementations

About the observation error covariance matrix

- For simplification, all ocean DA systems consider the observation error covariance matrix diagonal.
- To minimize the impact of the neglected correlations, it is common to inflate the variances (in the Norwegian operational system, they are multiplied by 2 for the update of the anomalies).
- On the other hand, many efforts are dedicated to the construction of the state error covariance matrix.

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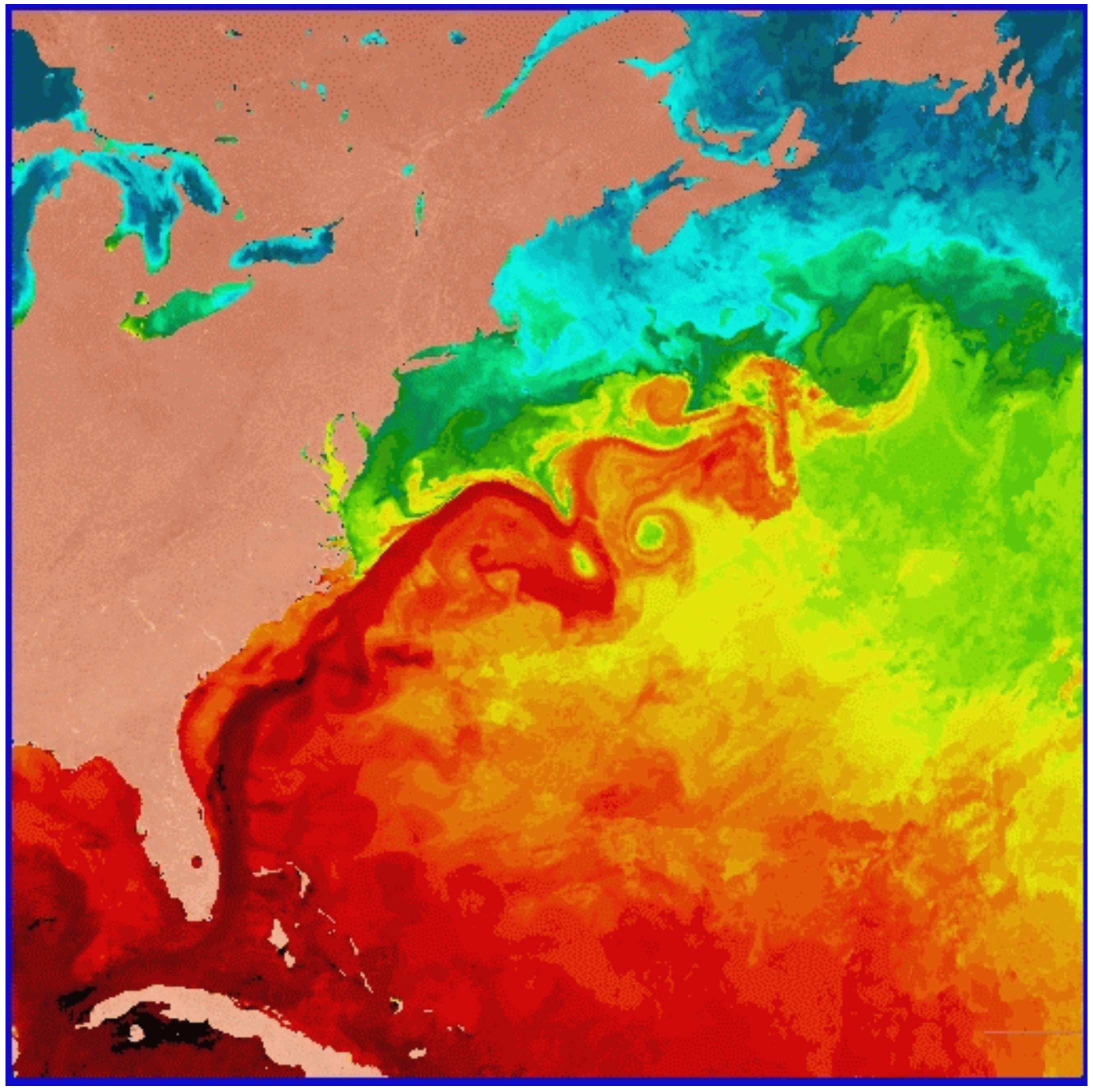
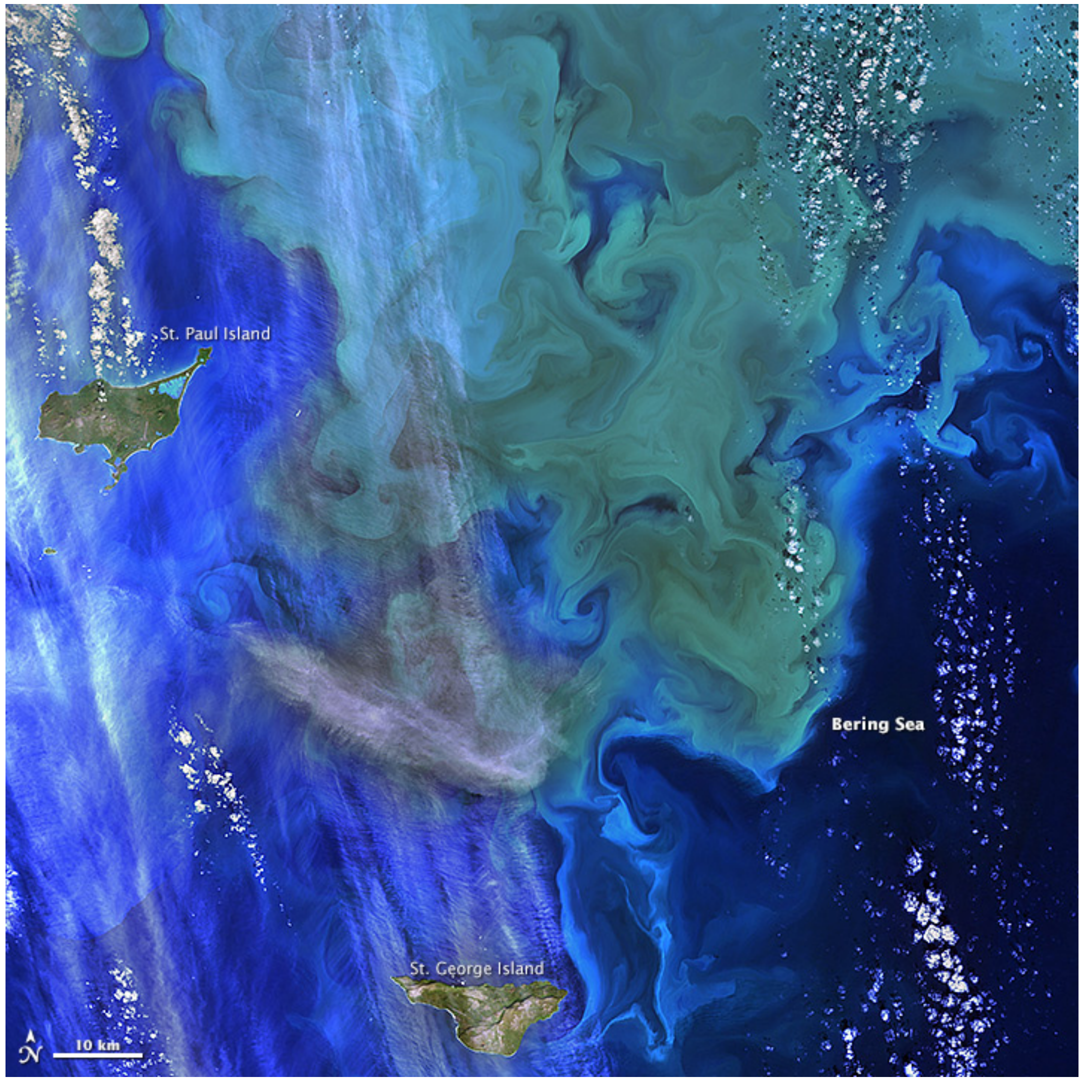
Assimilation of images

Optical images



AVHRR composite image of SST.

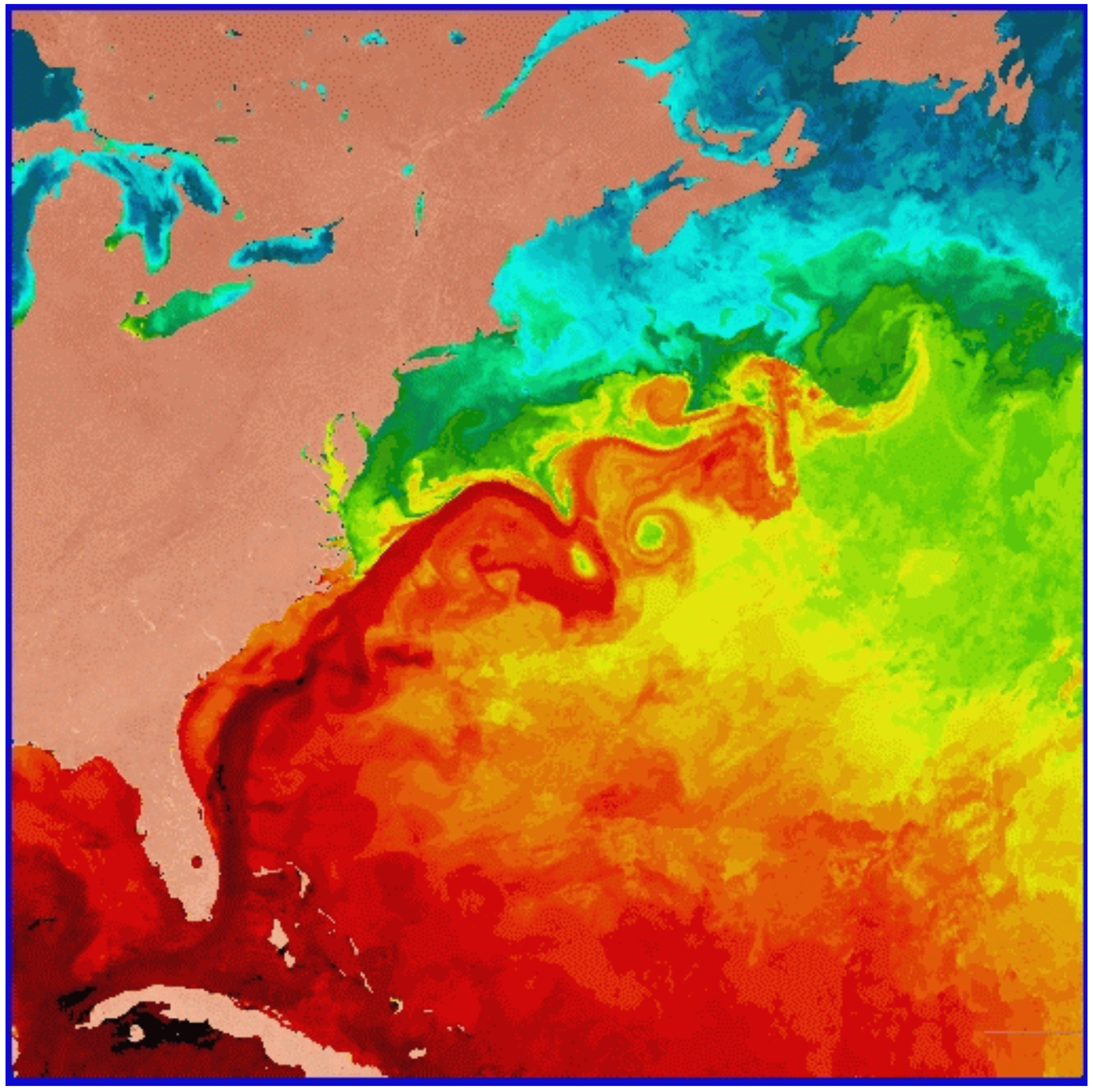


Ocean color sensors detect chlorophyll.

A phytoplankton bloom captured near Alaska by Operational Land Imager (OLI) on Landsat 8 (NASA).

Assimilation of images

Optical images



AVHRR composite image of SST.

Example: Optic flow methods

$$\frac{\partial T}{\partial t} + \nabla T \cdot \mathbf{w} = 0$$

with T observed and w driven by a shallow-water model:

$$\left\{ \begin{array}{rcl} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} & = & fv - g'\frac{\partial h}{\partial x} \\ \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} & = & -fu - g'\frac{\partial h}{\partial y} \\ \frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (vh)}{\partial y} & = & 0 \end{array} \right.$$

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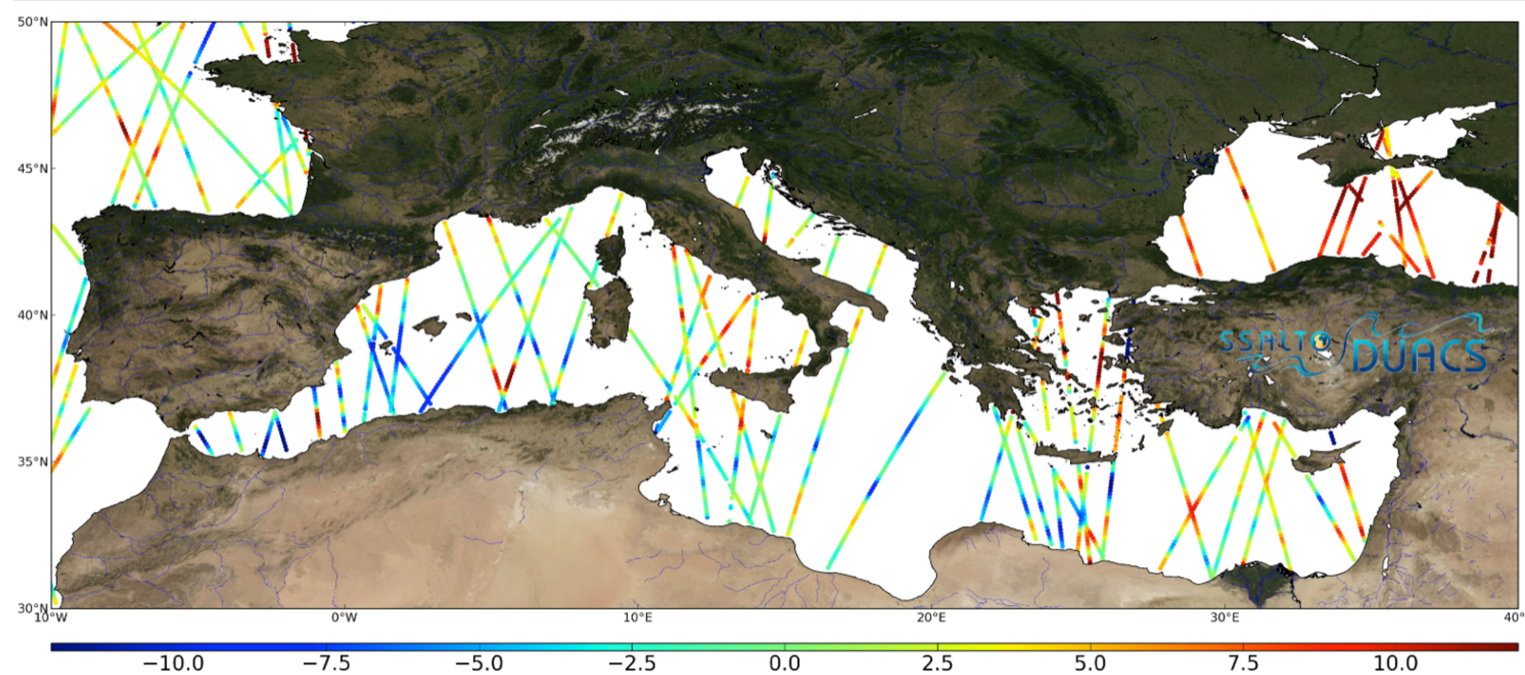
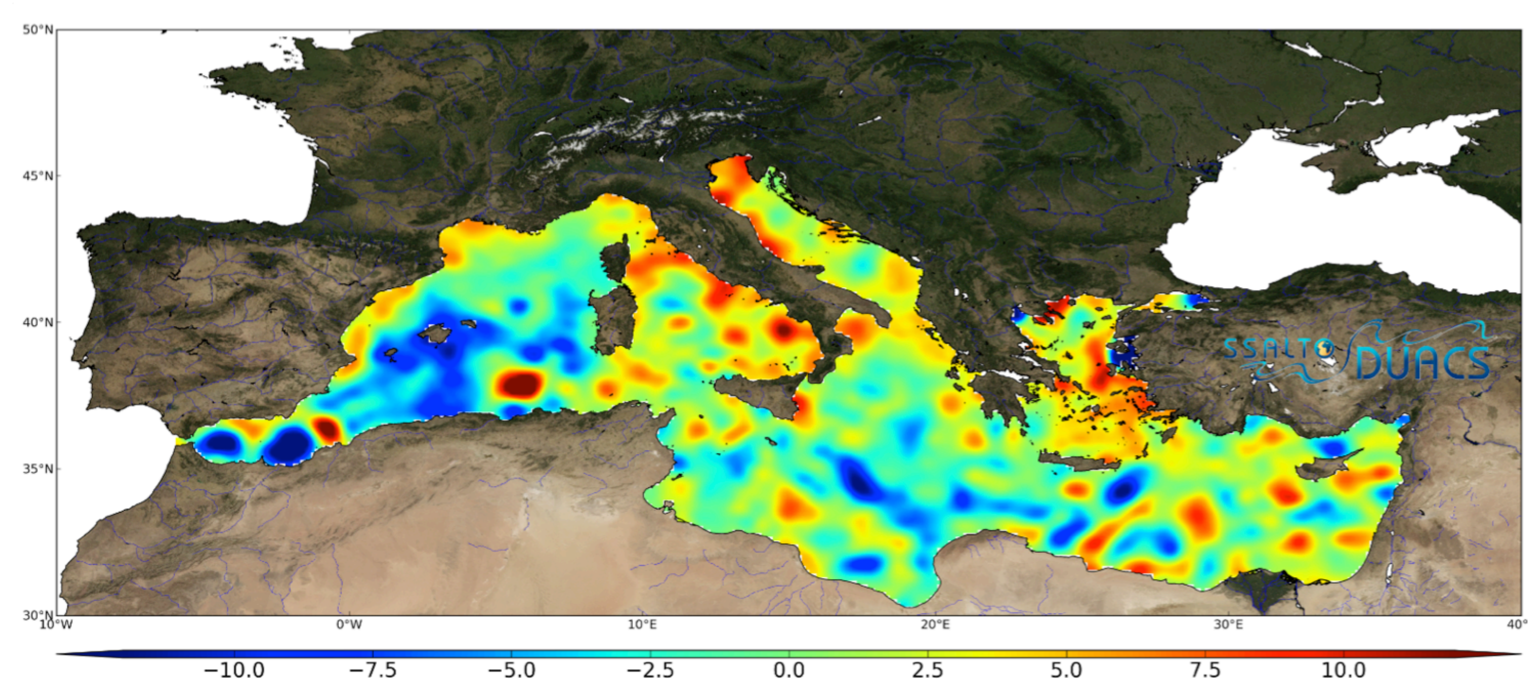
Altimetric products and the SWOT mission

Sea Level anomaly maps

Present-day nadir altimetry is processed to provide gridded maps of Sea Level Anomaly. This is done with the DUACS algorithm at CNES/CLS, implementing statistical interpolation.

These maps resolve scales of 200 km and 10 days.





https://www.aviso.altimetry.fr/en/home.html

Altimetric products and the SWOT mission

Sea Level anomaly maps

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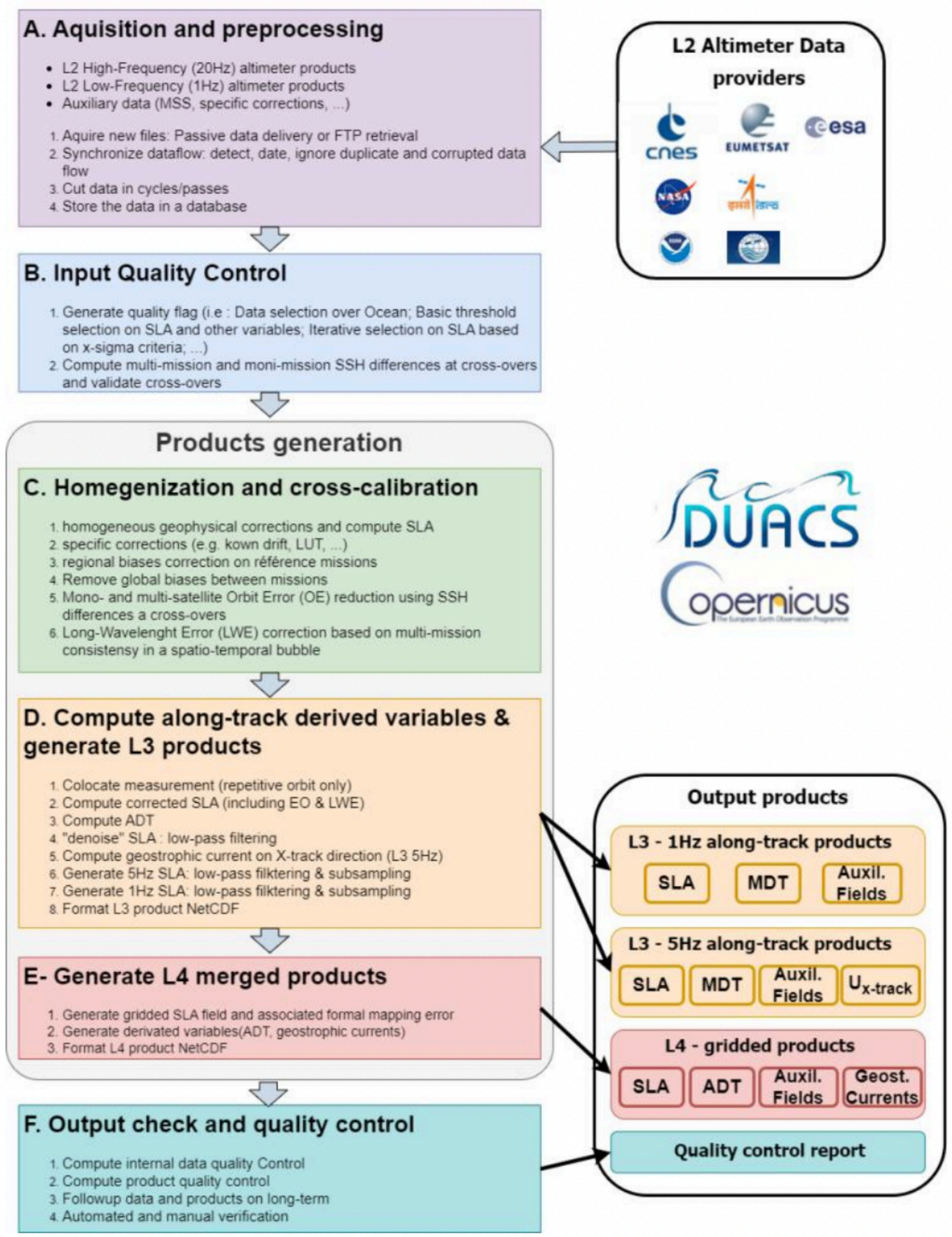
The DUACS processing system

- **DUACS** (https://duacs.cls.fr/) is the operational multimission production system of altimeter data developed by CNES/CLS
- Used **for operational production** of the Marine (**CMS** – Copernicus Marine Service) and Climate (**C3S**) services of the E.U. **Copernicus program**, for processing of **Sentinel-3 products** and for the production of demonstration and **pre-operational products** on behalf of CNES
- Near Real Time (**NRT**) goal: Deliver high-quality, directly usable altimeter data for operational applications.
- Delayed Time (**DT**) goal: Ensure a consistent, user-friendly data record based on altimetry community standards.

Figure: Evolution of the different DUACS product versions

- Updated component for the new **DT24** reprocessing
  - **New Altimetry standards** and geophysical corrections (e.g.Kocha et al, 2024).
  - Adoption of a new **multiscale mapping** methodology (Ubelmann et al., 2021)

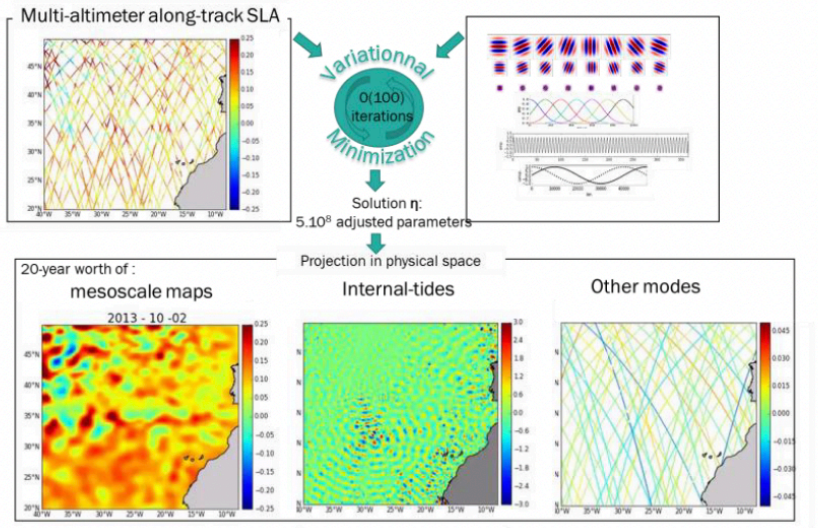


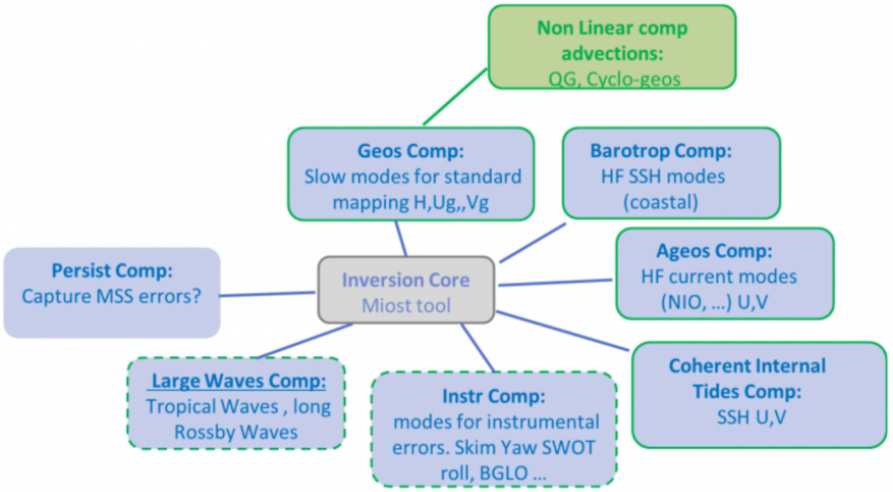
living planet symposium VIENNA 23–27 June 2025

From Maxime Ballarotta, CLS

L4 Mapping: From Optimal Interpolation to MIOST

- **MIOST** is a MultiScale and Multivariate mapping algorithm (Ubelmann et al. 2021)
- Similar to **Optimal interpolation**
- Designed originally to **handle large volume** of data (like SWOT data)
- Flexible enough to **manage** different types of ocean surface **variability** (geostrophic/ageostrophic, barotrope …) by defining specific covariance models in a single inversion
- The **MIOST** products has been validated with idealized and real systems, showing **effectiveness in mapping global surface topography and currents** (Ubelmann et al. 2021, 2022, Ballarotta et al., 2023).

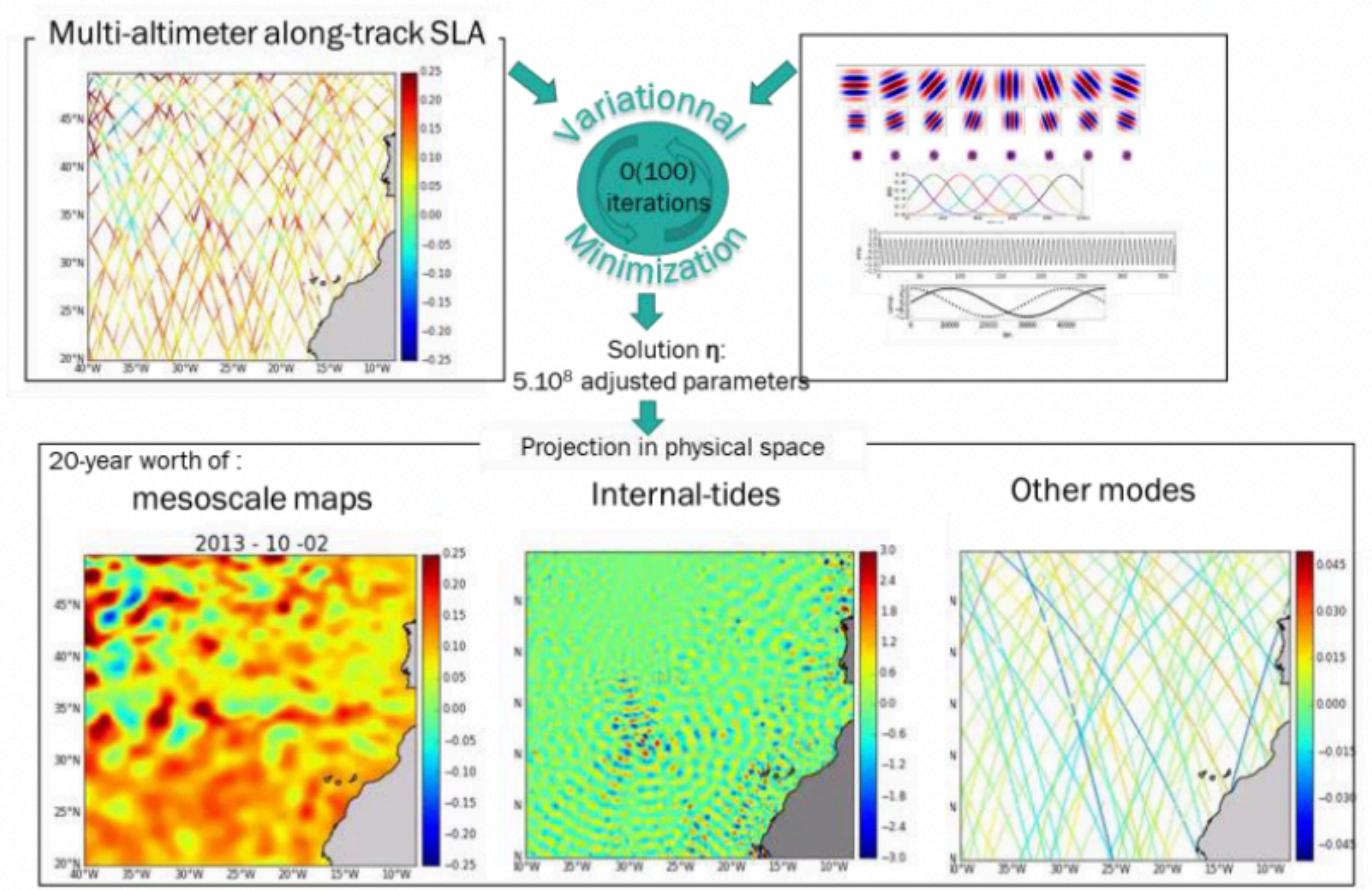




From Maxime Ballarotta, CLS

Altimetric products and the SWOT mission

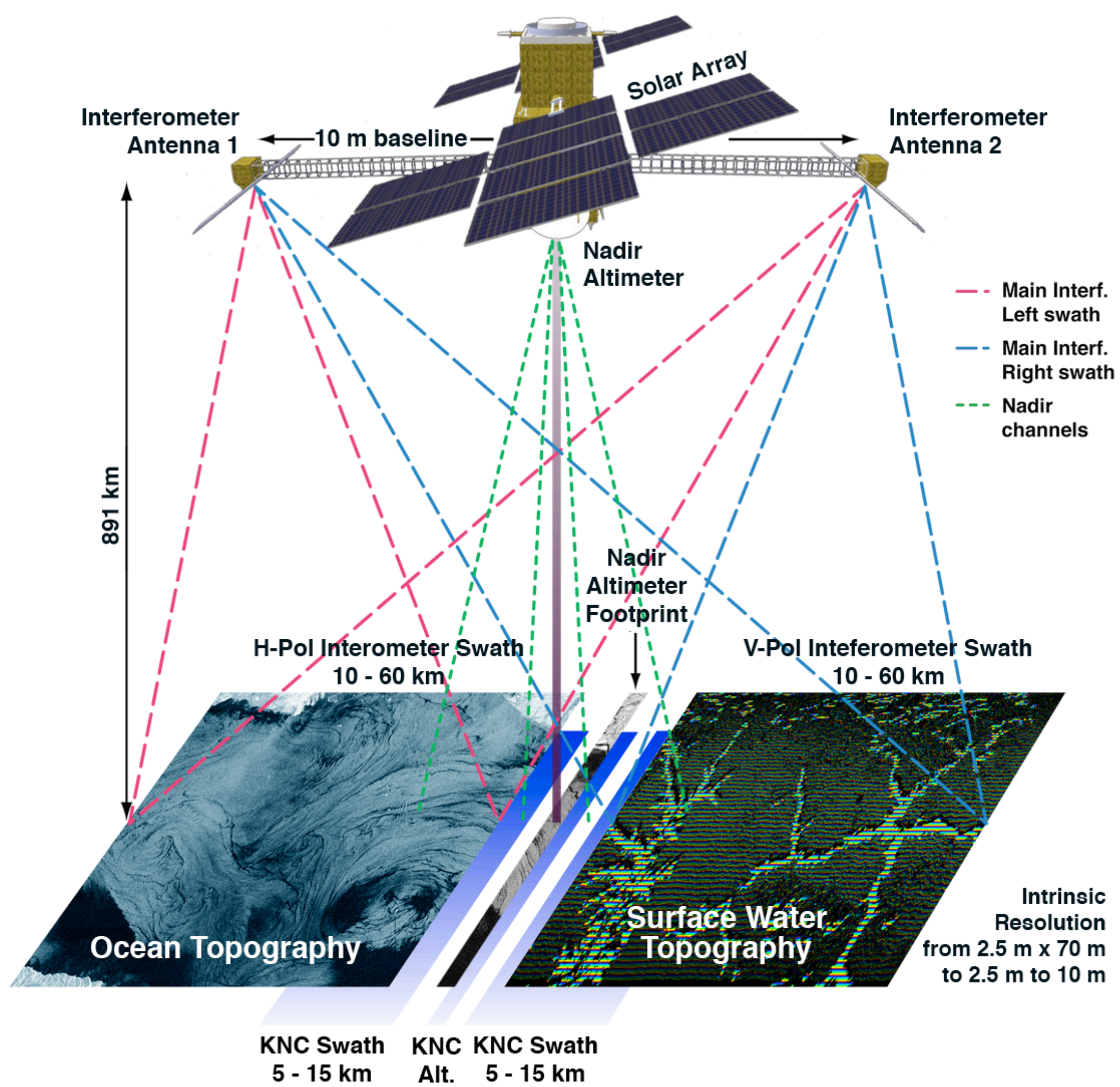
Sea Level anomaly maps



From Maxime Ballarotta, CLS

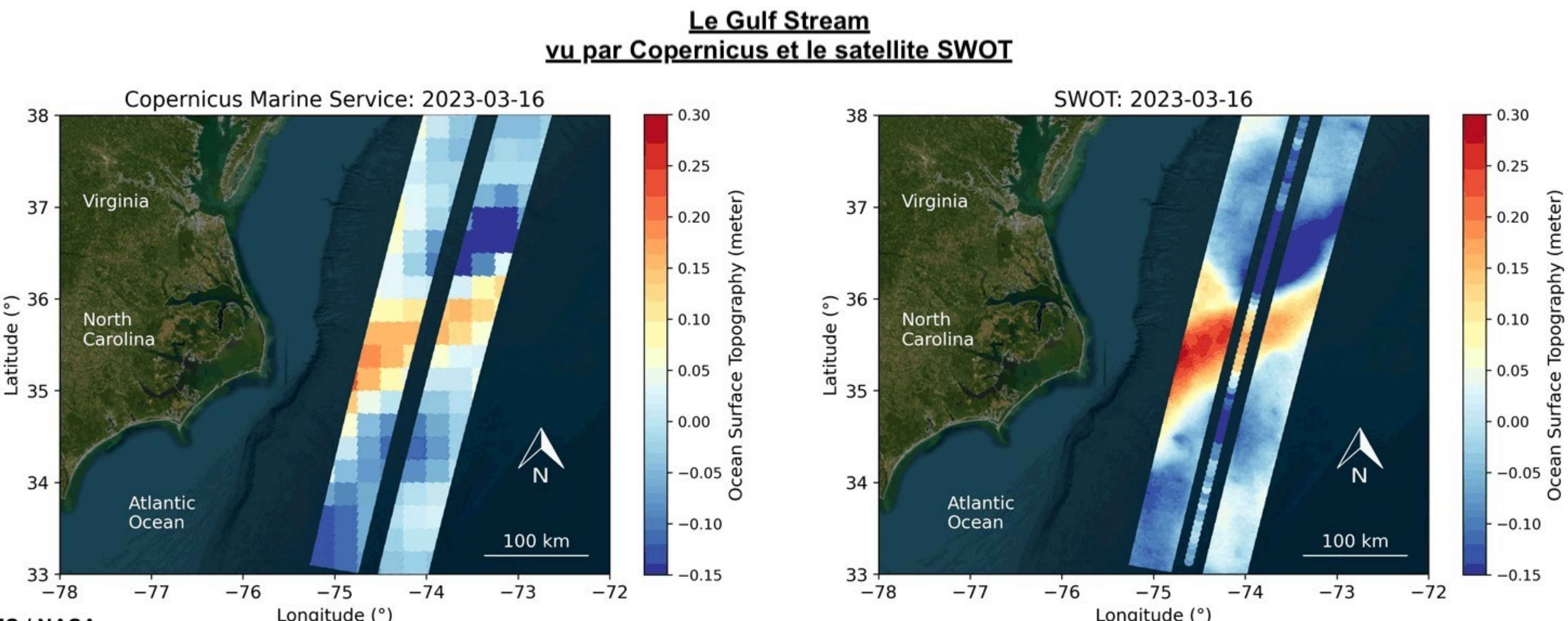
The SWOT mission

- SWOT: Surface Water and Ocean Topography
- Satellite mission launched in 2022
- Revolutionary altimetric observation: 120 km-wide swath
- Pixel of 2 km, revisit 10 days (mismatch)



https://swot.jpl.nasa.gov/

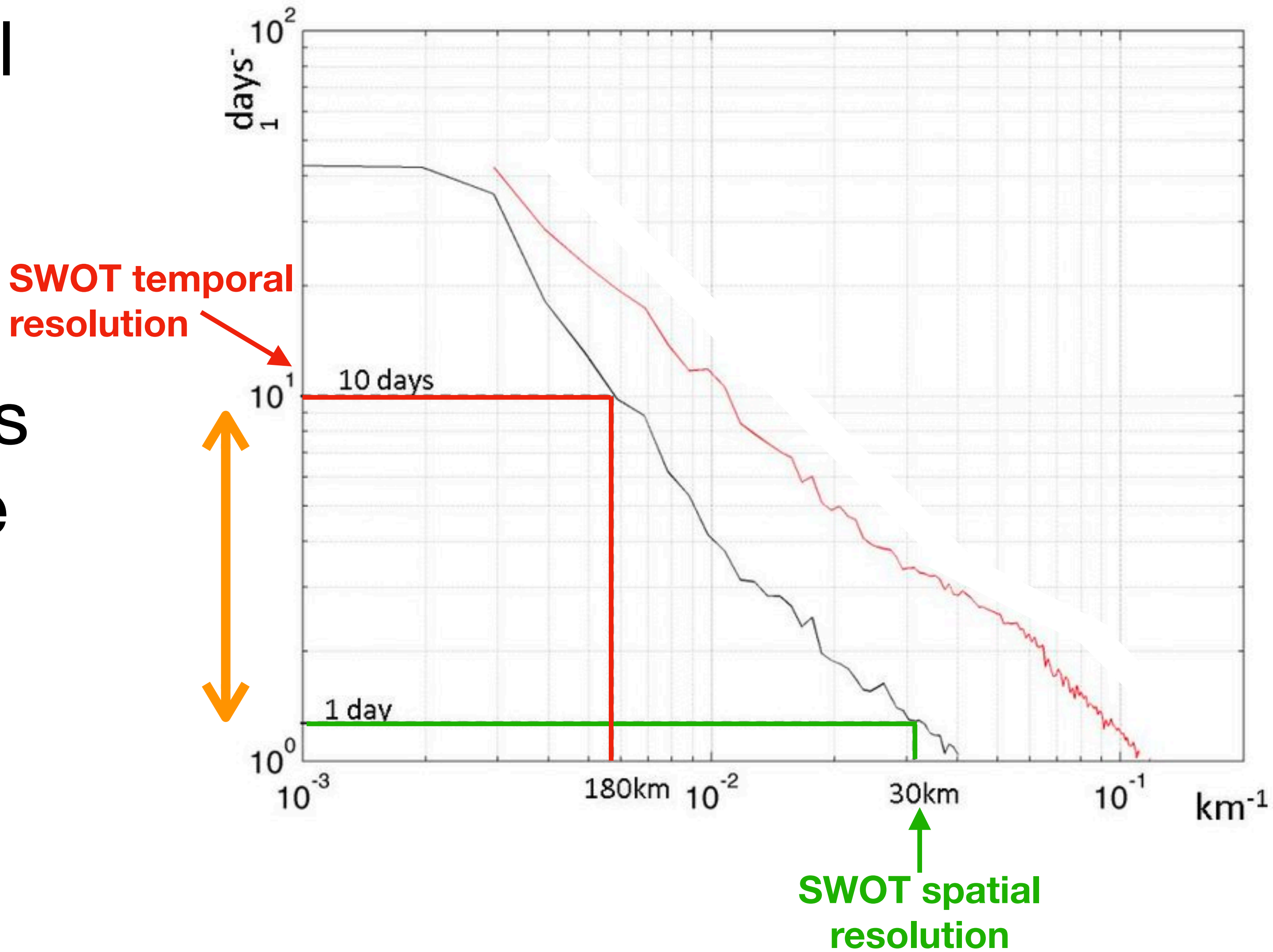
The SWOT mission



The SWOT mission

- Mismatch between spatial and temporal coverage of SWOT
- Expectation that dynamics must be considered in the interpolation
- ==> data assimilation

decorrelation time as a function of wavelength



Morrow et al., 2019

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Mapping balanced motions with a nudging technique

Methodology

- Method: Back-and-forth nudging (BFN) with a 1.5-layer quasi-geostrophic (QG) model.

  - Why a simple 1.5-layer QG model?

    It is a simple model able to capture a large part of mesoscale ocean dynamics as observed by altimetry.

  - Why BFN?

    It is a conceptually simple method.

    The QG dynamics is governed by a single variable, almost directly observed.

Le Guillou et al, 2020

Mapping balanced motions with a nudging technique

Methodology

(QG) Forward propagation: $$\frac{\partial X}{\partial t} = M(X,t) \qquad X(0) = x_0$$

Forward nudging: $$\frac{\partial X}{\partial t} = M(X,t) + K(y^{obs} - X)$$

(QG) Backward propagation: $$\frac{\partial X}{\partial t} = M(X,t) \qquad X(T) = x_T$$

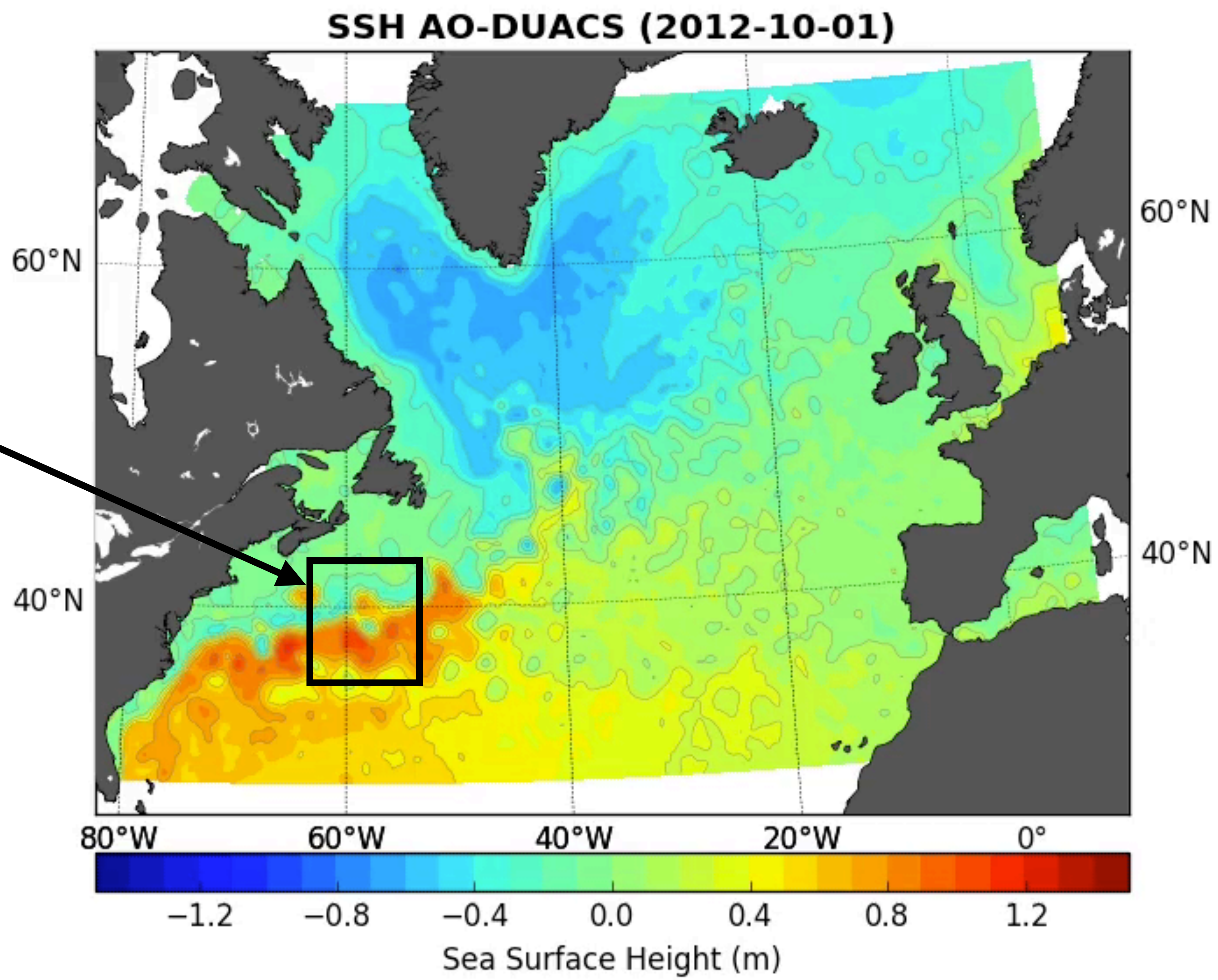
Backward nudging: $$\frac{\partial X}{\partial t} = M(X,t) - K(y^{obs} - X)$$

BFN algorithm (Auroux et al., 2008): combination of the **forward nudging** and the **backward nudging** in an **iterative** process over a temporal window

Mapping balanced motions with a nudging technique

Experimental setup

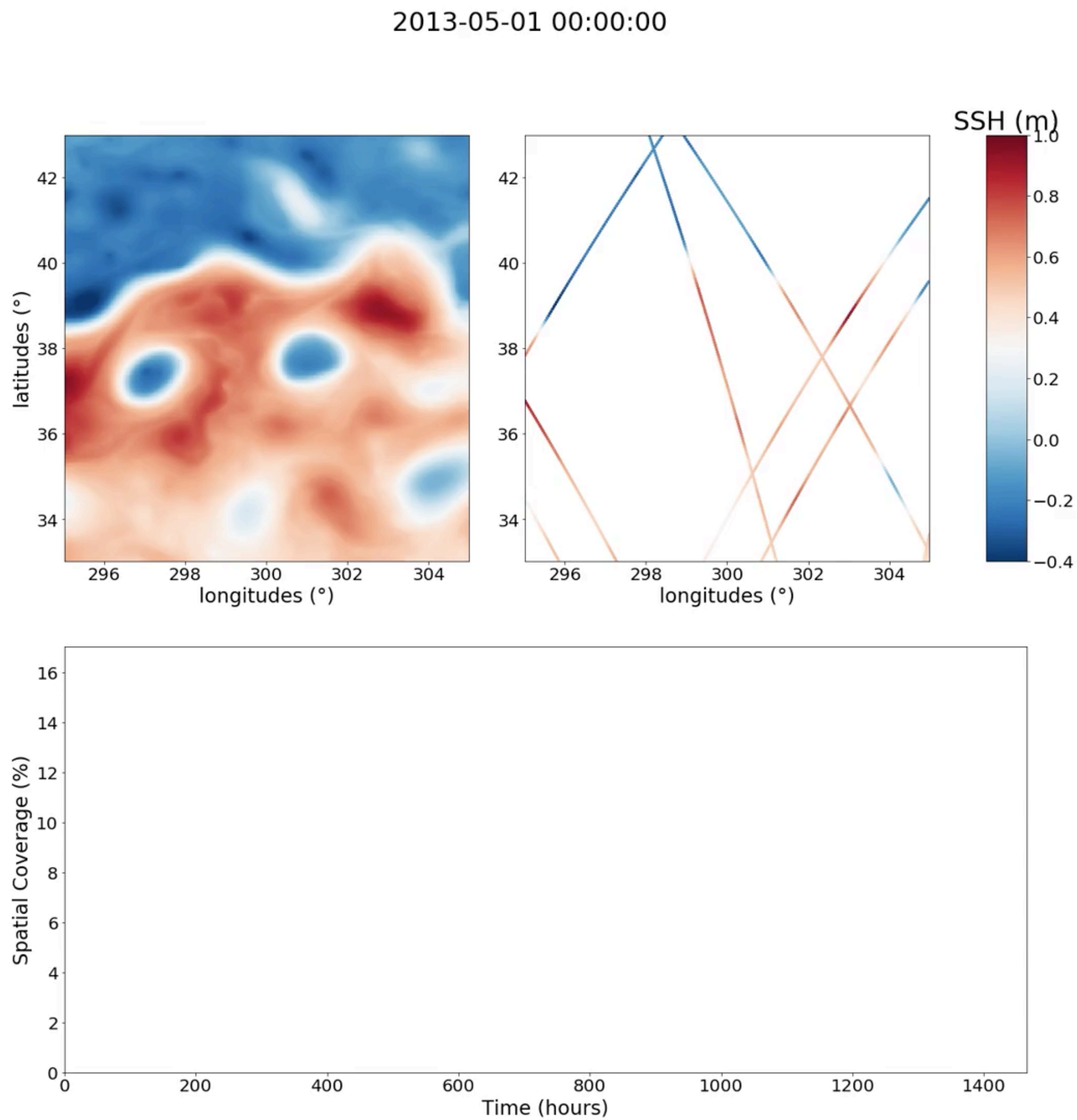
Our studied region



Mapping balanced motions with a nudging technique

Experimental setup

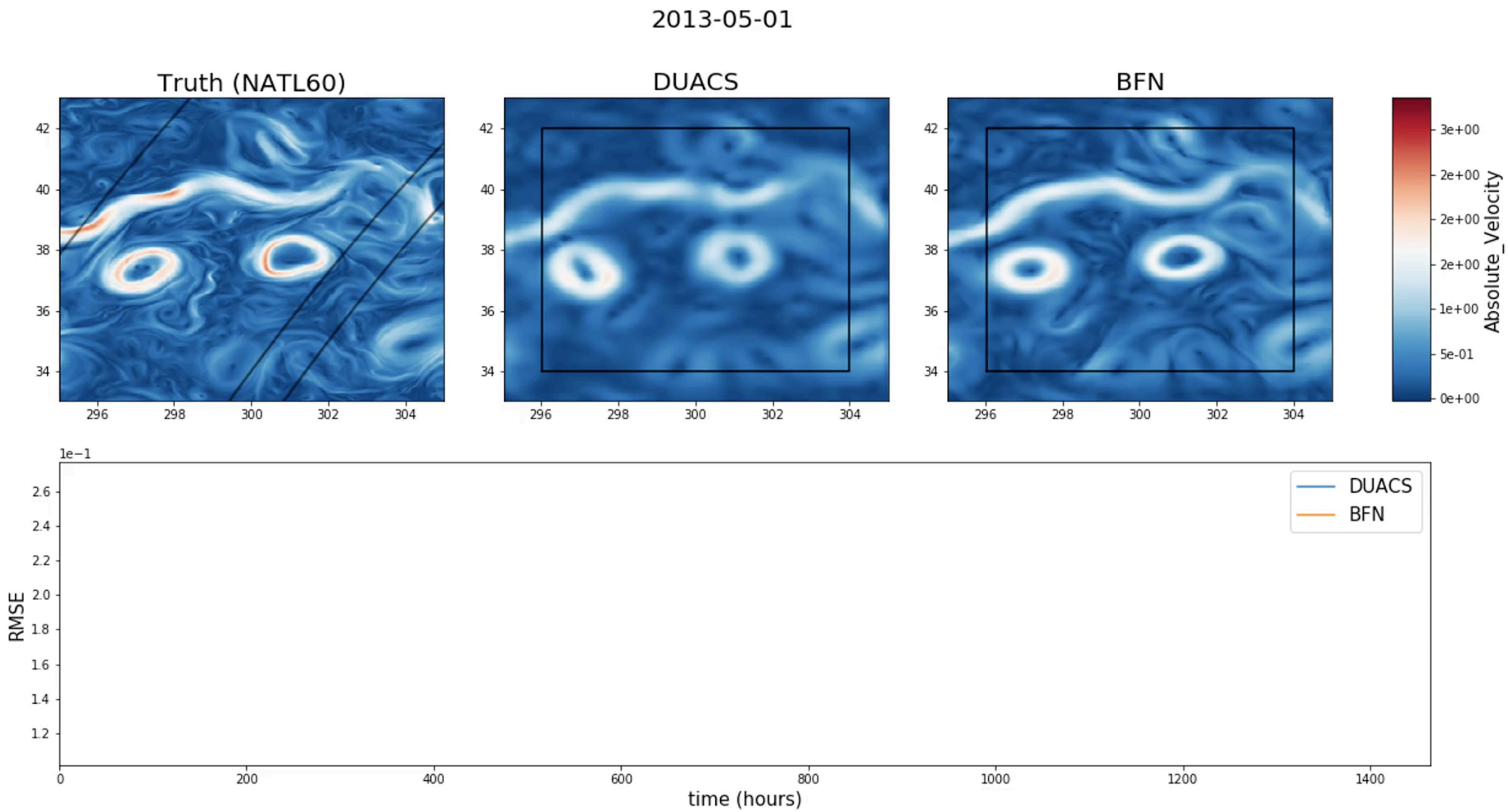
- 4 conventional along-track altimeters (Nadirs)
- SWOT
- No errors considered



Mapping balanced motions with a nudging technique

Results

Example with SWOT + Nadirs constellation



Le Guillou et al, 2020

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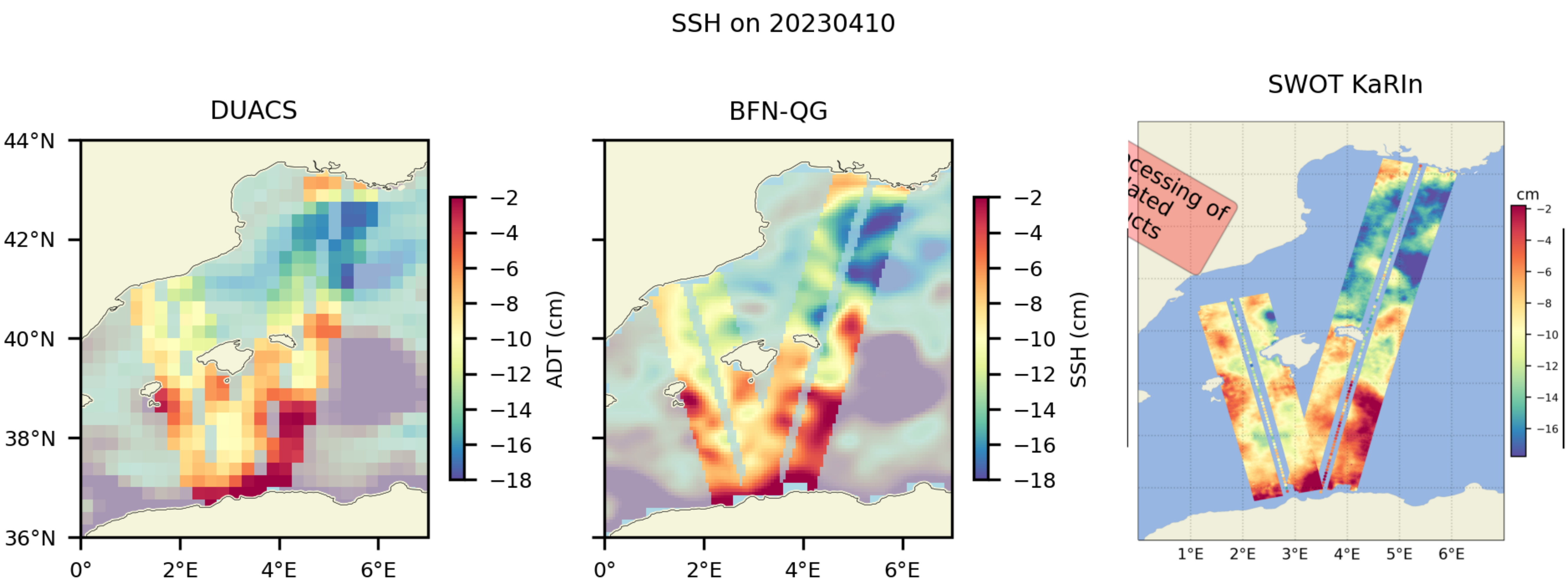
Mapping balanced motions with a nudging technique

Results



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Mapping balanced motions with 4DVar

Results

$$\mathrm{SSH} = M(\mathrm{F_{SSH}})$$

Variational cost function

$$\mathcal{J}(\mathrm{SSH}) = \|\mathrm{SSH} - \mathrm{SSH^b}\|_B^2 + \|\mathrm{SSH^{obs}} - \mathrm{SSH}\|_R^2$$
$$<=> \mathcal{J}(\phi) = \|\phi - \phi^\mathrm{b}\|_B^2 + \|\mathrm{SSH^{obs}} - \mathrm{M} \cdot W_{3D}(\phi)\|_R^2$$

Dynamical Model

Geostrophic balance

$$u = -\frac{g}{f}\frac{\partial \mathrm{SSH}}{\partial y} \; ; \; v = \frac{g}{f}\frac{\partial \mathrm{SSH}}{\partial x}$$

QG potential vorticity (q) conservation

(Ubelmann et al. 2015)

$$\frac{\partial \mathrm{q}}{\partial t} + u\frac{\partial \mathrm{q}}{\partial x} + v\frac{\partial \mathrm{q}}{\partial y} = \mathrm{F_{SSH}}$$

$\mathrm{F_{SSH}}$: Forcing flux, representing QG model error

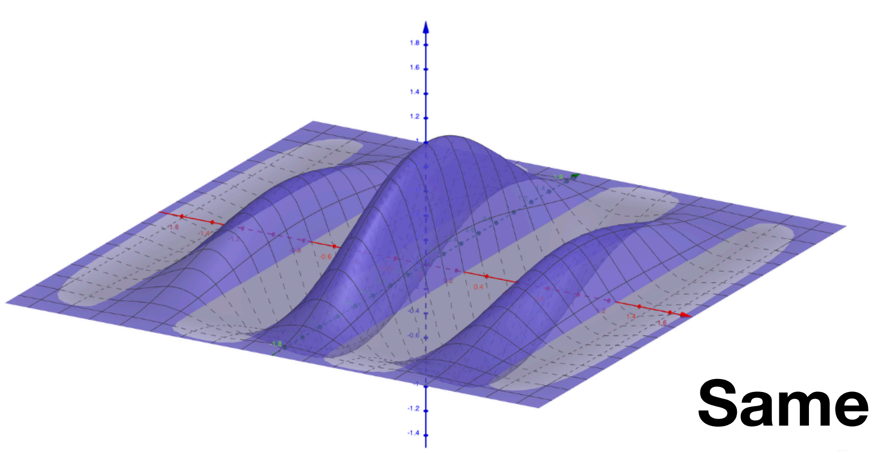
Wavelet Reduced Basis

(Ubelmann et al 2021)

$$\mathrm{F_{SSH}} = W_{3D}(\phi)$$
with size(ϕ) $\ll$ size($\mathrm{F_{SSH}}$)

Optimization scheme

L-BGFS-B gradient descent.
Gradient computed thanks to the JAX library, allowing GPU acceleration

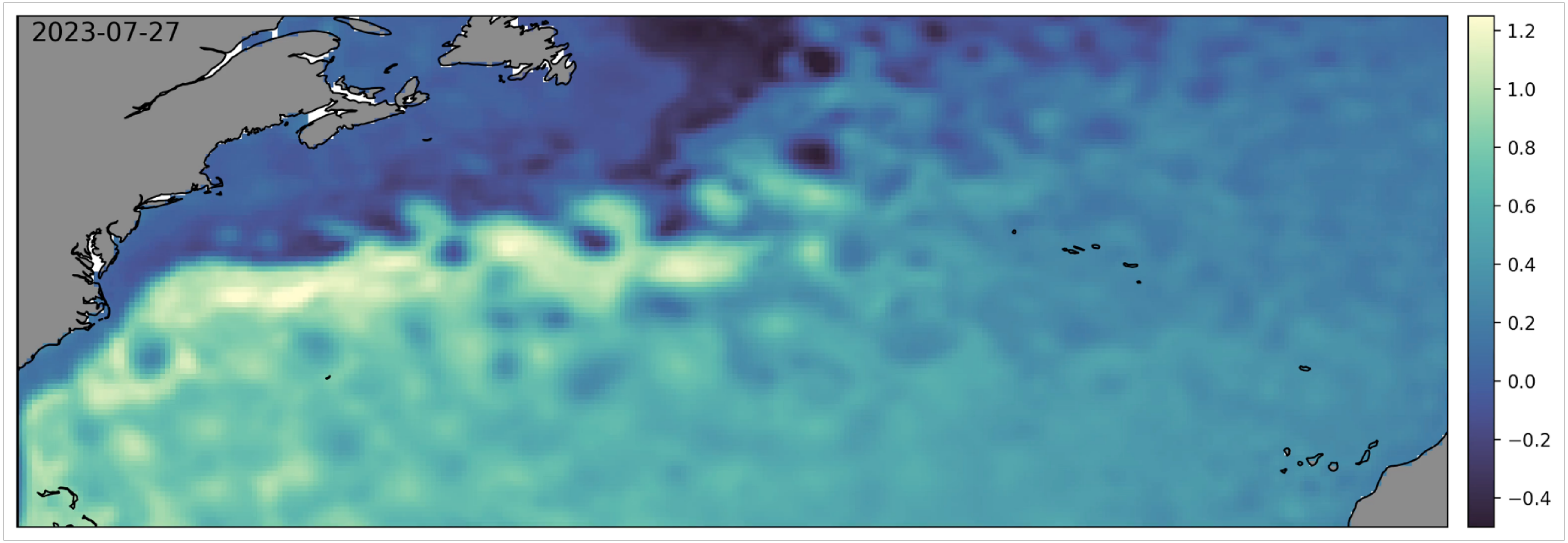


Same wavelets as in MIOST

Mapping balanced motions with 4DVar

Results

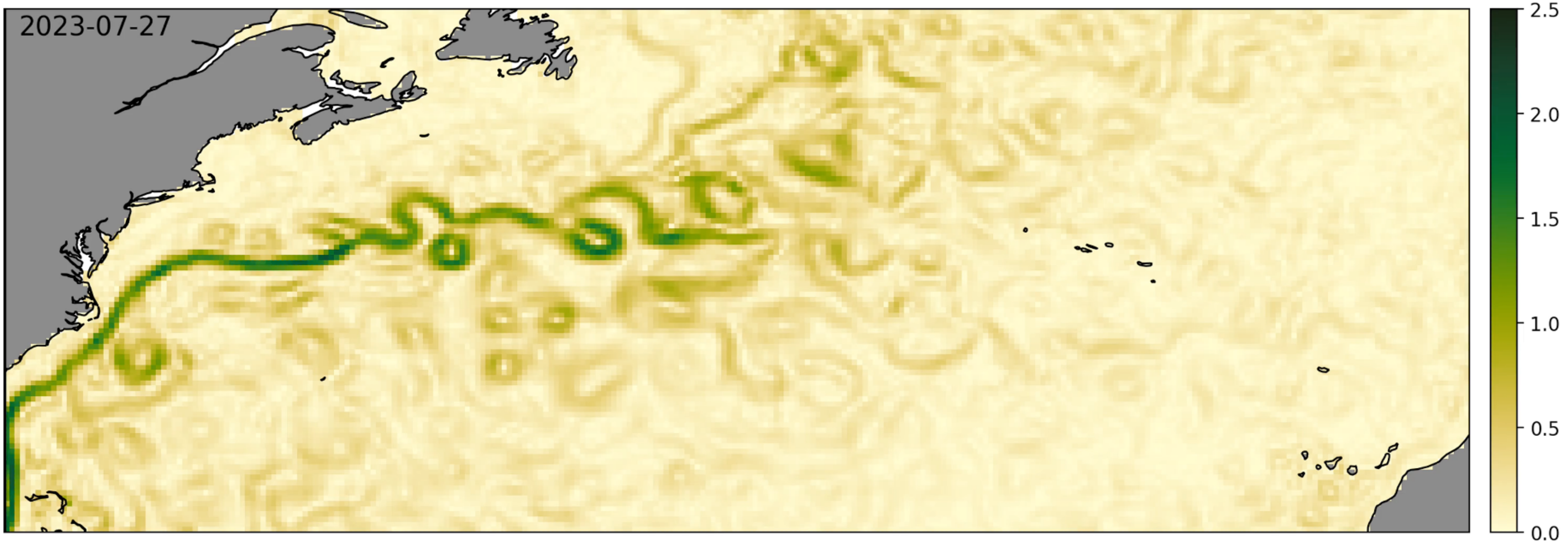
VarDyn | DUACS (old OI)



Mapping balanced motions with 4DVar

Results

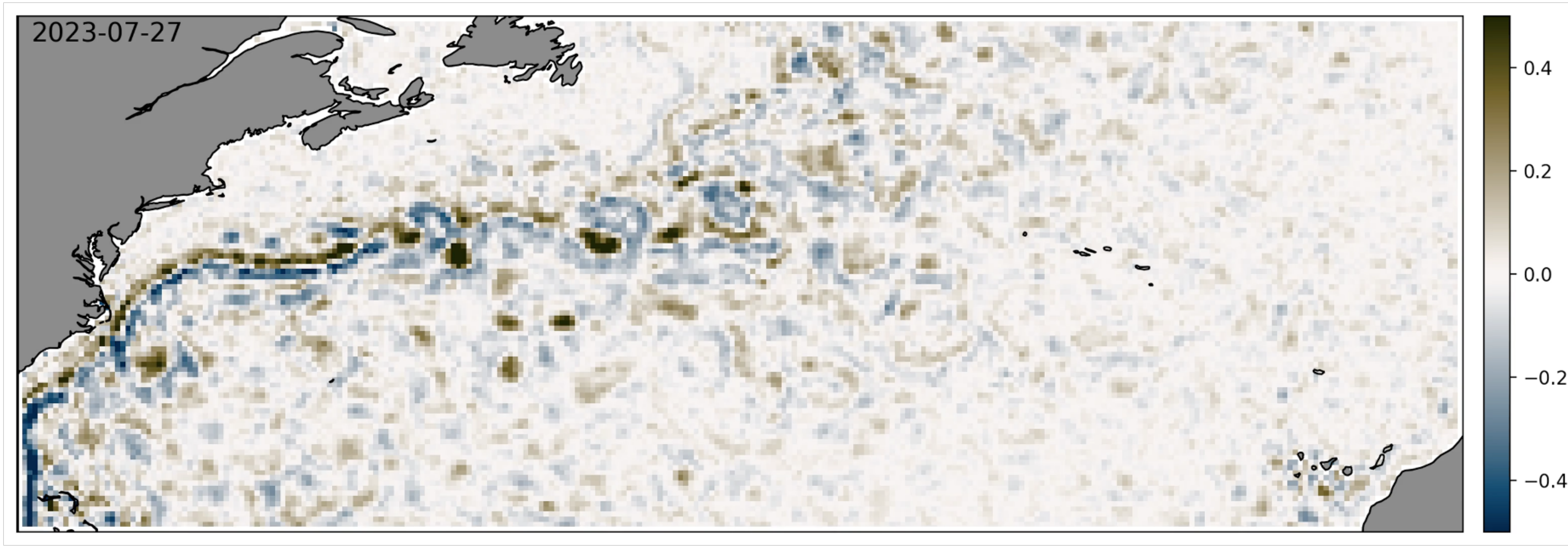
VarDyn | DUACS (old OI)



Mapping balanced motions with 4DVar

Results

VarDyn | DUACS (old OI)



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Outline

Texte du titre

Eddy/wave separation with a 4DVar technique

Problem statement

Phenomenon	Length scale L	Velocity scale U	Time scale T
Atmosphere:			
Sea breeze	5–50 km	1–10 m/s	12 h
Mountain waves	10–100 km	1–20 m/s	Days
Weather patterns	100–5000 km	1–50 m/s	Days to weeks
Prevailing winds	Global	5–50 m/s	Seasons to years
Climatic variations	Global	1–50 m/s	Decades and beyond
Ocean:			
SWOT: Internal waves	1–20 km	0.05–0.5 m/s	Minutes to hours
Coastal upwelling	1–10 km	0.1–1 m/s	Several days
Conventional nadir altimetry: Large eddies, fronts	10–200 km	0.1–1 m/s	Days to weeks
Major currents	50–500 km	0.5–2 m/s	Weeks to seasons
Large-scale gyres	Basin scale	0.01–0.1 m/s	Decades and beyond

Eddy/wave separation with a 4DVar technique

Problem statement and experimental setup

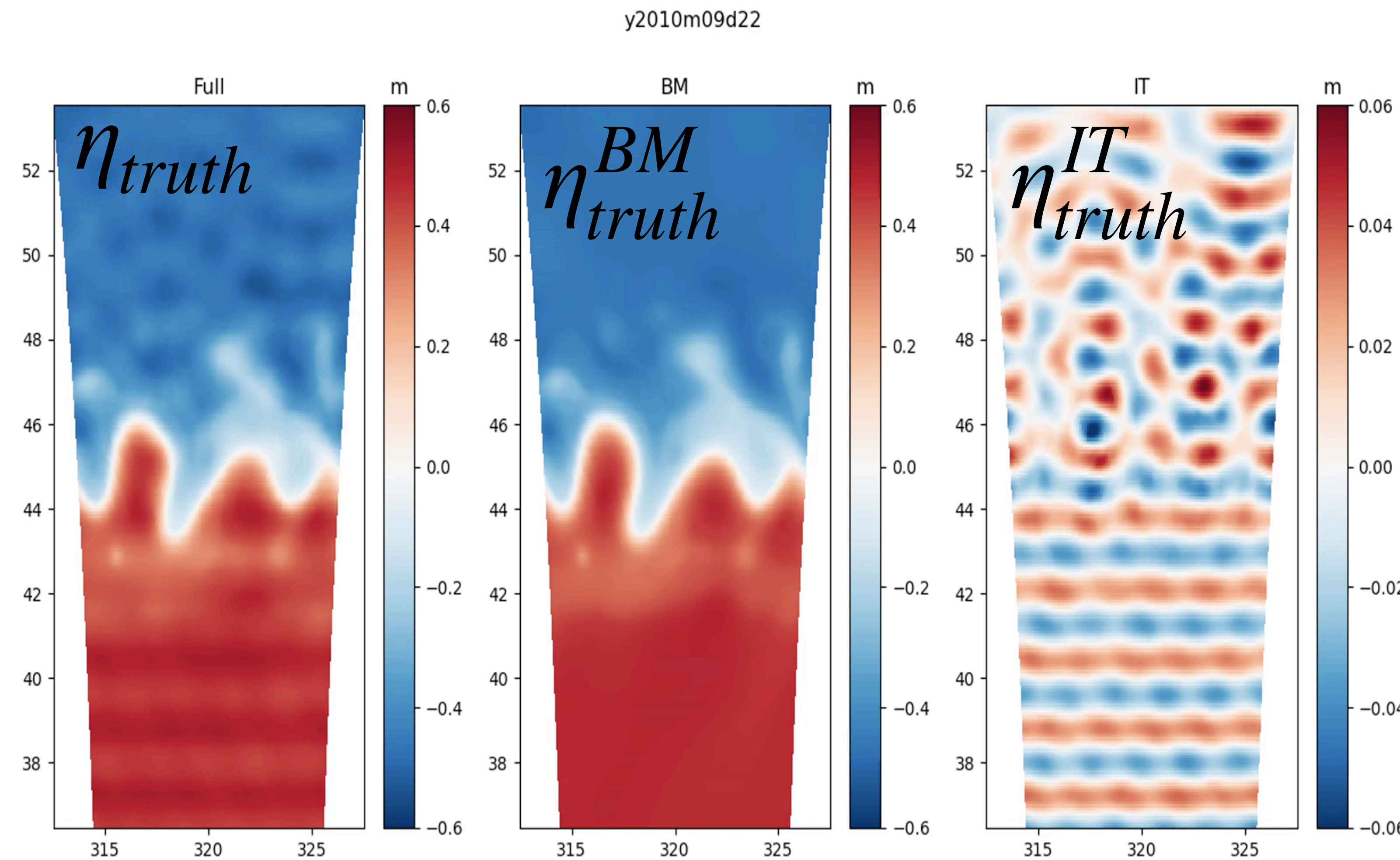
BM: Balanced motions IT: Internal tides

Reference

$$\eta_{truth} = \eta_{truth}^{BM} + \eta_{truth}^{IT}$$

$$\eta_{truth}^{BM}(t_0) = \frac{1}{2T}\int_{t_0-T}^{t_0+T} \eta_{truth}(t)dt$$

$$\eta_{truth}^{IT}(t_0) = \frac{1}{T}\int_{t_0-T}^{t_0+T} \eta_{truth}(t) \cdot cos\left(\frac{2\pi}{T}t\right) dt$$



Observation
One snapshot every 75 h (=3d+3h), free of noise

Simulation: Ponte et al, 2017

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Eddy/wave separation with a 4DVar technique

Method: coupled BM and IT estimations

BM estimation

- **Dynamics**

1.5-layer **quasi-geostrophic** model

$$\partial_t q + J(\psi, q) = 0$$

where: $\psi = \frac{g}{f}\eta$, $q = \nabla^2\psi - \frac{1}{{L_R}^2}\psi$

- **Data assimilation technique**

BFN, based on **nudging** equation:
$\partial_t q + J(\psi, q) - K(q_{obs} - q) = 0$

Le Guillou, F., Metref, S., Cosme, E., Ubelmann, C., Ballarotta, M., Le Sommer, J., & Verron, J. (2021). Mapping Altimetry in the Forthcoming SWOT Era by Back-and-Forth Nudging a One-Layer Quasigeostrophic Model, *Journal of Atmospheric and Oceanic Technology*, *38*(4), 697-710.

IT estimation

- **Dynamics**

1-layer **linear shallow water** model…

$$\partial_t u - fv = -g\partial_x\eta$$

$$\partial_t v + fu = -g\partial_y\eta$$

$$\partial_t\eta = -H_e(\partial_x u + \partial_y v)$$

…forced by **open boundary conditions**

- **Data assimilation technique:**

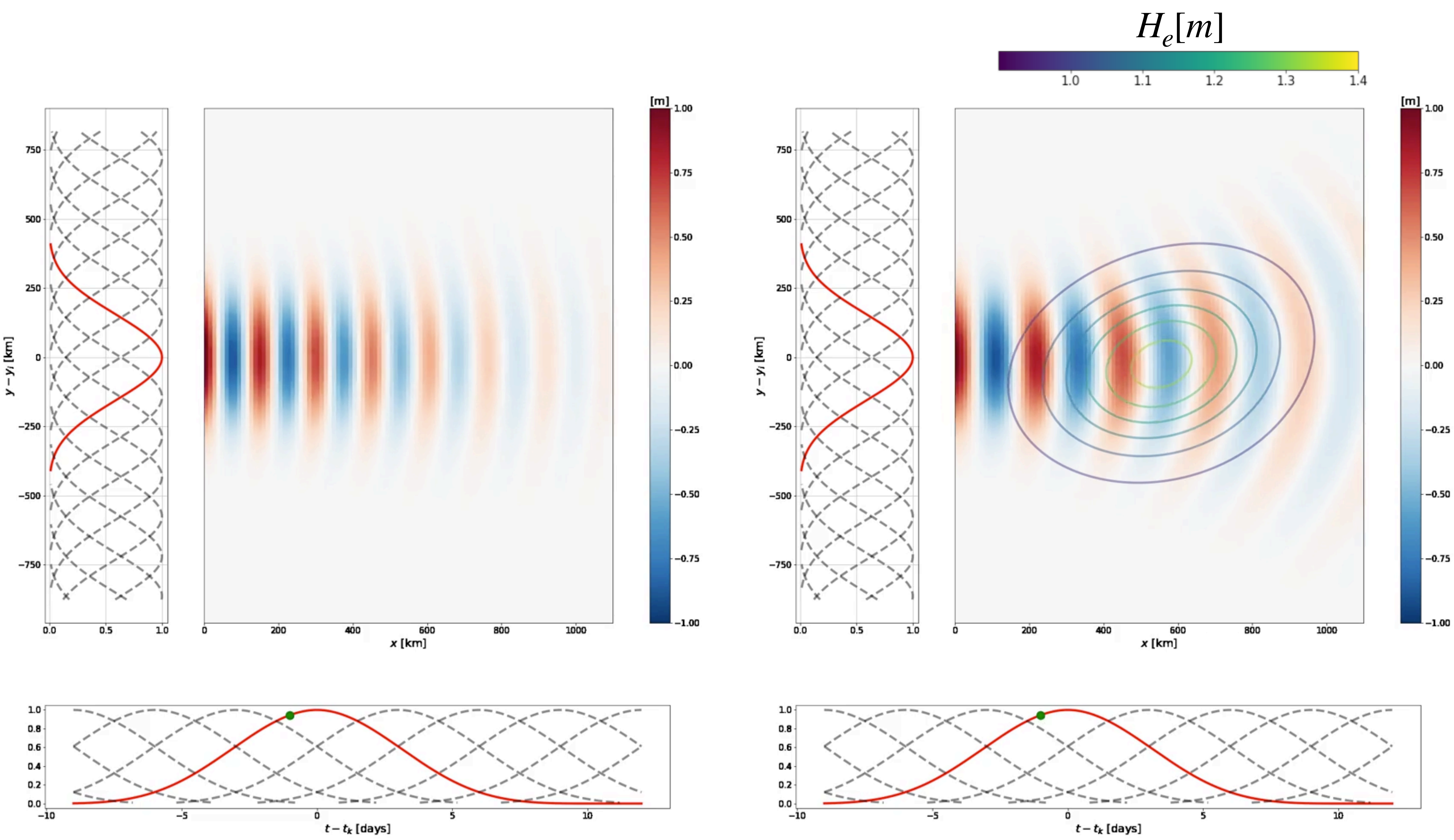
4Dvar, minimizing the cost function:

$$J(p) = ||\eta_{obs} - \eta||^2$$

where p : model parameters (H_e and boundary conditions)

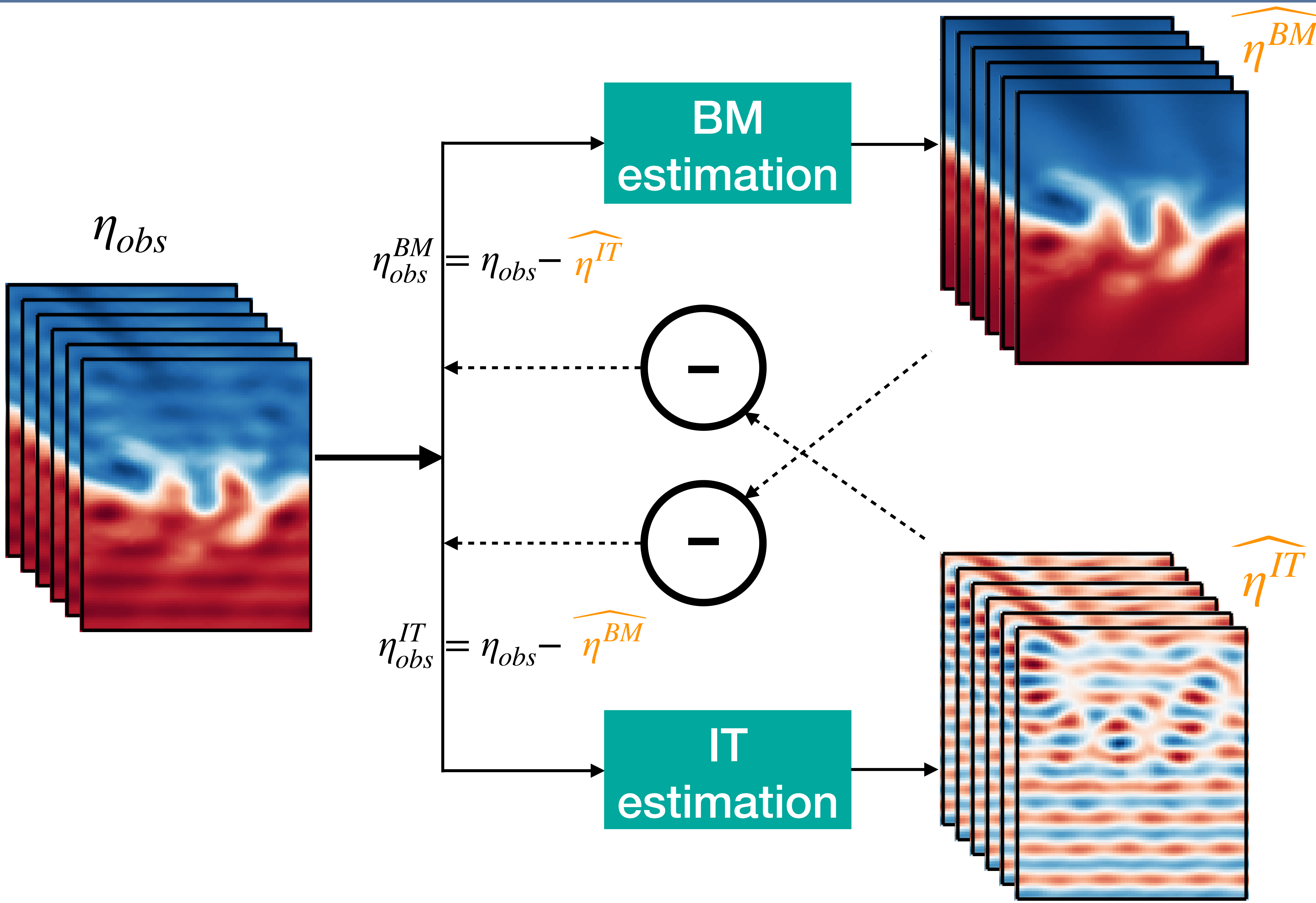
Eddy/wave separation with a 4DVar technique

4DVar control parameters: illustration



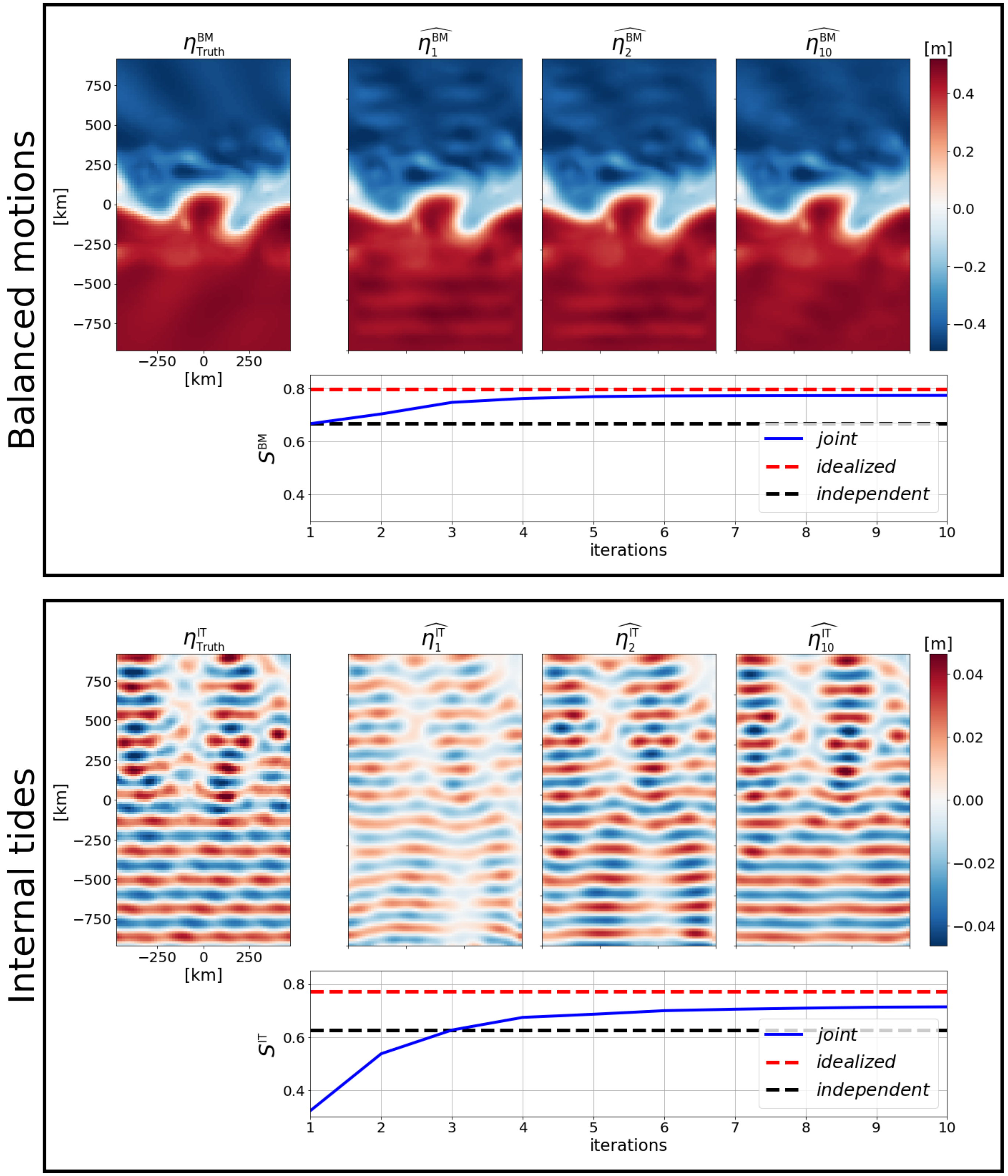
Eddy/wave separation with a 4DVar technique

Alternating minimization



Eddy/wave separation with a 4DVar technique

Results: convergence

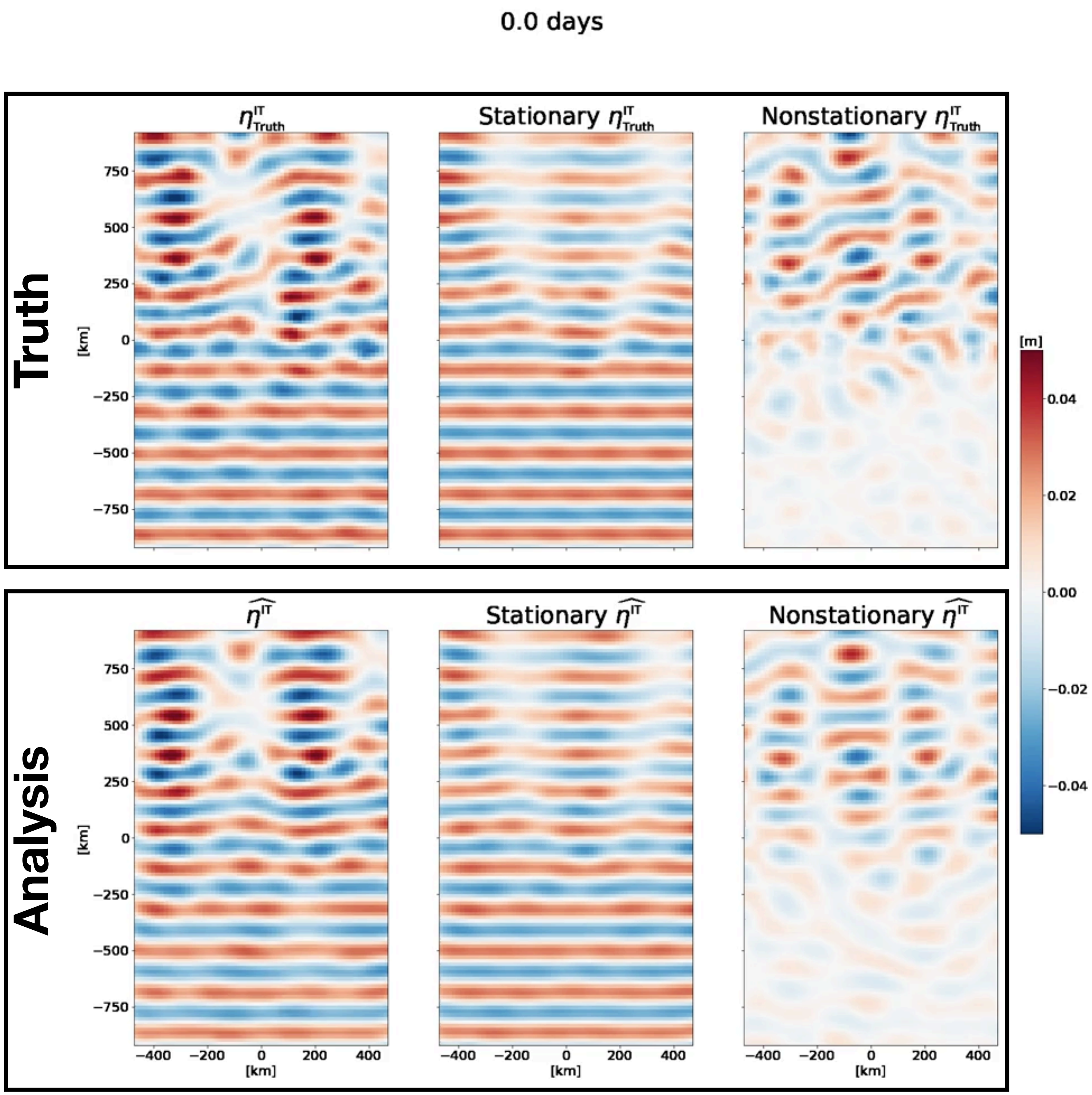


- Convergence reached after 10 iterations.
- Throughout iterations, both components are progressively separated.
- IT estimation looks very similar to the truth

Le Guillou et al, 2021

Eddy/wave separation with a 4DVar technique

Results: estimation of nonstationary IT



Le Guillou et al, 2021

Textbooks

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- Data Assimilation: Methods, Algorithms and Applications, M. Asch, M. Bocquet & M. Nodet, SIAM, 2016

- Advanced data assimilation for Geosciences, Eds. E. Blayo, M. Bocquet & E. Cosme, Oxford, 2014

- Data assimilation, Making sense of observations, Eds W. Lahoz, B. Khattatov & R. Ménard, Springer, 2010

- Ocean Weather Forecasting, Eds. E. Chassignet & J. Verron, Springer, 2006

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Eddy/wave separation with a 4DVar technique

Problem statement